Bose-Einstein Condensation in Weak and Strong Disorder Potentials

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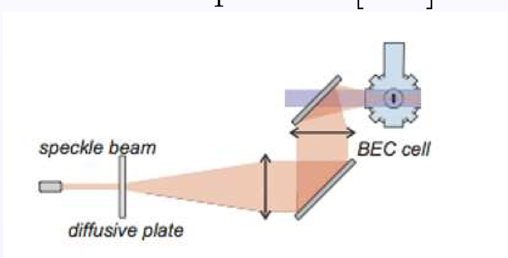
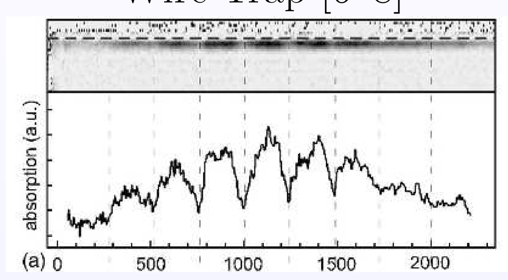
Perturbative Mean-Field Approach [1,2]

- **Experimental Realisation of Disorder Potentials**

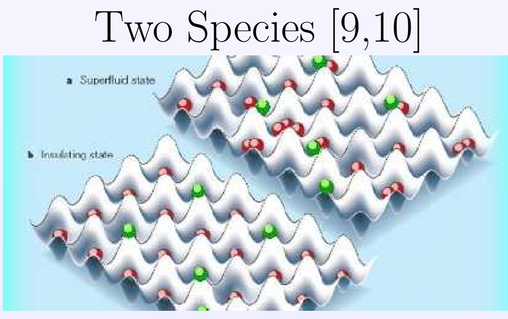
Laser Speckles [3–5]



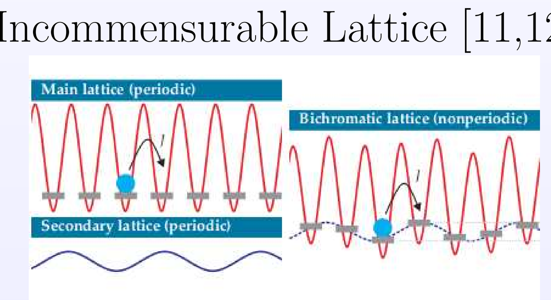
Wire Trap [6–8]



Two Species [9,10]



Incommensurable Lattice [11,12]



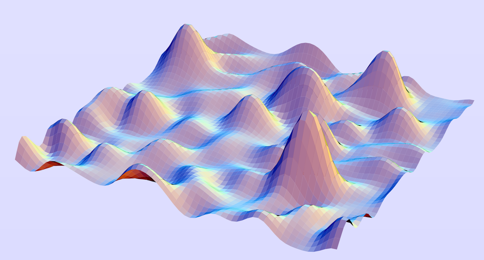
- **Theoretical Problem**
  - ⋆ Given: Disorder Potential
  - ⋆ Wanted: Determination of Condensate Depletion
- **Bose Gas**
  - ⋆ Grand-Canonical Hamiltonian

$$\hat{H} = \int d^3x \left\{ \hat{\psi}^\dagger(\mathbf{x}) \left[-\frac{\hbar^2\Delta}{2m} - \mu + U(\mathbf{x}) \right] \hat{\psi}(\mathbf{x}) + \frac{g}{2} \hat{\psi}^\dagger(\mathbf{x})\hat{\psi}^\dagger(\mathbf{x})\hat{\psi}(\mathbf{x})\hat{\psi}(\mathbf{x}) \right\}$$

  - ⋆ Quantum Average: $\langle \cdots \rangle$
  - ⋆ Mean Field Approach: $\psi(\mathbf{x}) = \left\langle \hat{\psi}(\mathbf{x}) \right\rangle$
    → Time-Independent Gross-Pitaevskii Theory

$$\left[-\frac{\hbar^2\Delta}{2m} - \mu + U(\mathbf{x}) \right] \psi(\mathbf{x}) + g|\psi(\mathbf{x})|^2\psi(\mathbf{x}) = 0$$

- **Ensemble Averages**
  - ⋆ Disorder Ensemble Average: $\overline{\cdots}$
  - ⋆ Homogeneous Disorder Potential
    $\overline{U(\mathbf{x})} = 0$, $\overline{U(\mathbf{x})U(\mathbf{x}')} = R(\mathbf{x} - \mathbf{x}')$



- **Condensate and Particle Density**
  - ⋆ Perturbative Expansion in Disorder Potential

$$\psi(\mathbf{x}) = \psi_0 + \psi_1(\mathbf{x}) + \psi_2(\mathbf{x}) + \cdots$$

  - ⋆ Particle Density: $n = \overline{|\psi(\mathbf{x})|^2}$
  - ⋆ Condensate Density: $n_0 = \overline{\psi^\star(\mathbf{x})}\ \overline{\psi(\mathbf{x})}$
  - ⋆ Condensate Depletion due to Disorder

$$n_0 = n - n \int \frac{d^3k}{(2\pi)^3} \frac{R(\mathbf{k})}{\left(\frac{\hbar^2\mathbf{k}^2}{2m} + 2gn \right)^2}$$

  - ⋆ Case of δ-correlated Disorder Potential: $R(\mathbf{k}) = R$ [13–17]

$$n_0 = n(1 - r)$$

  - ⋆ Dimensionless Disorder Strength: $r = \sqrt{\frac{\pi}{2ng}} R \left(\frac{m}{2\pi\hbar^2} \right)^{3/2}$

Non-Perturbative Mean-Field Approach [1]

- **Theoretical Problem**
  - ⋆ Goal: Quantum Phase Transition for strong Disorder?
  - ⋆ Gaussian Approximation for Homogeneous Random Fields $U(\mathbf{x})$ and $\psi(\mathbf{x})$: Characterisation by their First and Second Cumulants

$$\overline{U(\mathbf{x})} = 0 \qquad \overline{\psi(\mathbf{x})} = \sqrt{n_0}$$
$$\overline{U(\mathbf{x})U(\mathbf{x}')} = R(\mathbf{x} - \mathbf{x}') \qquad \overline{\psi(\mathbf{x})\psi(\mathbf{x}')}_c = G_{\psi\psi}(\mathbf{x} - \mathbf{x}')$$
$$\overline{U(\mathbf{x})\psi(\mathbf{x}')}_c = G_{U\psi}(\mathbf{x} - \mathbf{x}')$$

  - ⋆ Consequence: Express Higher Moments by First and Second Cumulant
    e. g. $\left\langle \psi(\mathbf{x})^3 \right\rangle = \sqrt{n_0}\,[n_0 + 3G_{\psi\psi}(\mathbf{0})]$
  - ⋆ Recursive Method
- **Order Parameters** [18]
  - ⋆ Superfluid Order Parameter: $n_0 = \lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{\psi(\mathbf{x})\psi^\star(\mathbf{x}')}$
  - ⋆ Edward-Anderson-Like Bose-Glass Order Parameter: q

$$(q + n_0)^2 = \lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{|\psi(\mathbf{x})\psi^\star(\mathbf{x}')|^2}$$

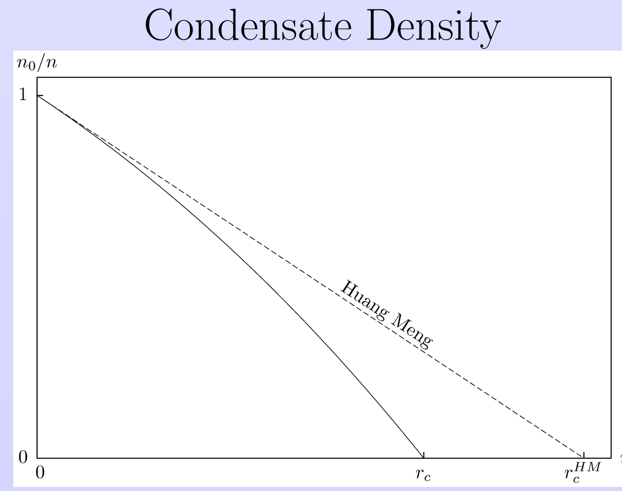
  - ⋆ Particle Density: $n = n_0 + q$
  - ⋆ Model: $\psi(\mathbf{x}) = \underbrace{\sqrt{n_0}}_{\text{Global Condensate with Fixed Phase}} + \underbrace{\sqrt{q}e^{i\phi(\mathbf{x})}}_{\text{Fragmented Condensates with Random Local Phase}}$

    → Disorder Average $\overline{\cdots} = \prod_{\mathbf{x}} \int_0^{2\pi} \cdots \frac{d\phi(\mathbf{x})}{2\pi}$
- **Disorder Averages of Gross-Pitaevskii Equation: two Self-Consistency Equations**

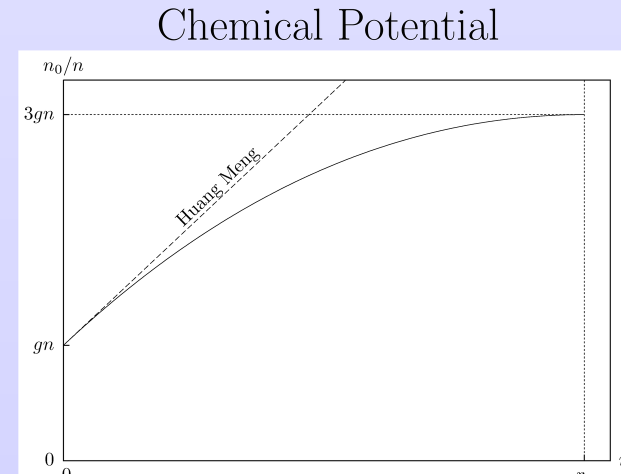
$$3gn - \mu - 2gn_0 = \int \frac{d^3k}{(2\pi)^3} \frac{R(\mathbf{k})}{\frac{\hbar^2\mathbf{k}^2}{2m} + 3gn - \mu}$$
$$q = n_0 \int \frac{d^3k}{(2\pi)^3} \frac{R(\mathbf{k})}{\left(\frac{\hbar^2\mathbf{k}^2}{2m} + 3gn - \mu \right)^2}$$

- **Case of δ-correlated Disorder Potential: $R(\mathbf{k}) = R$**

Condensate Density



Chemical Potential



$$n_0(n, r) = n \left(1 - \frac{r^2}{2} - \frac{r}{2}\sqrt{r^2 + 4} \right) \qquad \mu(n, r) = ng \left(1 - 5r^2 + 3r\sqrt{r^2 + 4} \right)$$

- **Quantum Phase Transition to a Bose Glass Phase**
  - ⋆ Critical Disorder Strength: $n_0(n, r_c) = 0$

$$r_c^{\mathrm{HM}} = 1, \qquad r_c = \frac{1}{\sqrt{2}}$$

  - ⋆ Compare with Literature: $r_c^{\mathrm{FNP}} \approx 0.04$ [19], $r_c^{\mathrm{NPG}} \approx 0.75$ [20]

Perturbative Beyond Mean-Field Approach [2]

- **Theoretical Problem**
  - ⋆ Literature [13–17]: Huang-Meng Condensate Depletion due to Disorder within Bogoliubov Theory
  - ⋆ Recent Work [21]: Derivable also within Gross-Pitaevskii Theory
  - ⋆ Explanation of Contradiction: Disorder Average at Different Stages of Calculation
  - ⋆ Goal: Determination of Influence of Quantum Fluctuations
- **Perturbative Double Expansion**
  - ⋆ Background Method [22–25]

$$\hat{\psi}(\mathbf{x}) = \underbrace{\psi(\mathbf{x})}_{\text{Global and Fragmented Condensates}} + \underbrace{\delta\hat{\psi}(\mathbf{x})}_{\text{Excitations}}$$

  - ⋆ Condensate Wave Function:

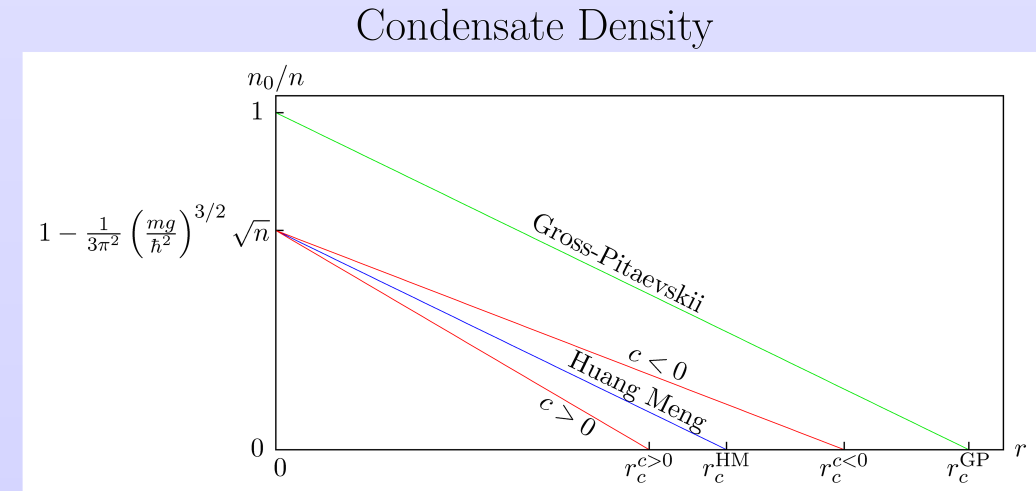
$$\psi(\mathbf{x}) = \psi_{00} + \psi_{01}(\mathbf{x}) + \psi_{02}(\mathbf{x}) + \psi_{10} + \psi_{11}(\mathbf{x}) + \psi_{12}(\mathbf{x}) + \ldots$$

  - ⋆ First Index ≙ Order of Fluctuations
    Second Index ≙ Order of Disorder Potential
  - ⋆ Chemical Potential: $\mu = \mu_{00} + \mu_{10} + \mu_{01} + \mu_{02} + \ldots$
  - ⋆ Correlations: $\langle \cdots \rangle = \langle \cdots \rangle_0 + \langle \cdots \rangle_U + \langle \cdots \rangle_{U^2} + \ldots$
  - ⋆ Bogoliubov Transformation [26]
  - ⋆ Dirac Interaction Picture
- **Case of δ-correlated Disorder Potential: $R(\mathbf{k}) = R$**

$$n_0 = n \left[1 - r - \frac{1}{3\pi^2} \left(\frac{mg}{\hbar^2} \right)^{3/2} \sqrt{n} - \frac{c}{\pi^2} \left(\frac{mg}{\hbar^2} \right)^{3/2} \sqrt{nr} \right]$$

  - ⋆ Gross-Pitaevskii Disorder Depletion [13–17]
  - ⋆ Bogoliubov Depletion [27]
  - ⋆ Bogoliubov Disorder Depletion

Condensate Density



  - ⋆ Critical Disorder Strength

$$r_c^{\mathrm{GP}} = 1, \; r_c^{\mathrm{HM}} = 1 - \frac{1}{3\pi^2} \left(\frac{mg}{\hbar^2} \right)^{3/2} \sqrt{n}, \; r_c^{c \gtrless 0} = \frac{1 - \frac{1}{3\pi^2} \left(\frac{mg}{\hbar} \right)^{3/2} \sqrt{n}}{1 + \frac{c}{\pi^2} \left(\frac{mg}{\hbar^2} \right)^{3/2} \sqrt{n}}$$

  - ⋆ Drawback: c Contains Infrared and Ultraviolet Divergencies [28]

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