Collective excitations in a trapped Bose-Einstein condensate with weak quenched disorder

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Dilute Bose gas with weak disorder

K. Huang and H. F. Meng, Phys. Rev. Lett. **69**, 644 (1992):

Dilute superfluid gas with random interaction
→ Model for ^{4}He in porous media
→ Understand qualitatively disorder effects on superfluidity
→ Condensation: not sufficient condition for having superfluidity

The action of the theory:

$$S[\psi^*,\psi;U] = \int_0^{\hbar\beta} d\tau \left(\int d\mathbf{x}\, \psi^*(\mathbf{x},\tau) \left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2\nabla^2}{2m} - \mu + U(\mathbf{x}) \right] \psi(\mathbf{x},\tau) + \frac{1}{2}\, g \int d\mathbf{x}\, \psi^*(\mathbf{x},\tau)\psi^*(\mathbf{x},\tau)\psi(\mathbf{x},\tau)\psi(\mathbf{x},\tau) \right) \qquad g = 4\pi\hbar^2 a/m$$

Ensemble averages assumed:

$$\langle U(\mathbf{x})\rangle = 0,$$

$$\langle U(\mathbf{x})U(\mathbf{x}')\rangle = R\left(\mathbf{x}-\mathbf{x}'\right) = R_0 \frac{\exp\{-\frac{(\mathbf{x}-\mathbf{x}')^2}{2\xi^2}\}}{(2\pi\xi^2)^{3/2}}$$

Delta correlated disorder: $\xi \to 0$

Perturbative approach: uniform gas

Bogoliubov particle number equation: disorder-induced condensate depletion

$$T=0 \qquad n = n_0 + n_g + n_{\rm R} \equiv n_0 + \frac{8}{3\sqrt{\pi}}(na)^{3/2} + R_0 \frac{m^2}{8\,\pi^{3/2}\,\hbar^4}\sqrt{\frac{n}{a}} f(4\pi n_{\mathbf{0}} a \xi^2)$$

n_g → Bogoliubov depletion due to the atomic interaction g
$n_{\rm R}$ → Disorder-induced depletion

Normal component due to the disorder:

$$n_n = n - n_s = (4/3)\, n_{\rm R} \quad \to \quad n_0 > n_s$$

Conditions of validity of the theory:

- Dilute gas: $n^{1/3} a \ll 1$
- Weak disorder approximation:

$$n_{\rm R} \ll n \Leftrightarrow R_0' \equiv m^2 R_0/8\pi^{3/2}\hbar^4\,(na)^{1/2} \ll 1$$

- Self-averaging condition:

$$\xi \ll \xi_{\rm heal} = 1/\sqrt{8\pi n_{\mathbf{0}} a},$$

$\xi_{\rm heal}$ is the healing length of the superfluid

Hydrodynamic equations

$\xi \ll \xi_{\rm heal}$ → "Two-fluids" collisionless hydrodynamic equations at $T=0$:

$$\frac{\partial}{\partial t} n + \nabla\left(\mathbf{v}_s n_s + \mathbf{v}_n n_n\right) = 0$$

$$m\frac{\partial}{\partial t}\mathbf{v}_s + \nabla\left(\mu + \frac{1}{2} m \mathbf{v}_s^2\right) = 0$$

Normal component in linear regime is stationary, $\mathbf{v}_n = 0$

→ "Fourth sound" linearized hydrodynamic equation:

$$m\frac{\partial^2}{\partial t^2}\delta n - \nabla\left[n_s \nabla\left(\frac{\partial\mu}{\partial n}\delta n\right)\right] = 0$$

Disorder affects collective excitations in two ways:
$(\partial\mu/\partial n)^{-1}$ → change of the macroscopic compressibility
$n_s \neq n$ → disorder-induced superfluid depletion

Homogeneous gas → phonon dispersion: $\quad \delta n \sim e^{i\omega t}, \quad \hbar\omega = c q_z$
For $\xi = 0$ reproduces the correct disorder-induced shift on the velocity of sound:
S. Giorgini, L. Pitaevskii, and S. Stringari, Phys. Rev. B **49**, 12938 (1994).

$$c^2 = c_0^2\,(1 + 5n_{\rm R}/3n)\,; \quad c_0^2 = 4\pi\hbar^2 a n/m^2$$

What We Calculate

We use the hydrodynamic approach to calculate perturbatively the effects of weak disorder on the collective modes in trapped 3D Bose-Einstein condensates

Thomas-Fermi approximation in harmonic traps

Harmonic isotropic trap:

$$V(\mathbf{x}) = m\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)/2$$

$$\omega_x = \omega_y = \omega_z = \omega_{\rm HO}$$

Harmonic oscillator length: $a_{\rm HO} = \sqrt{\hbar/m\omega_{\rm HO}}$

Large N Thomas-Fermi limit: $\quad \frac{Na}{a_{\rm HO}} \gg 1$

Thomas-Fermi radius $R_{\rm TF}$: $\quad \frac{1}{2}\omega_{\rm HO}^2 R_{\rm TF}^2 = \mu$

The Thomas-Fermi approximation: $\quad R_{\rm TF} \gg a_{\rm HO}, \quad \frac{\xi_{\rm heal}}{R_{\rm TF}} = (\frac{a_{\rm HO}}{R_{\rm TF}})^2 \quad \Rightarrow$

$$\xi_{\rm heal} \ll R_{\rm TF}$$

⇓

local density approximation

where the healing length in the trap is defined as $\xi_{\rm heal} = 1/\sqrt{8\pi n(\mathbf{0})a}$

Total number of particles in local density approximation in presence of disorder:
Example: case $\xi = 0$

$$N = \int d\mathbf{r}\; n_0(\mathbf{r}) + \int d\mathbf{r} \frac{8}{3\sqrt{\pi}} \left[n(\mathbf{r})\, a\right]^{3/2} + N_{\rm R}$$

$$N_{\rm R} = \int d\mathbf{r}\; n_{\rm R}(\mathbf{r}) = N \frac{15\pi}{32} \frac{n_{\rm R}(\mathbf{0})}{n_0(\mathbf{0})}$$

Local equilibrium condition:

include beyond mean-field disorder corrections in the equation of state:

Example: case $\xi = 0$

$$\mu_{local}(n) = g\, n(\mathbf{r}) \left(1 + \frac{32}{3}\sqrt{\frac{n(\mathbf{r})\, a^3}{\pi}}\right) + 6\, g\, n_{\rm R}(\mathbf{r})$$

↓

modified Thomas-Fermi density:

$$n(\mathbf{r}) = n_{\rm TF}(\mathbf{r}) - 6\, n_{\rm R}(\mathbf{r})$$

Collective modes in the collisionless regime $\hbar\omega_{\rm HO} \lesssim \hbar\omega_{\mathbf{k}} \ll \mu$

When

$$n_g(\mathbf{r}) \ll n_{\rm R}(\mathbf{r}) \Leftrightarrow R_0'(\mathbf{0}) \gg \sqrt{a^3 n(\mathbf{0})}$$

→ beyond mean-field effects for the atom-atom interaction can be neglected

Hydrodynamic equation in the trap with disorder:

Example : delta-correlated disorder $\xi = 0$,
(the structure of the equations does not change for finite ξ)

$$m\frac{\partial^2}{\partial t^2}\delta n(\mathbf{r}) - \nabla\left[g\, n_{\rm TF}(\mathbf{r})\, \nabla \delta n(\mathbf{r})\right] = -\nabla^2\left[3\, g\, n_{\rm R}(\mathbf{r})\;\; \delta n(\mathbf{r})\right] - \nabla\left[\frac{4n_{\rm R}(\mathbf{r})}{3}\nabla\left[g\, \delta n(\mathbf{r})\right]\right].$$

$R_0 = 0$ → Stringari's hydrodynamic equation:

$$m\omega_0^2 \delta n(\mathbf{r}) + \nabla\left[g\, n_{\rm TF}(\mathbf{r})\, \nabla\delta n(\mathbf{r})\right] = 0$$

Stringari, PRL **77**, 2360 (1996):

$$\omega_0(n_r, l) = \omega_{\rm HO}\left(2n_r^2 + 2n_r l + 3n_r + l\right)^{1/2}$$

Results for delta-correlated disorder ($\xi = 0$)

Shift in the compressional modes

- Monopole mode, $n_r = 1$, $l = 0$, $\omega_{\rm M} = \sqrt{5}\omega_{\rm HO}$, $\delta n \propto 1 - (5/3)r^2$:

$$\frac{\delta\omega_M}{\omega_M} = -\frac{469\pi}{768} R_0'(\mathbf{0})$$

To compare with the beyond-mean-field correction of the Beliaev theory:
$\frac{\delta\omega_M}{\omega_M} = (63\pi/128)\sqrt{a^3 n(\mathbf{0})}$ (L. Pitaevskii and S. Stringari, PRL **81**, 4541 (1998))

Shifts in the surface modes

$$n_r = 0,\;\; \omega_l = \sqrt{l}\;\omega_{\rm HO}, \quad \delta n(\mathbf{r}) = r^l Y_{lm}, \quad \nabla^2 \delta n(\mathbf{r}) = 0$$

→ No shift due to the beyond-mean-field effects in the atomic interaction

- Dipole mode, $l = 1$ (Kohn's theorem states $\omega_{dip} = \omega_{\rm HO}$)

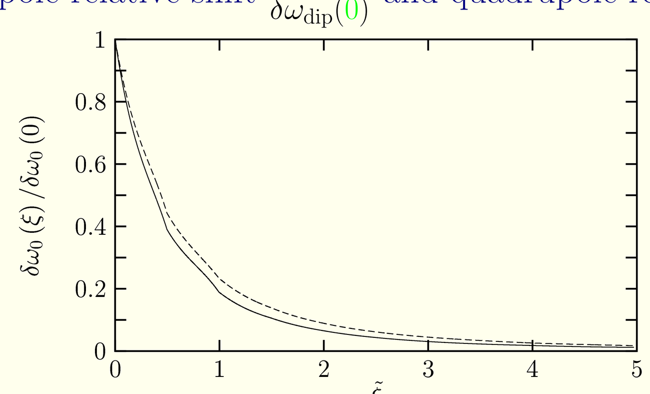
$$\frac{\delta\omega_{\rm dip}}{\omega_{\rm dip}} = -\frac{5\pi}{16} R_0'(\mathbf{0})$$

- Quadrupole mode $l = 2$, $m = 2$

$$\frac{\delta\omega_{\rm Q}}{\omega_{\rm Q}} = -\frac{35\pi}{96} R_0'(\mathbf{0})$$

Effects of finite disorder correlation length ($\xi \neq 0$)

Dipole relative shift $\frac{\delta\omega_{\rm dip}(\xi)}{\delta\omega_{\rm dip}(0)}$ and quadrupole relative shift $\frac{\delta\omega_{\rm Q}(\xi)}{\delta\omega_{\rm Q}(0)}$



$$\tilde{\xi} = \xi R_{\rm TF}/a_{\rm HO}^2\sqrt{2}$$

Hydrodynamic approach valid when $\xi \ll \xi_{\rm heal} \Leftrightarrow \tilde{\xi} \ll 1$

Current experiments in Thomas-Fermi limit ($R_{\rm TF}/a_{\rm HO} \simeq 10$):
$\tilde{\xi} \geq 6 \Leftrightarrow \xi \geq \xi_{\rm heal}$

Anisotropy effects for delta correlated disorder

Anisotropic trap:

$$V(\mathbf{x}) = m\left(\omega_\perp^2 r_\perp^2 + \omega_z^2 z^2\right)/2$$

Anisotropy factor: $\lambda = \omega_z/\omega_\perp$

$$\omega_{\rm dip}(\lambda) = \sqrt{2}\omega_z,\; \omega_{\rm Q}(\lambda) = \sqrt{2}\omega_\perp \Rightarrow \frac{\delta\omega_{\rm dip}(\lambda)}{\omega_{\rm dip}(\lambda)} = \frac{\delta\omega_{\rm dip}(1)}{\omega_{\rm dip}(1)},\; \frac{\delta\omega_{\rm Q}(\lambda)}{\omega_{\rm Q}(\lambda)} = \frac{\delta\omega_{\rm Q}(1)}{\omega_{\rm Q}(1)}$$

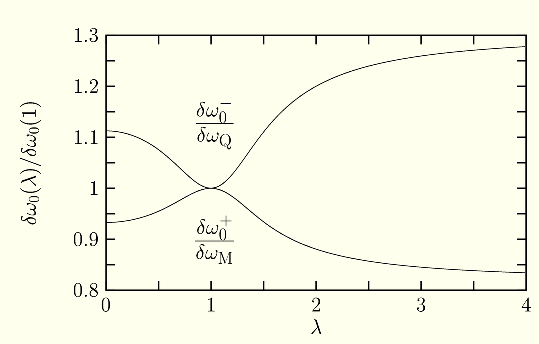
Anisotropy:

quadrupole mode with $m = 0$ couple to monopole mode

mean-field: $[\omega_0^{\pm}(m=0)]^2 = \omega_\perp^2(2 + \frac{3}{2}\lambda^2 \pm \frac{1}{2}\sqrt{9\lambda^4 - 16\lambda^2 + 16})$

$\lambda = 1 \Rightarrow \quad \omega_0^+ = \omega_{\rm M},\; \omega_0^+ = \omega_{\rm Q}$

Disorder effects:



$$\frac{\delta\omega_0^{\pm}}{\omega_0^{\pm}} = -\pi R_0' \frac{7(\pm 72 \pm 9\lambda^2 + 107\sqrt{16 - 16\lambda^2 + 9\lambda^4})}{1536\sqrt{16 - 16\lambda^2 + 9\lambda^4}}$$

Conclusions and Perspectives

- Collective modes can be measured with great accuracy in trapped BECs. beyond-mean-field effects can be measured.
  A. Altmeyer *et al.*, PRL **98**, 040401 (2007).
- Realistic test for the predictions of the Huang and Meng theory (see also G. E. Astrakharchik *et al.*, PRA **66**, 023603 (2002)).
- New effects due to disorder for trapped condensates: deviation from Kohn's theorem can be observed