Thermodynamics of Ideal and Dipolar Interacting Bose Gases

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1. CANONICAL APPROACH TO BEC

- N-particle partition function [1,2]:

$$Z_N^B(\beta) = \frac{1}{N!}\sum_P \prod_{n=1}^N \left(\int d^3x_n \int_{\boldsymbol{x}_n(0)=\boldsymbol{x}_n}^{\boldsymbol{x}_n(\hbar\beta)=\boldsymbol{x}_{P(n)}} \mathcal{D}^3x_n \right) e^{-\mathcal{A}[\boldsymbol{x}_1,\ldots,\boldsymbol{x}_N]/\hbar}$$

- Action:

$$\mathcal{A}[\boldsymbol{x}_1,\ldots,\boldsymbol{x}_N] = \sum_{n=1}^N \int_0^{\hbar\beta} d\tau \left[\frac{M}{2}\dot{\boldsymbol{x}}_n^2 + U(\boldsymbol{x}_n) + {\sum_{m=1}^N}' V^{(\mathrm{int})}(\boldsymbol{x}_n - \boldsymbol{x}_m) \right]$$

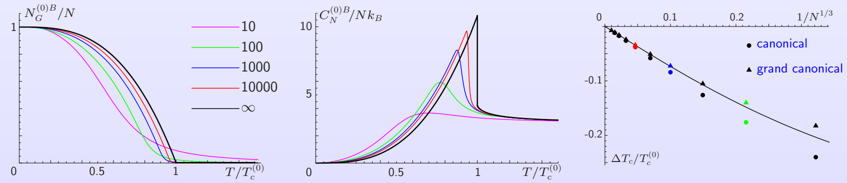
- Recursion for interaction-free partition function [3–6]:

$$Z_N^{(0)B}(\beta) = \frac{1}{N}\sum_{n=1}^N Z_1(n\beta)\, Z_{N-n}^{(0)B}(\beta) \quad \text{with} \quad Z_1(\beta) = \sum_{\boldsymbol{k}} e^{-\beta E_{\boldsymbol{k}}}$$

- Ground-state occupancy and heat capacity without interaction [4,6]:

$$N_G^{(0)B} = \sum_{n=1}^N e^{-n\beta E_{\boldsymbol{0}}}\, Z_{N-n}^{(0)B}(\beta) \,/\, Z_N^{(0)B}(\beta) \quad \text{and} \quad C_N^{(0)B} = \frac{1}{k_B}\frac{\partial^2}{\partial\beta^2}\ln Z_N^{(0)B}(\beta)$$

- Interaction-free results for harmonic case [7]:



2. GRAND CANONICAL DESCRIPTION

- Partition function [1]:

$$Z_{GC}^B(\beta,\mu) = \int_{\psi(\boldsymbol{x},\hbar\beta)=\psi(\boldsymbol{x},0)}^{\psi^*(\boldsymbol{x},\hbar\beta)=\psi^*(\boldsymbol{x},0)} \mathcal{D}\psi^*(\boldsymbol{x},\tau)\,\mathcal{D}\psi(\boldsymbol{x},\tau)\, e^{-\mathcal{A}_{GC}[\psi^*,\psi]/\hbar}$$

- Grand canonical action:

$$\mathcal{A}_{GC}[\psi^*,\psi] = \int_0^{\hbar\beta} d\tau \int d^3x \left\{ \psi^*(\boldsymbol{x},\tau)\left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2\boldsymbol{\nabla}^2}{2M} + U(\boldsymbol{x}) - \mu\right]\psi(\boldsymbol{x},\tau) + \int d^3x' \left|\psi(\boldsymbol{x}',\tau)\right|^2 V^{(\mathrm{int})}(\boldsymbol{x}-\boldsymbol{x}') \left|\psi(\boldsymbol{x},\tau)\right|^2 \right\}$$

- Ideal gas result:

$$Z_{GC}^{(0)B}(\beta,\mu) = \prod_{\boldsymbol{k}} \frac{1}{1-e^{-\beta(E_{\boldsymbol{k}}-\mu)}} \quad \Longrightarrow \quad N^{(0)B} = \sum_{\boldsymbol{k}} \frac{1}{e^{\beta(E_{\boldsymbol{k}}-\mu)}-1}$$

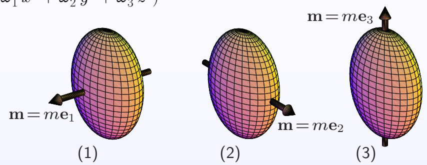
- Critical temperature for harmonic case [7]:

$$\frac{T_c^{(\mathrm{FS})}}{T_c^{(0)}} = 1 - \frac{\zeta(2)}{2N^{1/3}\,\zeta^{2/3}(3)} + \frac{\ln[\zeta(3)/N] + 9\zeta^2(2)/4\zeta(3) + 19/8 - 3\gamma}{9N^{2/3}\,\zeta^{1/3}(3)} + \ldots \quad \text{with} \quad T_c^{(0)} = \frac{\hbar\omega}{k_B}\left[\frac{N}{\zeta(3)}\right]^{1/3}$$

3. DIPOLAR INTERACTING BEC

- Stuttgart experiment [8]: $U(\boldsymbol{x}) = \frac{M}{2}\left(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2\right)$

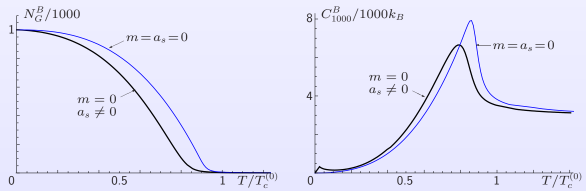
$\omega_1 = 2\pi\cdot 581\,\mathrm{Hz}$
$\omega_2 = 2\pi\cdot 406\,\mathrm{Hz}$
$\omega_3 = 2\pi\cdot 138\,\mathrm{Hz}$



- Interaction potential: $m = 6\,m_B,\ a = 105\,a_B$ [9]

$$V^{(\mathrm{int})}(\boldsymbol{x}) = \frac{4\pi\hbar^2 a}{M}\,\delta(\boldsymbol{x}) + \frac{\mu_0}{4\pi}\left\{\frac{\mathbf{m}^2}{|\boldsymbol{x}|^3} - \frac{3\,[\mathbf{m}\,\boldsymbol{x}]^2}{|\boldsymbol{x}|^5}\right\}$$

- Canonical results for 1000 particles [10]:



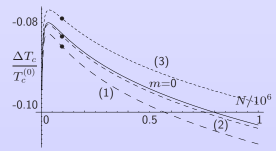
- Critical temperature: FS [11–14] + δ [15,16] + dd [17,18]

$$\frac{\Delta T_c^{(j)}}{T_c^{(0)}} = -0.73\,\frac{\omega_1+\omega_2+\omega_3}{3\,(\omega_1\omega_2\omega_3)^{1/3}}\,\frac{1}{N^{1/3}} - 3.43\,\frac{a}{\lambda_c^{(0)}} - 1.715\,f^{(j)}\!\left(\frac{\omega_1}{\omega_2},\frac{\omega_1}{\omega_3}\right)\frac{\mu_0 m^2 M}{12\pi\hbar^2\lambda_c^{(0)}}$$

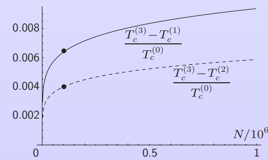
- Anisotropy factor [19,20]:

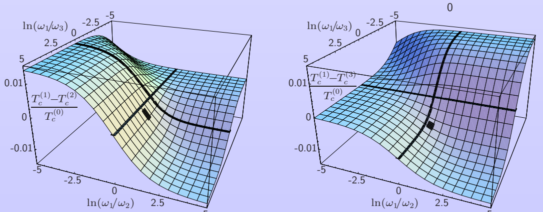
$$f^{(1)}(\eta,\kappa) = 1 + \frac{3\,\kappa\,\eta}{\sqrt{1-\kappa^2}\,(1-\eta^2)}\left\{E\left(\arcsin\sqrt{1-\kappa^2},\sqrt{\frac{1-\eta^2}{1-\kappa^2}}\right) - F\left(\arcsin\sqrt{1-\kappa^2},\sqrt{\frac{1-\eta^2}{1-\kappa^2}}\right)\right\}$$

$$f^{(2)}(\eta,\kappa) = f^{(1)}\left(\frac{\kappa}{\eta},\frac{1}{\eta}\right), \quad f^{(3)}(\eta,\kappa) = f^{(1)}\left(\frac{1}{\kappa},\frac{\eta}{\kappa}\right)$$



[18]





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