Quantum Mechanical Description of Thermo-Optic Interaction in Photon BECs

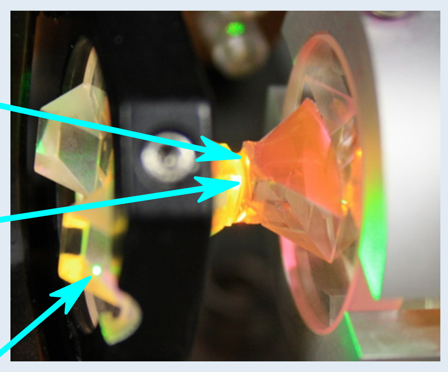
TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Enrico Stein, Axel Pelster

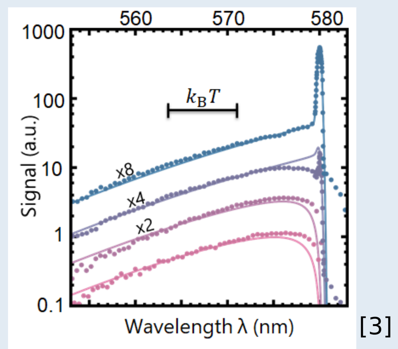
Introduction

Bose-Einstein Condensation of Photons

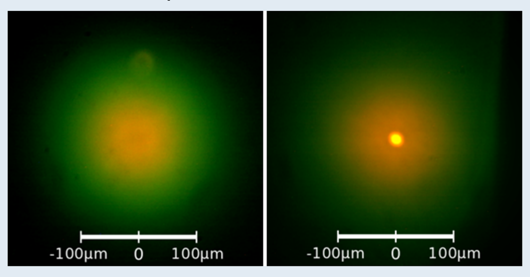
- High finesse cavity: Provides energy cutoff
- Dye solution: Heat and particle reservoir
- Pump radiation: Provides chemical potential



Energy distribution ⇒ Thermal distribution

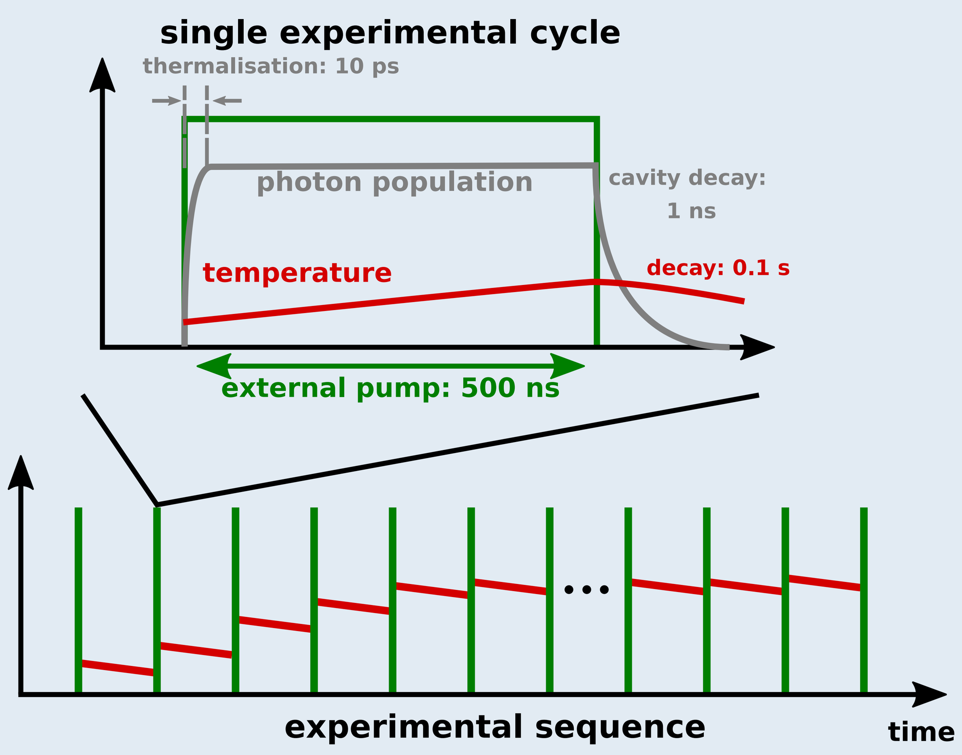


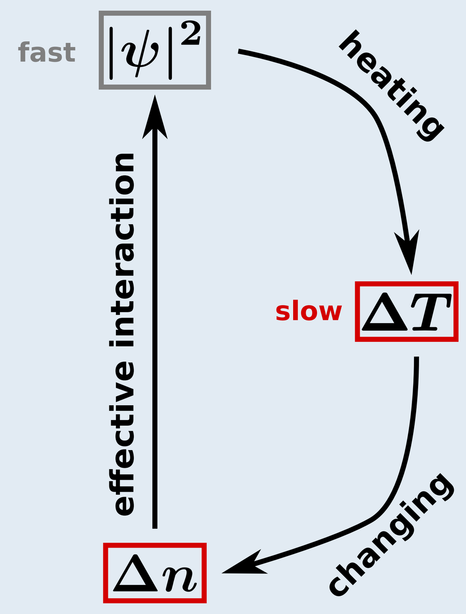
Spatial information



[1] J. Klaers et al., Nature **468**, 545 (2010)
[2] J. Klaers et al., Nat. Phys. **6**, 512 (2010)
[3] T. Damm et al., Nature Commun. **7**, 11340 (2016)

Time-Scales



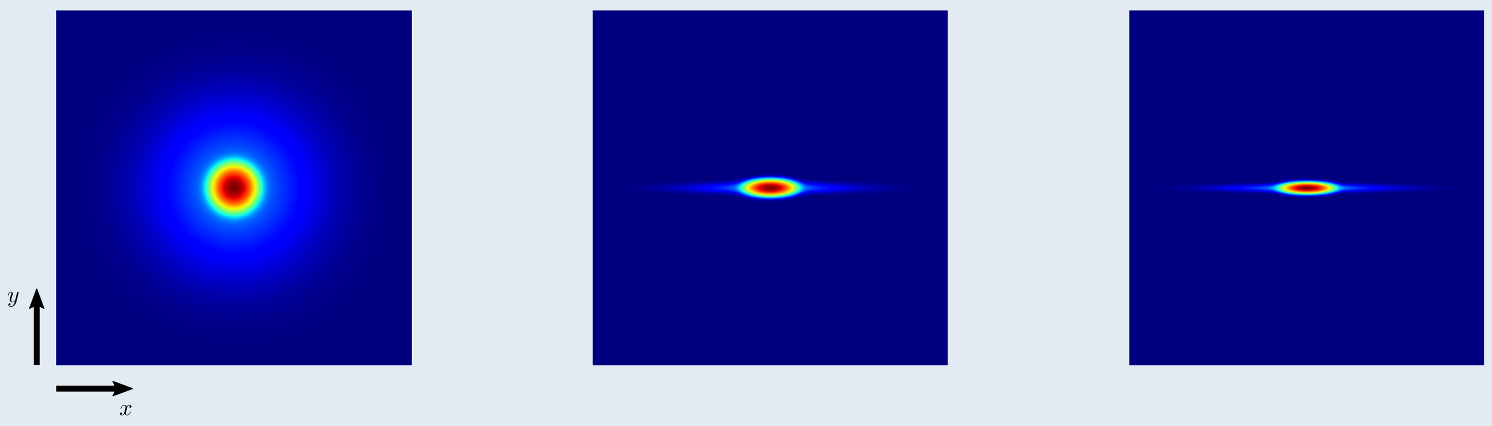


Ideal Bose Gas: Dimensional Crossover

NJP **23**, 023013 (2022)

Aim: Effective system dimension at dimensional crossover

trap-aspect ratio: $\lambda = \Omega_y/\Omega$ $\quad V = \frac{m\Omega^2}{2}(x^2+\lambda^2 y^2)$



2D $\lambda = 1$ → 1D $\lambda \to \infty$

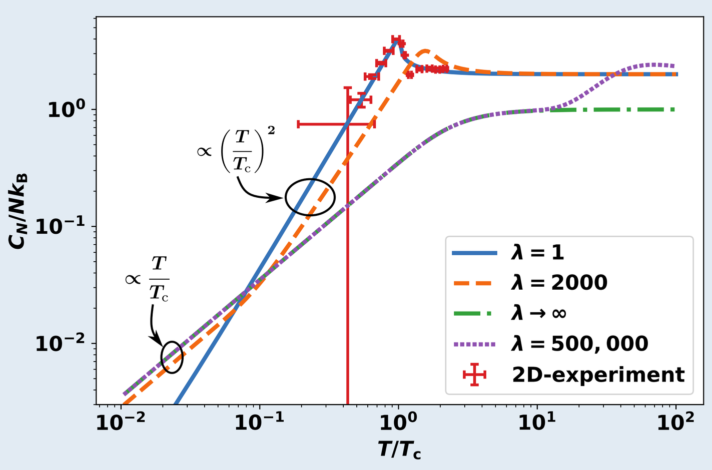
Energy Levels: $E_{jn}(\lambda) = \hbar\Omega\left(j + \lambda n + \frac{1+\lambda}{2}\right)$

Grand-Canonical Partition Function: $\Pi = \Pi_{1D} + \Delta\Pi$

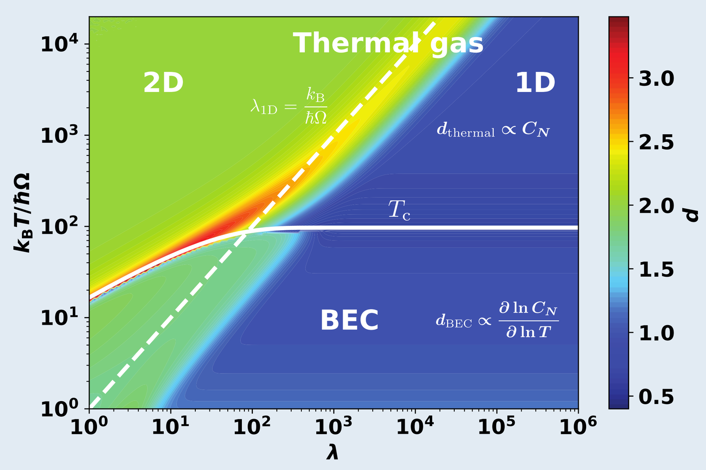
pure 1D part — corrections to 2D

Thermodynamic Quantities:

Specific Heat



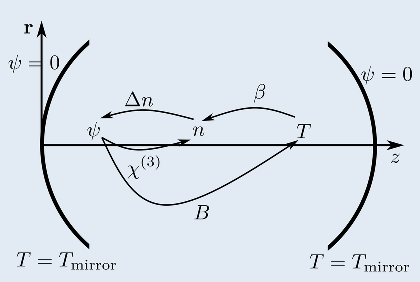
Effective Dimension



Photon BEC with Interaction

NJP **24**, 023032 (2022)

Aim: Behaviour of photon BEC ground state at dimensional crossover with interaction



Stein et al., NJP 21, 103044 (2019)

Photon-Energy Functional:

$$E[\psi,\psi^*] = \int d^2x \left\{ \frac{\hbar^2}{2m}|\nabla\psi|^2 + \frac{m\Omega^2}{2}(x^2+\lambda^4 y^2)|\psi|^2 + \frac{g_K}{2}|\psi|^4 + \frac{g_T}{2}\int d^2x'\, \mathcal{G}(\mathbf{x}-\mathbf{x}')|\psi(\mathbf{x}')|^2|\psi(\mathbf{x})|^2 \right\}$$

Kerr effect — thermo-optic effect

Minimisation with Gaussian Ansatz:

$$\psi = \sqrt{\frac{\lambda N}{\pi l_x^2 \alpha_x \alpha_y}} \exp\left[-\frac{1}{2l_x^2}\left(\frac{x^2}{\alpha_x^2} + \lambda^2\frac{y^2}{\alpha_y^2}\right)\right]$$

Oscillator length: $l_x = \sqrt{\frac{\hbar}{m\Omega}}$

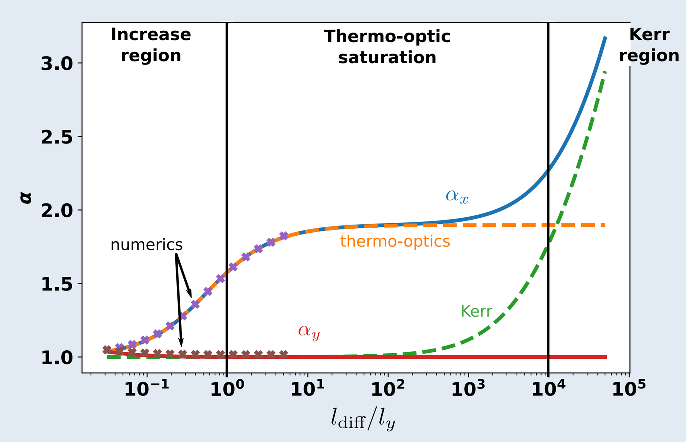
Variational parameters: $\alpha_x,\ \alpha_y$

Results:

Diffusion length: $l_{\mathrm{diff}} = \sqrt{\tau\mathcal{D}}$

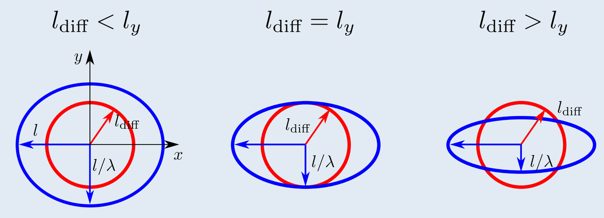
relaxation time — diffusion constant

Variational Parameters

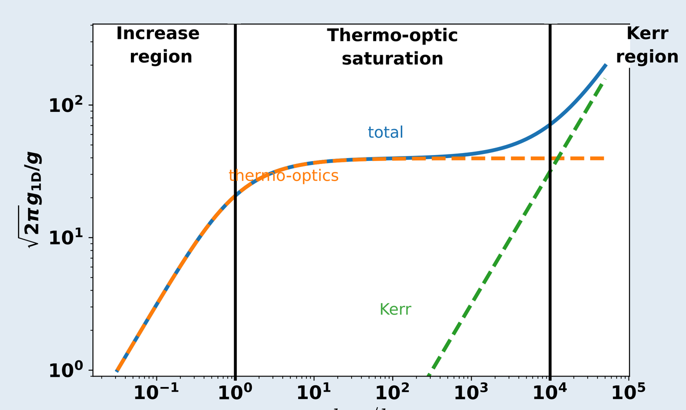


Competition between diffusion and thermo-optics

$l_{\mathrm{diff}} < l_y$ $\quad l_{\mathrm{diff}} = l_y$ $\quad l_{\mathrm{diff}} > l_y$



Effective 1D Interaction Strength



Large trap anisotropies:

$$\lim_{\lambda\to\infty} g_{1D}(\lambda) = \frac{1}{\sqrt{2\pi}}\left(g_K\lambda + \frac{g_T l_x}{l_{\mathrm{diff}}}\right)$$

Quantum Mechanics of Thermo-Optic Interaction

arXiv: 2203:16955 (2022)

Aim: Description of thermo-optic interaction during a single cycle

Thermo-Optic Hamiltonian: $\hat{H}(t) = \int d^2x\ \hat{\Psi}^\dagger\left\{h_0(\mathbf{x}) + \gamma T(\mathbf{x},t)\right\}\hat{\Psi}$

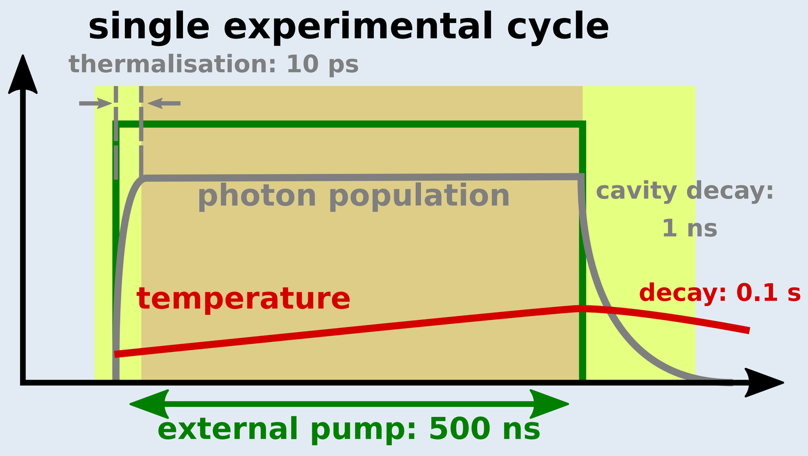
energy shift from heating

Temperature Diffusion: $\partial_t T = \left\{\mathcal{D}\nabla^2 - \frac{1}{\tau}\right\}T + B\left\langle\hat{\Psi}^\dagger\hat{\Psi}\right\rangle(\mathbf{x},t)$

slow dynamics ⇒ neglect — photons as heat source

Adiabatic Hamiltonian: $\hat{H}(t) = \sum_{mn}\left[E_m(0)\delta_{mn} + g(t)F_{mn}\right]\hat{a}_m^\dagger\hat{a}_n$

eigenenergies at beginning — eigenmodes of $h_0(\mathbf{x})$ — time-dependent interaction strength $g(t) = t\gamma B$



- Neglect dissipative dynamics
- Only thermal steady state

Interaction Matrix:

$$F_{mn} = \int d^2x\ \psi_m^* \sum_l N_l(0)|\psi_l|^2\psi_n$$

thermal density: $N_l(0) = \frac{1}{e^{\beta[E_l(0)-\mu]}-1}$

⇒ **Hartree-Fock analogue for thermo-optic interaction**

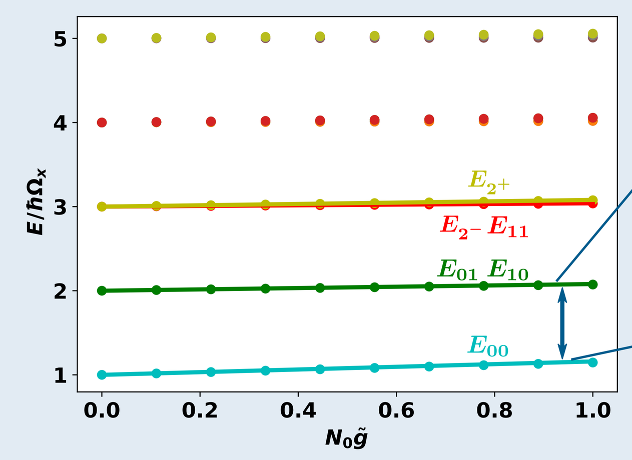
ED of Photon BEC with Thermo-Optic Interaction

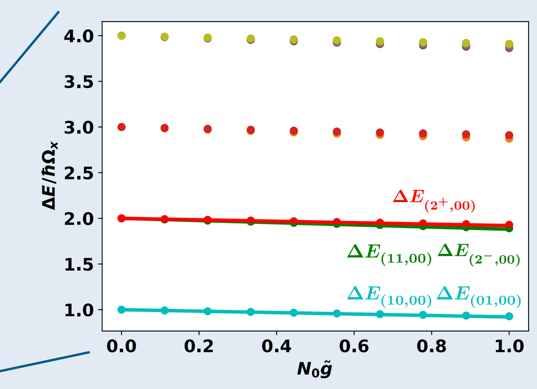
arXiv: 2204:08818 (2022)

Aim: Provide theoretical support for spectroscopic measurement of effective photon-photon interaction strength

Method: ED of thermo-optic Hamiltonian in harmonic trap

Energy Spectrum





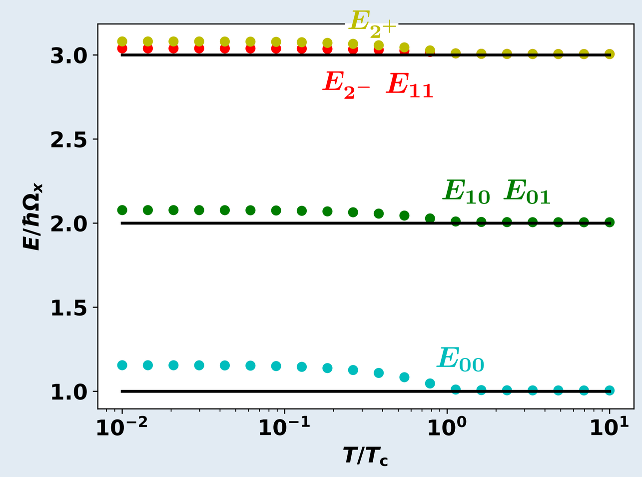
@ $N = 10.000,\ N_0/N \approx 1$

$\tilde{g} = \frac{m}{\hbar^2}g$

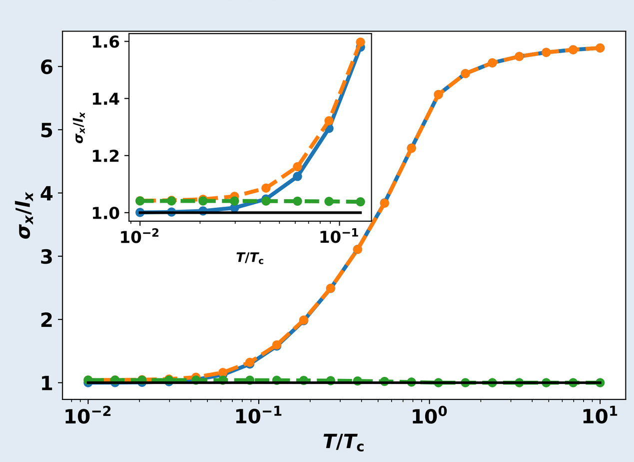
- Dots: Exact diagonalisation
- Lines: Perturbation theory

Finite Temperatures

Energy Spectrum

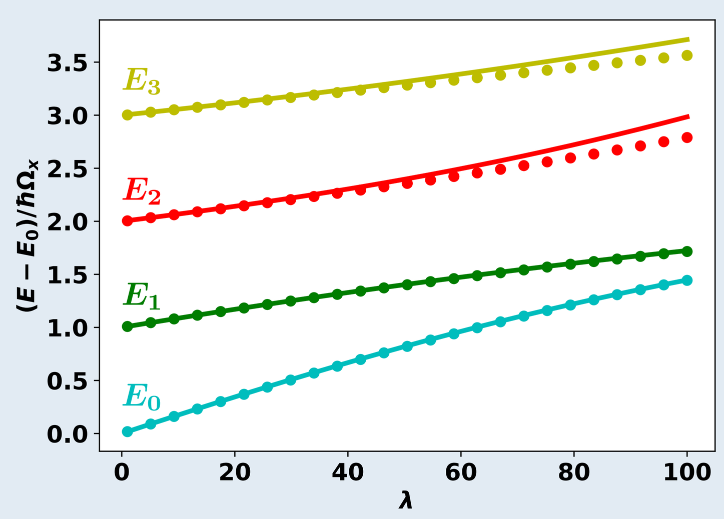


Photon Gas Width

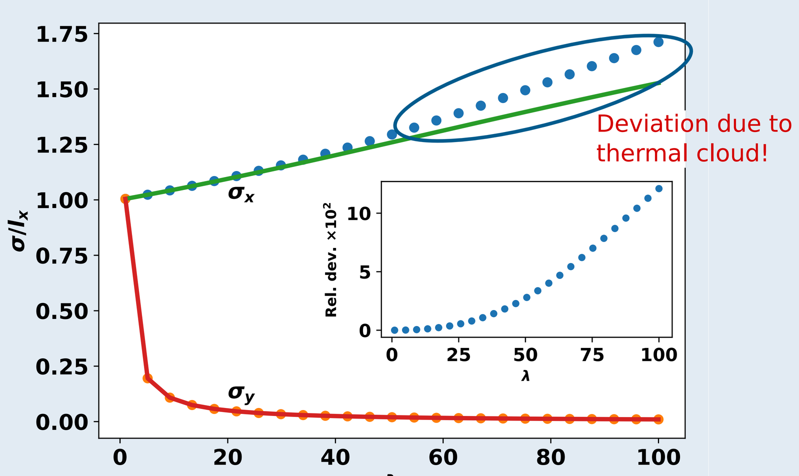


Dimensional Crossover

Energy Spectrum



Condensate Width



- Dots: Exact diagonalisation
- Lines: Variatonal approach

OPTIMAS — SFB/TR 185