Tuning of Scattering by Periodic Modulation

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Theory of Condensed Matter and Many Body Systems

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Feshbach Resonances in Ultracold Quantum Gases

Feshbach resonances provide a powerful tool to control the scattering length and therefore the interaction strength in ultracold atom experiments. In the case of magnetic Feshbach resonances the scattering length a can be modified by changing a magnetic field [1]:

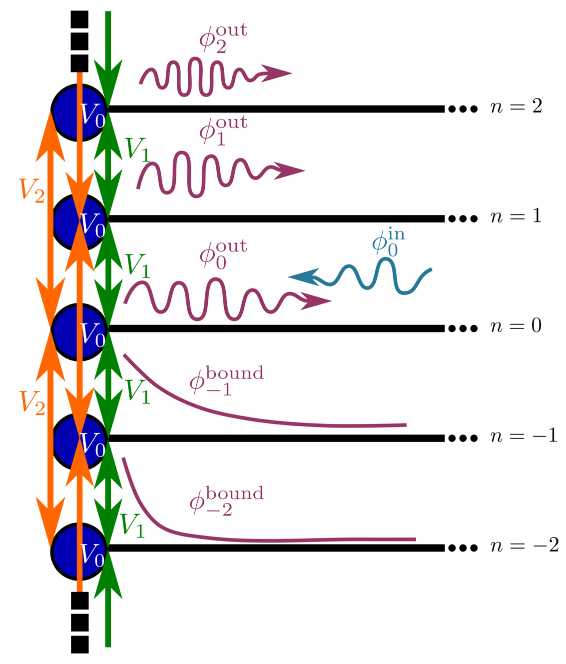
$$a = a_{\rm bg}\left(1 - \frac{\Delta}{B - B_0}\right)$$

Here we show that a time periodic magnetic field $B(t) = B_1 + B_2\cos(\omega t)$ [2, 3] can be used to induce a "Feshbach-like" resonance at any given magnetic field B_1 by tuning the driving frequency ω.

Floquet-Partial Wave Expansion

Floquet theory is used to calculate steady-states of a time-periodic Hamiltonian $\hat{H}(t) = \hat{H}(t+T)$

- Wave function: Floquet state $|\psi(t)\rangle = e^{-i\frac{\epsilon}{\hbar}t}|\phi(t)\rangle$
- Floquet equation $\left(\hat{H} - i\hbar\frac{\partial}{\partial t}\right)|\phi\rangle = \epsilon|\phi\rangle$
- Floquet mode $|\phi(t)\rangle = |\phi(t+T)\rangle$
- Floquet energy ϵ



Scattering by time-periodic potential described by

$$H(\mathbf{r},t) = -\frac{\hbar^2}{2\mu}\Delta + V(r,t).$$

Fourier transform $\phi_l(\mathbf{r},t) = \sum_{n=-\infty}^{\infty} e^{-in\omega t}R_{l,n}(r)P_l(\cos(\theta))$ and $V(r,t) = \sum_{n=-\infty}^{\infty} V_n(r)e^{-in\omega t}$ in order to derive radial Floquet-equation

$$\left(\Delta_r + k_n^2 - \frac{l(l+1)}{r^2} - v_0(r)\right)R_{l,n}(r) = \sum_{m\neq 0} v_m(r)R_{l,n-m}(r),$$

where $\Delta_r = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}$, $\frac{\hbar^2}{2\mu}k_n^2 = \epsilon + n\hbar\omega$, $v_j = \frac{2\mu}{\hbar^2}V_j$ and l angular momentum quantum number

Asymptotic wave function of a scattering state is given by

$$\phi_n(\mathbf{x}) = \delta_{n,0}e^{i\mathbf{kr}} + f_n\frac{e^{ik_n r}}{r},$$

with the scattering amplitude in n-th Floquet channel $f_n = f_n(\epsilon,\omega,\theta)$. Only modes above a critical index $n_c = \lceil -\frac{\epsilon}{\hbar\omega}\rceil$ contribute to scattering. Important scattering quantities are [2, 4]:

- Time averaged scattering length $a_{\rm scatt} = -\lim_{\epsilon\to 0} f_0$
- Elastic cross section $\langle\langle\sigma\rangle\rangle_{\rm el} = \int_\Omega d\Omega |f_0|^2$
- Total cross section (via Floquet-optical theorem [4]) $\langle\langle\sigma\rangle\rangle = \frac{4\pi}{k_0}{\rm Im}\ f_0(\theta=0)$

Model: Contact Potential

s-wave scattering, low energies: Absorb interaction in contact potential with time-dependent scattering length $a(t) = \bar{a} + a_1\cos(\omega t)$, valid if $k_n r_{\rm pot} \ll 1$.

$$v_0(r) = 2\frac{\bar{a}}{r^2}\delta(r)\frac{\partial}{\partial r}r,\quad v_{\pm 1}(r) = \frac{a_1}{r^2}\delta(r)\frac{\partial}{\partial r}r$$

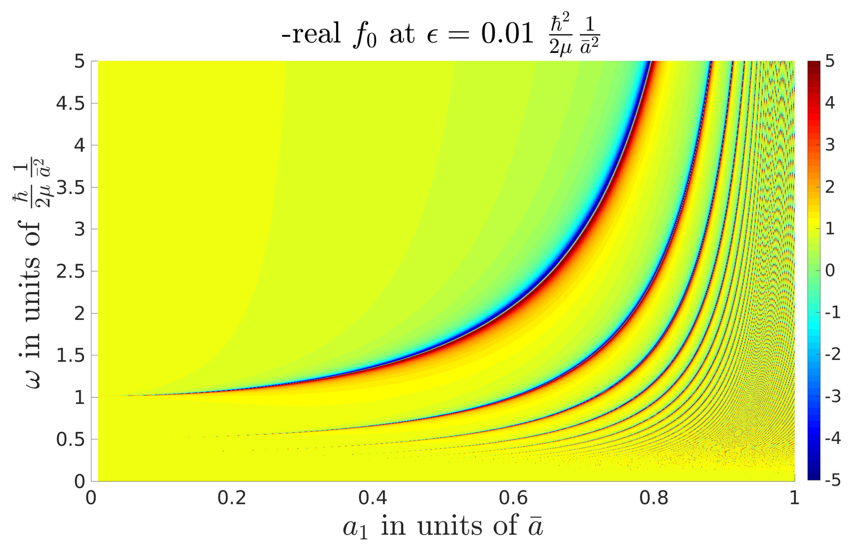
Insert the solution $R_{0,n}(r) = \frac{\delta_{n,0}}{2}\frac{ie^{-ik_n r}}{k_n r} - D_n\frac{ie^{ik_n r}}{k_n r}$ into radial Floquet equation, integrate around origin and get recursion relation

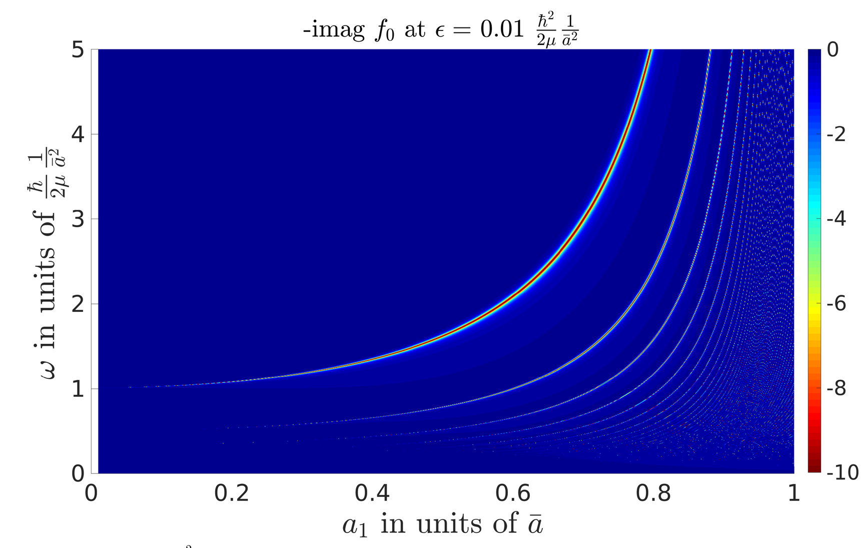
$$\left(\frac{i}{k_n\bar{a}} - 1\right)D_n - \frac{a_1}{2\bar{a}}\left(D_{n+1} + D_{n-1}\right) = \lambda_n$$

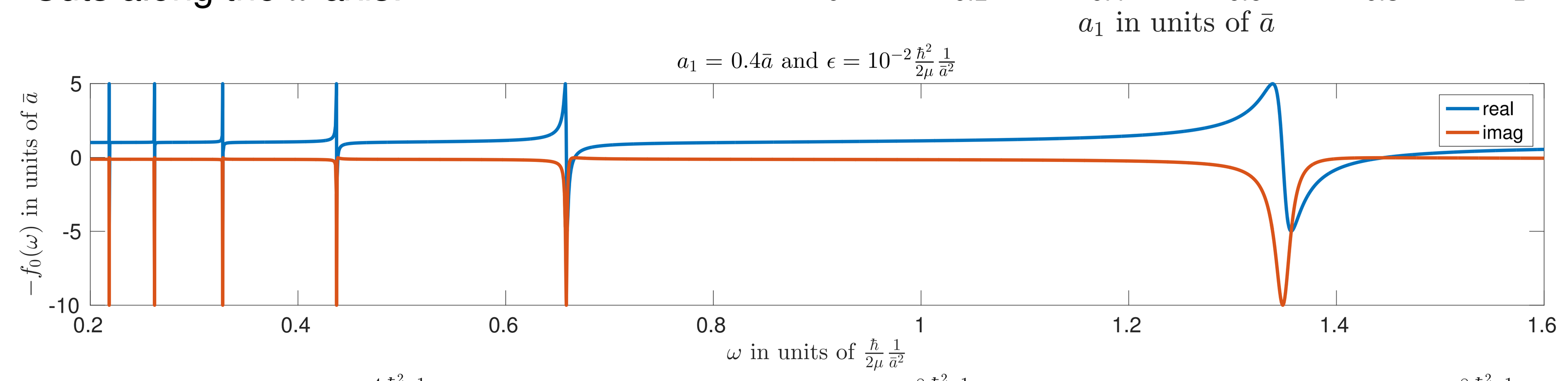
- $\lambda_n = \left(\frac{i}{k_n\bar{a}} + 1\right)\frac{\delta_{n,0}}{2} + \frac{a_1}{2\bar{a}}\left(\frac{\delta_{n+1,0}}{2} + \frac{\delta_{n-1,0}}{2}\right)$
- Length scale $\bar{a}$
- Energy scale $E_D = \frac{\hbar^2}{2\mu}\frac{1}{\bar{a}^2}$ (dimer energy)

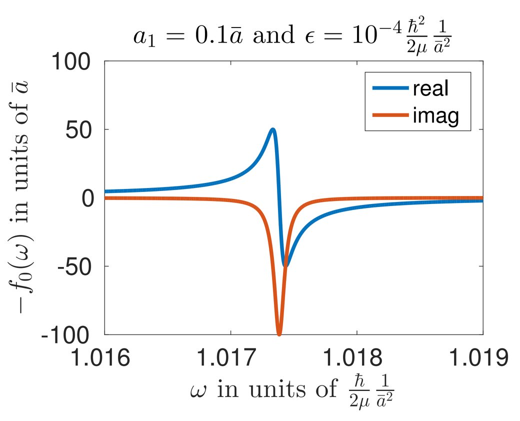
Scattering amplitudes $f_n = \frac{-i}{k_n}\left(D_n - \frac{\delta_{n,0}}{2}\right)$

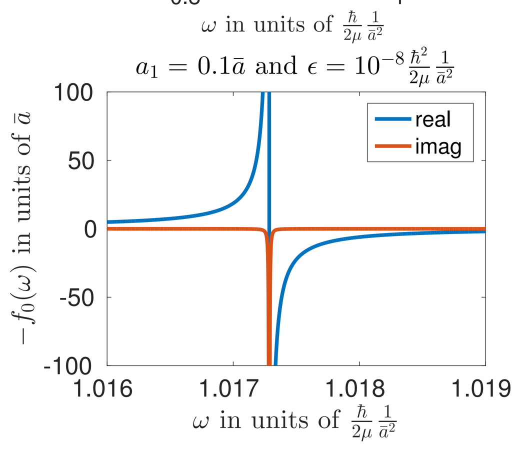
- Result: Scattering resonances (large f_0) in the ω-a_1-plane [5] if $\bar{a} > 0$
- Cuts along the ω axis:

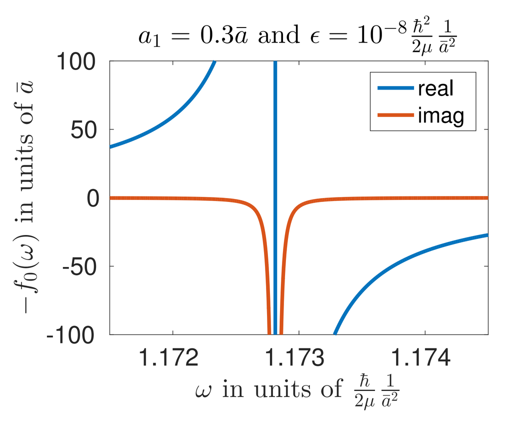












Tunable Enhancement of Scattering Length

- Scattering resonances occur along lines, which are enumerated by $n = 1, \ldots, \lfloor 1 + 1/(\epsilon/E_D)\rfloor$
- Scattering amplitude in vicinity of resonance is approximated by [2]:

$$-\frac{1}{f_0(\omega)} = \frac{1}{a_{\rm BG}}\frac{\omega - \omega_n}{\omega - \omega_n - \delta_n} + i\gamma_n$$

  - resonance frequency ω_n
  - resonance width δ_n
  - background scattering length $a_{\rm BG}$
  - amount of maximal scattering $\max|f_0| \propto \frac{1}{\gamma_n}$, with $\gamma_n \approx \sqrt{\epsilon}$

- Limit of vanishing ϵ: $a_{\rm scatt}(\omega) = a_{\rm BG}\left(1 - \frac{\delta_n}{\omega - \omega_n}\right) - i\pi|\delta_n|a_{\rm BG}\delta_{\rm Dirac}(\omega - \omega_n)$
- f_0 fulfils the Kramers-Kronig relations for anti-causal susceptibilities

Possibilities of controlling:

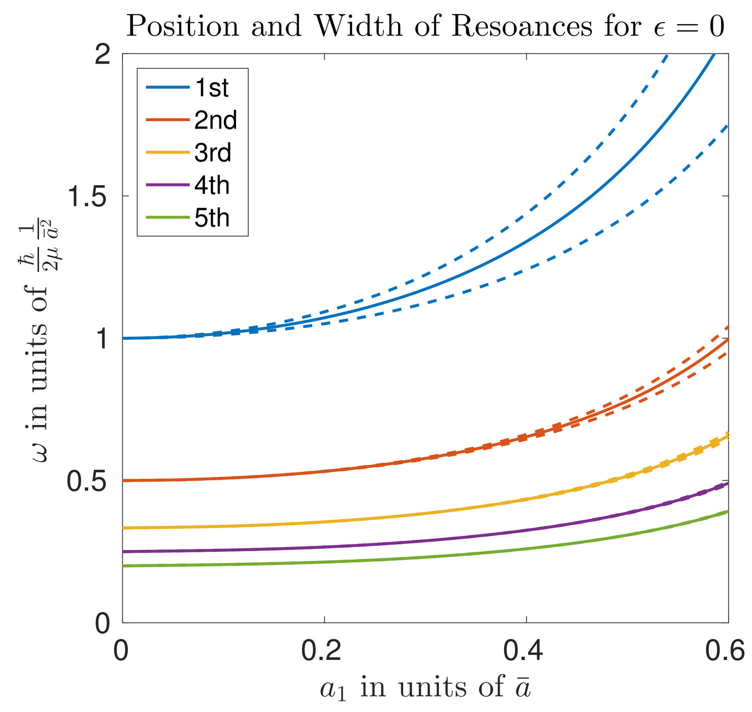
- Frequency scale of ω_n by $\bar{a}$
- Width δ_n by choosing a_1
- Enhancement of scattering length by choosing ω relative to ω_n

For small a_1 resonances occur, if

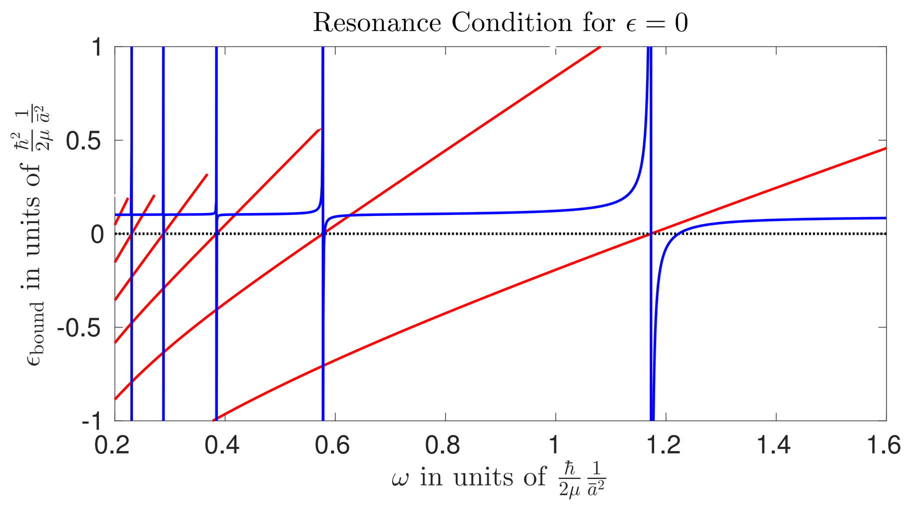
$$\epsilon \approx E_D + n\hbar\omega_n + C_n a_1^2$$

General resonance condition is

$$\epsilon = \epsilon_{\rm bound}(\omega, a_1/\bar{a})$$



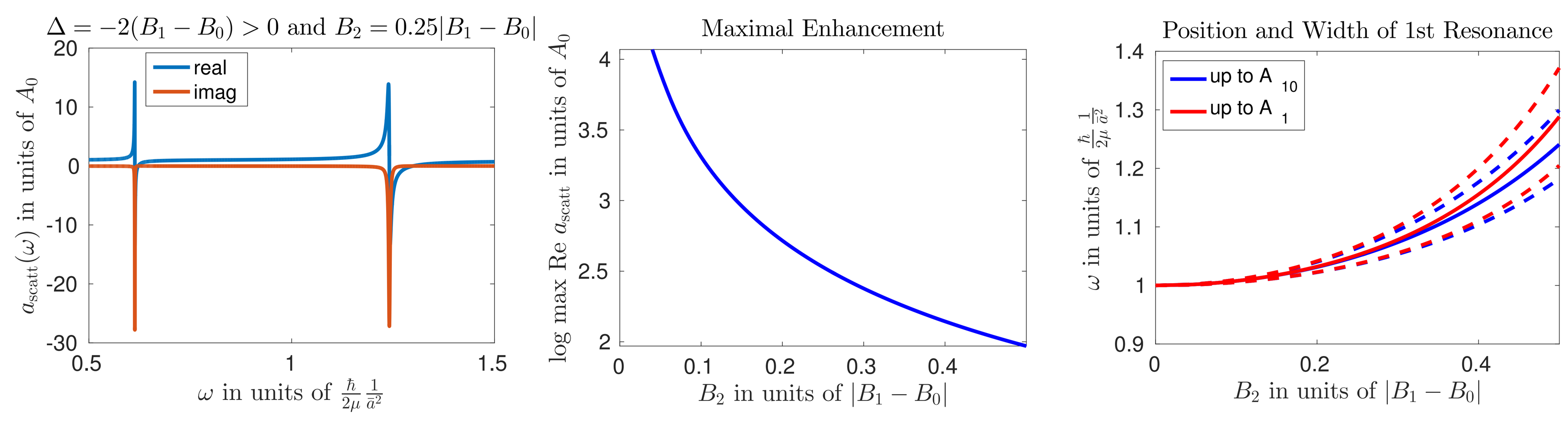
- $\epsilon_{\rm bound}$ is the energy of a dynamically created bound state in the continuum [5] in Floquet-side system
- Scattering resonances induced by Fano-Feshbach resonance in Floquet-coupled-channel picture
- No scattering resonances for negative $\bar{a}$, as no bound states in continuum



Influence of Higher Harmonics

Consider higher Fourier modes of $a(t) = a_{\rm bg}\left[1 - \frac{\Delta}{B_1 - B_0 + B_2\cos(\omega t)}\right] = \sum_{n=-\infty}^{\infty} e^{-in\omega t}A_n,$

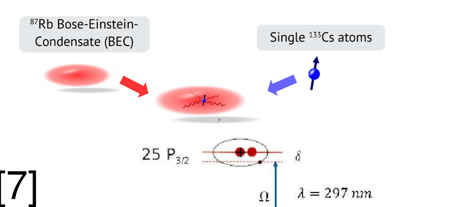
$$A_n = a_{\rm bg}\left\{\delta_{n,0} - \frac{\Delta}{(B_1 - B_0)\sqrt{1^2 - y^2}}\left[\frac{\sqrt{1 - y^2} - 1}{y}\right]^{|n|}\right\},\ y = \frac{B_2}{B_1 - B_0},\ \text{if } B_2 < |B_1 - B_0|,\ \epsilon = 0$$



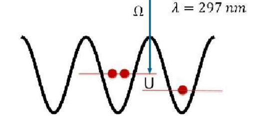
- Resonance approximated by $\frac{1}{a_{\rm scatt}(\omega)} = \frac{1}{a_{\rm BG}}\frac{\omega - \omega_n}{\omega - \omega_n - \delta_n} + i\gamma_n$ for small a_1
- Finite maximal amount of scattering (finite γ_n), it decreases with B_2
- Resonant frequency ω_n shifted downwards compared to harmonic drive
- Width δ_n increases with B_2, but smaller compared to harmonic drive

Connection to Experimental Groups

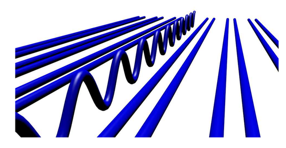
- AG Widera: Magnetic Rb-Cs Feshbach resonance [6]
  - include spinor structure of scattering atoms



- AG Ott: Optical Feshbach resonance with Rydberg molecules [7]
  - investigate scattering in driven-dissipative environment



- AG von Freymann and AG Linden: Probing stability of topological protection in a 1D SSH-model realized with optical wave guides [8]



References

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[4] A. G. Sykes, H. Landa, and D. S. Petrov. *Two- and three-body problem with Floquet-driven zero-range interactions.* Phys. Rev. A **95** 062705 (2017).

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[6] F. Schmidt, D. Mayer, Q. Bouton, D. Adam, T. Lausch, N. Spethmann, and A. Widera. *Quantum spin dynamics of individual neutral impurities coupled to a Bose-Einstein condensate.* arXiv:1802.08702 (2018).

[7] O. Thomas, C. Lippe, T. Eichert, and H. Ott. *Experimental realization of a Rydberg optical Feshbach resonance in a quantum many-body system.* Nat. Commun. **9** (2018).

[8] Z. Cherpakova, C. Jörg, C. Dauer, F. Letscher, M. Fleischhauer, S. Eggert, S. Linden, and G. von Freymann. *Breakdown of topological protection under local periodic driving.* arXiv:1807.02321 (2018).