Parametric Destabilisation for Collective Oscillations in Ultracold Boses Gases

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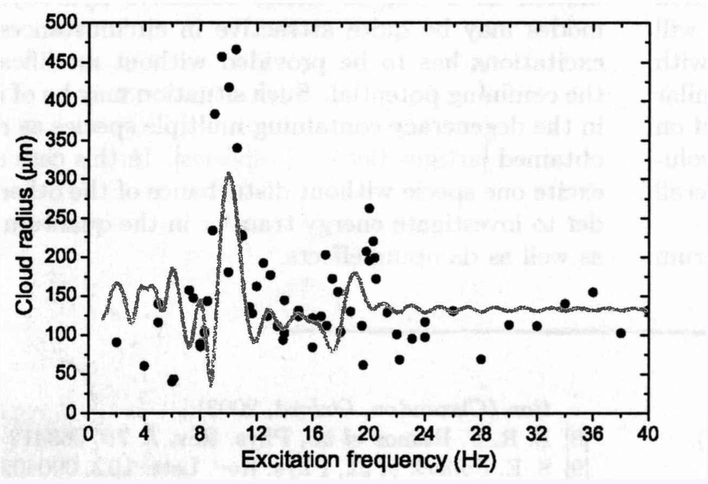
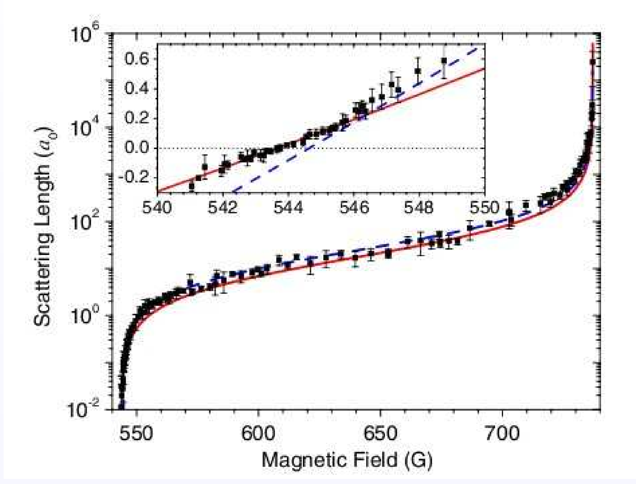
1. Experiment [1, 2]





2.Variational Principle [3]

- Gross-Pitaevskii equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{x},t) = \left\{-\frac{\hbar^2}{2M}\Delta + V(\boldsymbol{x}) + \frac{4\pi\hbar^2 a_s(t)}{M}\|\Psi(\boldsymbol{x},t)\|^2\right\}\Psi(\boldsymbol{x},t)$$

Harmonic trap potential: $V(\boldsymbol{x}) = \frac{M}{2}\sum_{i=1}^{3}\omega_i^2 x_i^2$

- Feshbach resonance: $a_s(t) = a_0 + a\,\cos\Omega t$
- Equations of motion for condensate widths:

$$\ddot{\alpha}_i(\tau) = -\frac{\partial V_{\text{eff}}(\alpha_x(\tau),\alpha_y(\tau),\alpha_z(\tau))}{\partial\alpha_i(\tau)}$$

- with effective potential:

$$V_{\text{eff}}(\alpha_x,\alpha_y,\alpha_z) = \sum_{i=1}^{3}\frac{1}{2}\left(\lambda_i^2\alpha_i^2 + \frac{1}{\alpha_i^2}\right) + \frac{P(\tau)}{\alpha_x\alpha_y\alpha_z}$$

λ: anisotropy factor, interaction strength: $P(\tau) = \sqrt{\frac{2}{\pi}}\frac{a_s(\tau)N}{\sqrt{\hbar/M\omega}}$

- Cylindric trap symmetry: $\lambda_x = \lambda_y = 1,\ \lambda_z = \lambda,\ \alpha_x = \alpha_y = \alpha$
- Equilibrium positions:

$$\alpha_0^4 = 1 + \frac{P_0}{\alpha_{z0}}, \quad \lambda^2\alpha_{z0}^4 = 1 + \frac{P_0\alpha_{z0}}{\alpha_0^2}$$

- Expansion for small deflections:

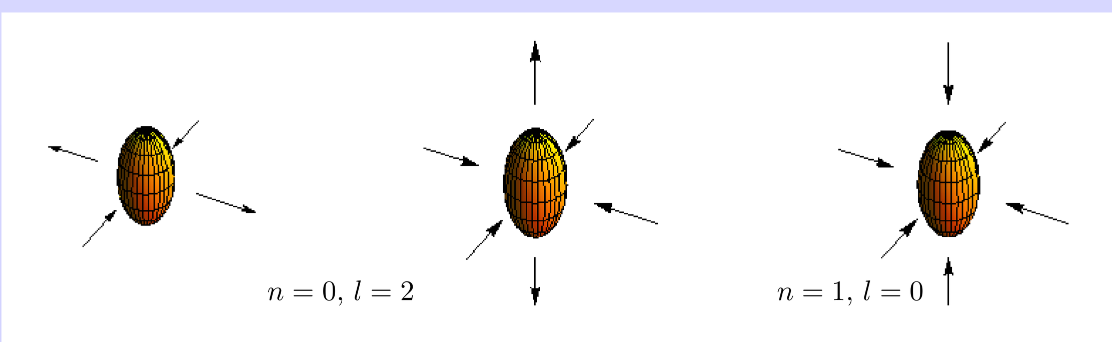
$$V_{\text{eff}}(\alpha_0+\delta\alpha_x,\alpha_0+\delta\alpha_y,\alpha_{z0}+\delta\alpha_z) = V_{\text{eff}}(\alpha_0,\alpha_{z0}) + \frac{1}{2}\delta\boldsymbol{\alpha}^T M\delta\boldsymbol{\alpha} + \ldots$$

- Frequencies of collective oscillations [3]:

$$\nu_{\text{a}} = 2\omega\sqrt{1 - \frac{P_0}{2\alpha_0^4\alpha_{z0}}},$$

$$\nu_{\text{b,c}} = 2\omega\left\{\frac{1}{2}\left(1+\lambda^2 - \frac{P_0}{4\alpha_0^2\alpha_{z0}^3}\right) \mp \frac{1}{2}\sqrt{\left(1-\lambda^2+\frac{P_0}{4\alpha_0^2\alpha_{z0}^3}\right)^2 + \frac{P_0^2}{2\alpha_0^6\alpha_{z0}^4}}\right\}^{1/2}$$

- Eigenmodes:



- For isotropic trap:

$$\nu_{\text{a,b}} = \omega\sqrt{2 + \frac{2}{\alpha_0^4}}, \quad \nu_{\text{c}} = \omega\sqrt{5 - \frac{1}{\alpha_0^4}}$$

3. Parametric Resonance

- Equations of motion for isotropic trap after rescaling of time:

$$\delta\ddot{\alpha}(t') + \frac{4\omega^2}{\Omega^2}\left[5 - \frac{1}{\alpha_0^4} + \frac{4P_1\cos 2t'}{\alpha_0^5}\right]\delta\alpha(t') = 0$$

- Mathieu equation:

$$\ddot{x}(t) + [c - 2q\,\cos 2t]\,x(t) = 0$$

- Floquet theory:

$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{(\lambda+2in)t}$$

- Trilinear recurrence relation:

$$x_n[(\lambda+2in)^2 + c] - qx_{n-1} - qx_{n+1} = 0$$

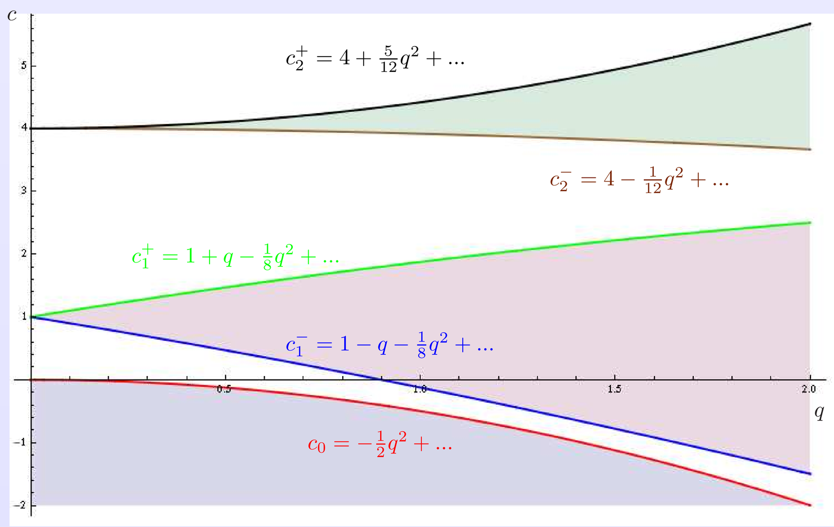
- Ladder operators [4]:

$$x_{n-1} = S_n^- x_n\,, \quad x_{n+1} = S_n^+ x_n$$

- Continued fractions:

$$\left[\lambda^2 + c - \frac{q^2}{(\lambda+2i)^2 + c - \frac{q^2}{(\lambda+4i)^2+c-\ldots}} - \frac{q^2}{(\lambda-2i)^2 + c - \frac{q^2}{(\lambda-4i)^2+c-\ldots}}\right]x_0 = 0$$

- Stability diagram:

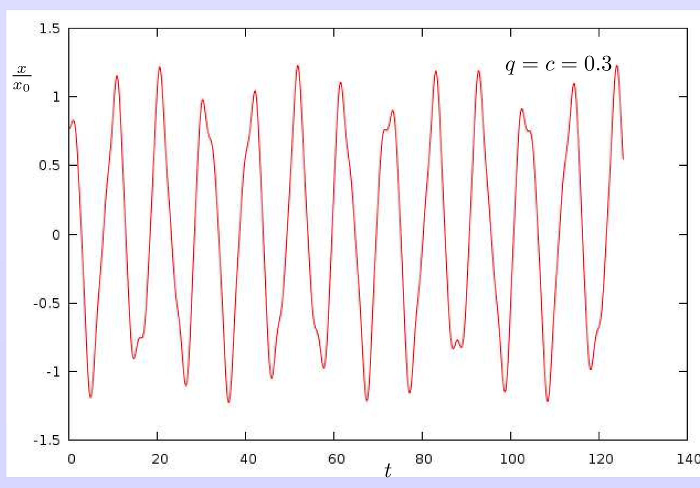


white regions: stable, colored regions: unstable

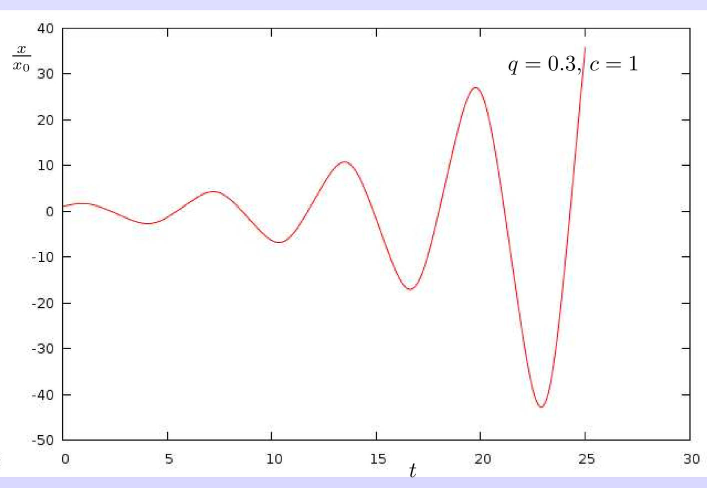
- Amplitudes:

$$x(t) = x_0 e^{\lambda t}\left[1 + e^{2it}\frac{q}{(\lambda+2i)^2 + c - \frac{q}{(\lambda+4i)^2+c-\ldots}} + e^{-2it}\frac{q}{(\lambda-2i)^2 + c - \frac{q}{(\lambda-4i)^2+c-\ldots}}\right]$$

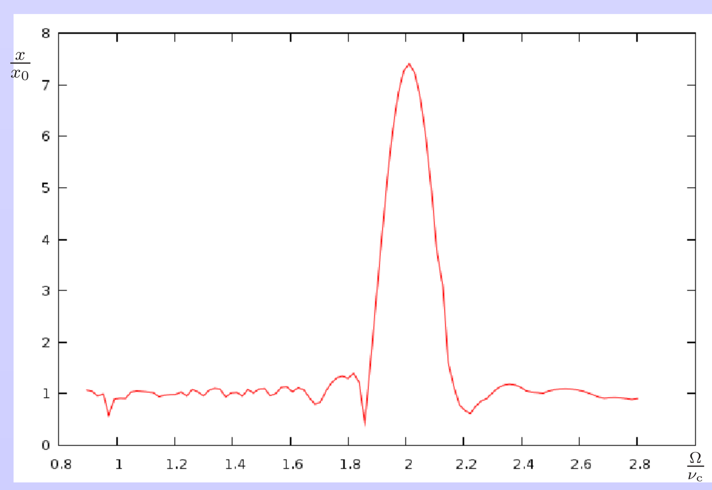
with λ determined from continued fractions



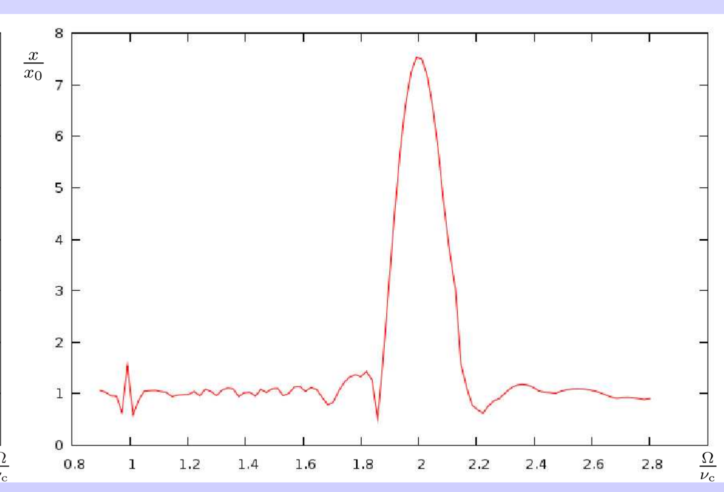
stable solution



unstable solution



resonance spectrum in first approximation



resonance spectrum in second approximation

Parametric destabilisation at resonance frequencies: $\omega_n = \frac{2\omega_0}{n}$

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