TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

SFB/Transregio 49
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Condensed matter systems with variable many-body interactions

SFB/TR 185

DIPOLAR BOSE-EINSTEIN CONDENSATES WITH WEAK DISORDER

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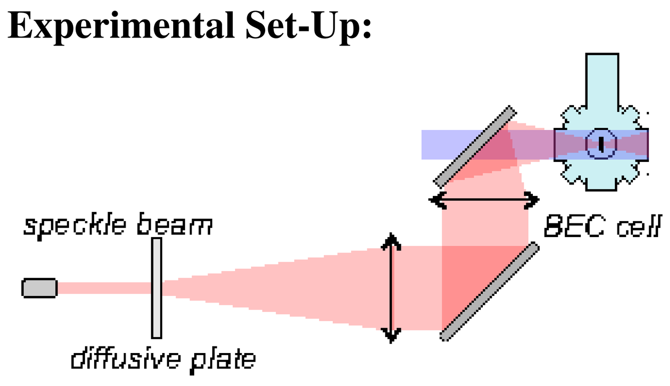
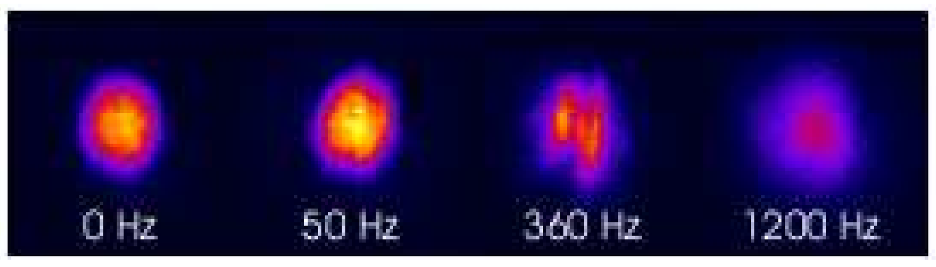
On the Dirty Boson Problem

- **Laser Speckles: Controlled Randomness** [1]

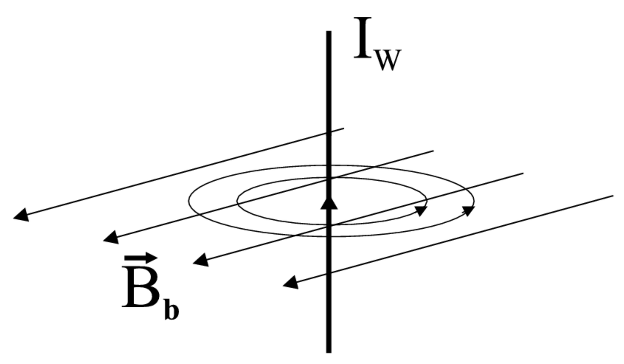
Experimental Set-Up:

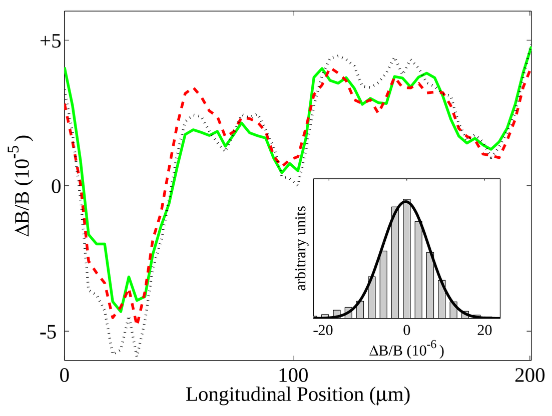


Fragmentation:



- **Wire Trap: Undesired Randomness** [2,3]



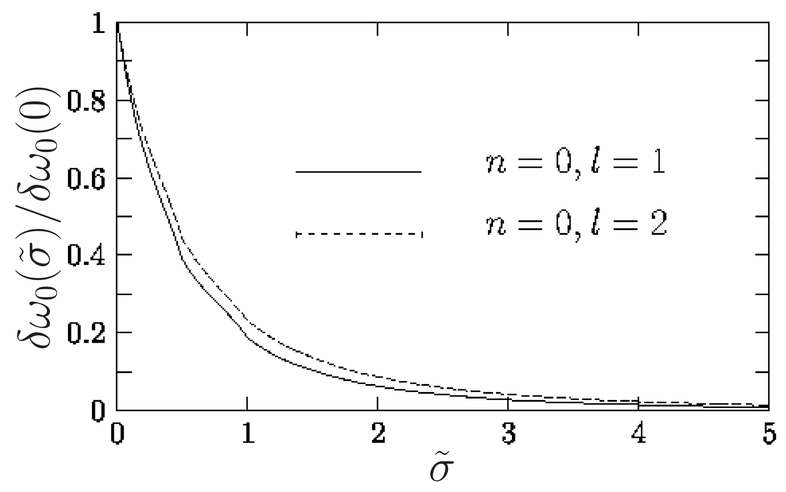


distance: $d = 10\,\mu$m , **wire width:** $100\,\mu$m , **magnetic field:** 10 G, 20 G, 30 G

- **Bogoliubov Theory of Dirty Bosons** [4,5]
  - **Assumptions:** homogeneous Bose gas: $U(\mathbf{x}) = 0$, δ-correlated disorder: $R(\mathbf{x}) = R\,\delta(\mathbf{x})$
  - **Condensate Depletion:** $n_0 = n - \frac{8}{3\sqrt{\pi}}\sqrt{a\,n_0}^3 - \frac{M^2 R}{8\pi^{3/2}\hbar^4}\sqrt{\frac{n_0}{a}}$
  - **Superfluid Depletion:** $n_s = n - n_n = n - \frac{4}{3}\frac{M^2 R}{8\pi^{3/2}\hbar^4}\sqrt{\frac{n_0}{a}}$

- **Collective Excitations**
  - **Disorder effect vanishes in previous laser speckle experiment:** [1,6]

$$\left.\begin{array}{l}\sigma\ = 10\,\mu\text{m}\\ R_{\text{TF}} = 100\,\mu\text{m}\\ l_{\text{HO}} = 10\,\mu\text{m}\end{array}\right\}\quad \tilde\sigma = \frac{\xi R_{\text{TF}}}{l_{\text{HO}}^2\sqrt{2}} \approx 7$$



  - **Disorder effect should be measurable for smaller correlation length** [6,7]

- **Hartree-Fock Mean-Field Theory: Replica Symmetry** [8]

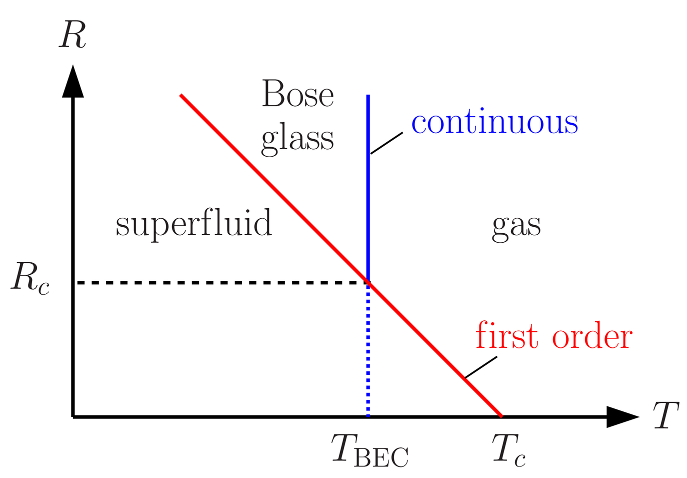
Phase classification: $n = n_0 + q + n_{\text{th}}$

$$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{\langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\rangle} = n_0$$

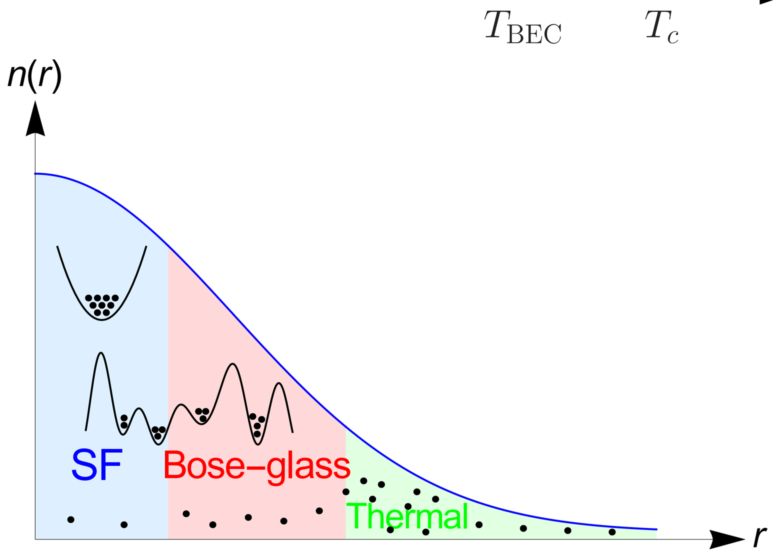
$$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{|\langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\rangle|^2} = (n_0+q)^2$$

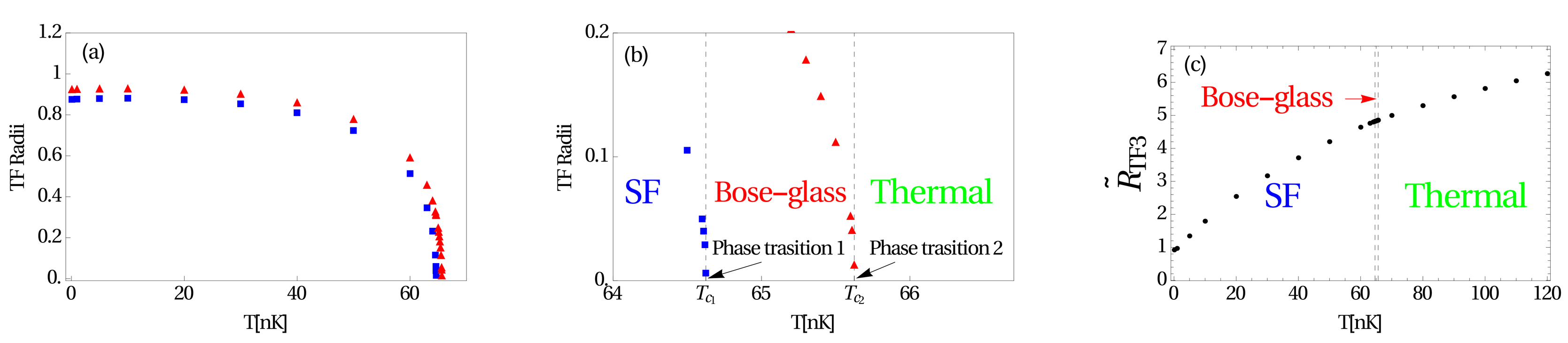
thermal gas	Bose-glass	superfluid
$q = n_0 = 0$	$q > 0, n_0 = 0$	$q > 0, n_0 > 0$

Phase diagram:



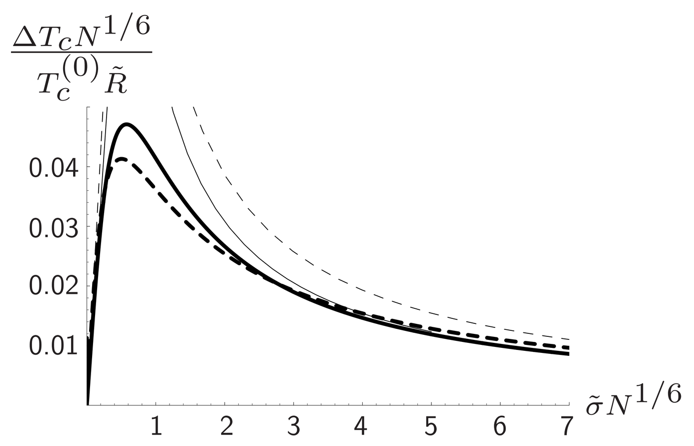
- **Harmonically Trapped Dirty Bose-Einstein-Condensate** [9-11]:
  - Thomas-Fermi approximation:
    $n(r) = n_0(r) + q(r) + n_{\text{th}}(r)$
    $\Longrightarrow$ Self-consistency equations
  - 1D, $T = 0$ [10]: redistribution of densities
  - 3D, $T = 0$ [11]: QFT from SF to BG
  - 3D, thermal phase transitions [11]:





(a) Condensate radius $\tilde R_{\text{TF1}}$ (square, blue) and Bose-glass radius $\tilde R_{\text{TF2}}$ (triangle, red); (b) blow-up of Bose-glass region; (c) cloud radius $\tilde R_{\text{TF3}}$ (dotted, black) as functions of temperature T.

- **Shift of Condensation Temperature** [12]
  - **Length scale:** $l_{\text{HO}} = \sqrt{\frac{\hbar}{M\omega_g}}$, $\omega_g = (\omega_1\omega_2\omega_3)^{1/3}$
  - **Dimensionless units:** $\tilde\sigma = \frac{\sigma}{l_{\text{HO}}}$, $\tilde R = R\sqrt{\frac{M^3}{\hbar^7\omega_g}}$
  - **Shape of correlation function does not matter:** solid (Gaussian), dashed (Lorentzian)



Superfluid Density as Tensor

- **Linear Response Theory [13]: Landau-Khalatnikov 2-fluid model**

$$p_i = VM\left(n_{nij}v_{nj} + n_{sij}v_{sj}\right) + \ldots\,,\qquad n\delta_{ij} = n_{sij} + n_{sij}$$

  - **Spin-orbit coupling: elliptic vortices [14]**
  - **Tunable anisotropic superfluidity in Kagome superlattice [15]**
  - **Dipolar interaction at finite temperature [16]**
  - **Dipolar interaction and isotropic disorder at zero temperature [17,18]**

- **Josephson Sum Rule**
  - **Superfluid density: tensor** $\Longleftrightarrow$ **condensate density: scalar**
  - **Linear response theory, isotropic case [13,19]:** $A(\mathbf{k},\omega)$: Fourier transformed Green's function

$$n_s = \frac{m^2 n_0}{\lim\limits_{\mathbf{k}\to 0}\hbar\mathbf{k}^2\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}A(\mathbf{k},\omega)}$$

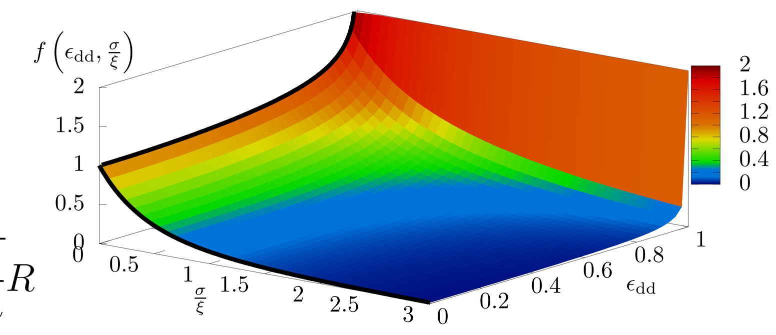
  - **Consequence for critical exponents [20]:** $\beta_s = \beta_0 - \eta\,\nu$
  - **Questions:** experimental verfication?, anisotropic case?

Dipolar BEC and Gaussian Correlated Disorder [17]

- **Weak Disorder:** perturbative solution of Gross-Pitaevskii equation $\Longleftrightarrow$ Huang-Meng theory
- **Condensate Depletion:**

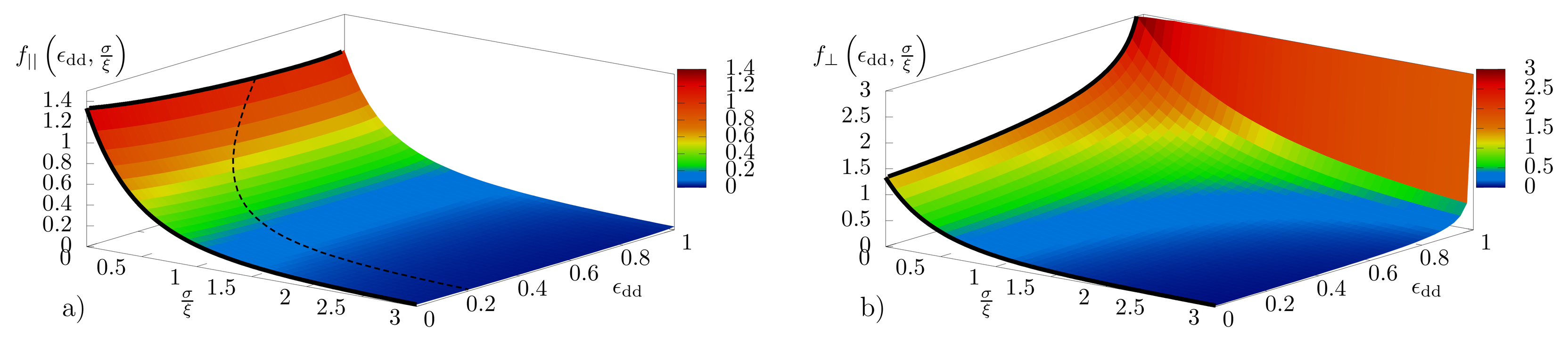
$$n - n_0 = n_{\text{HM}}\, f\left(\epsilon_{\text{dd}}, \frac{\sigma}{\xi}\right)$$

  - **Coherence length:** $\xi = \sqrt{\frac{\hbar^2}{4Mng}}$
  - **Huang-Meng depletion:** $n_{\text{HM}} = \left(\frac{M}{2\pi\hbar^2}\right)^{\frac{3}{2}}\sqrt{\frac{\pi}{2gn}}R$



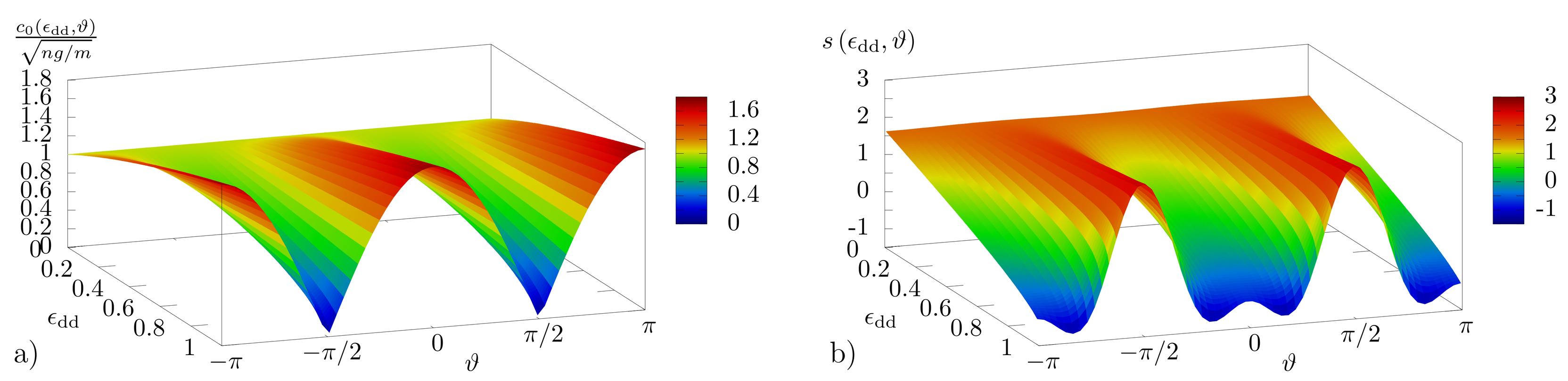
- **Superfluid Depletion:**

$$n - n_S = n_{\text{HM}}\begin{pmatrix} f_\perp\left(\epsilon_{\text{dd}},\frac{\sigma}{\xi}\right) & 0 & 0\\ 0 & f_\perp\left(\epsilon_{\text{dd}},\frac{\sigma}{\xi}\right) & 0\\ 0 & 0 & f_{||}\left(\epsilon_{\text{dd}},\frac{\sigma}{\xi}\right)\end{pmatrix}$$



$\Longrightarrow$ **Finite localization time [8]**

- **Speed of Sound:**
  - **Delta correlated disorder:** $c^2(\epsilon_{\text{dd}},\vartheta) = c_0^2(\epsilon_{\text{dd}},\vartheta) + \frac{n_{\text{HM}}g}{M}s(\epsilon_{\text{dd}},\vartheta)$

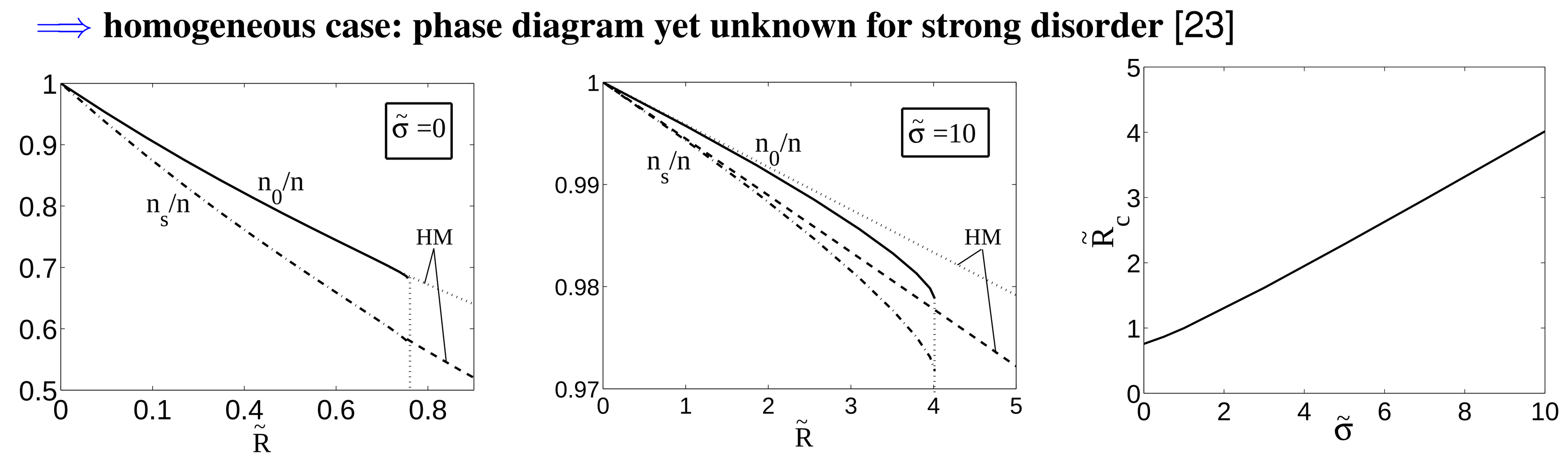


  - **Special case of contact interaction [21,22]:** $s(0,\vartheta) = \frac{5}{3}$

$\Longrightarrow$ **Measurable via Bragg spectroscopy**

Outlook

- **On the dirty boson problem:**
  global condensate + local condensates in minima + thermally excited
  $\Longrightarrow$ **homogeneous case: phase diagram yet unknown for strong disorder [23]**



$\Longrightarrow$ **trapped case: consequences for time-of-flight absorption pictures**

- **Anisotropic superfluidity:**
  interplay between anisotropic disorder and dipolar interaction:
  $\Longrightarrow$ **necessitates anisotropic 3-fluid model**

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