Using a space-time mapping for probing heating suppression in periodically driven many-body quantum systems: a mean-field example with Bose gases

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Abstract

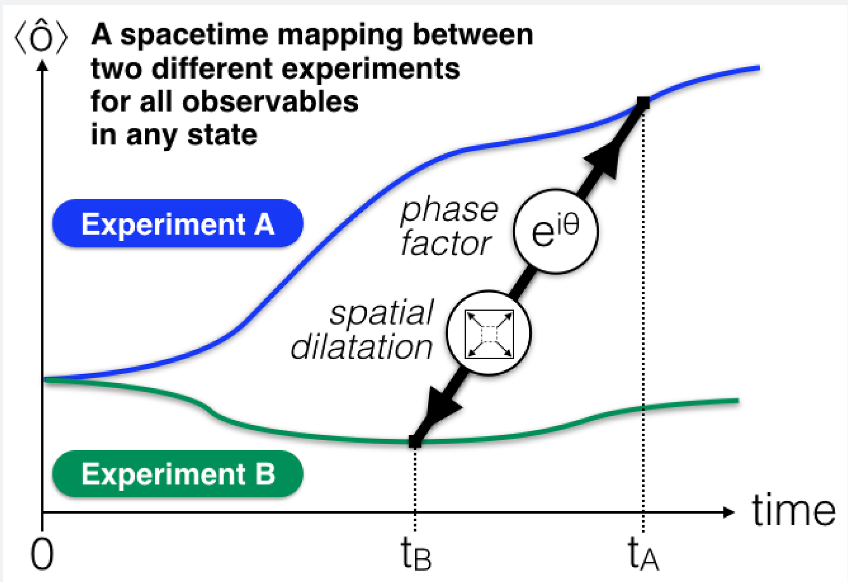
We construct a mean-field model of many-body systems with rapid periodic driving. Then the evolutions of the model system are mapped onto evolutions with slowly varying parameters. Such a mapping between a Floquet evolution and a slow process allows us to investigate non-equilibrium many-body dynamics and examine how rapidly driven systems may avoid heating up, at least when mean-field theory is still valid. We learn that rapid periodic driving may not yield to heating because the time evolution of the system has a kind of hidden adiabaticity, inasmuch as it can be mapped exactly onto that of an almost static system.

Mapping scheme and identities

- The ruling (Heisenberg) equation for a quantum gas experiment is:

$$i\hbar\frac{\partial}{\partial t}\hat{\psi}_n(\mathrm{r},t) = \left(-\frac{\hbar^2}{2M_n}\nabla^2 + V(\mathrm{r},t)\right)\hat{\psi}_n(\mathrm{r},t) + \sum_{k\ell m}\int d^D\mathrm{r}' U_{k\ell mn}(\mathrm{r},\mathrm{r}',t)\hat{\psi}_k^\dagger(\mathrm{r}',t)\hat{\psi}_\ell(\mathrm{r}',t)\hat{\psi}_m(\mathrm{r},t). \quad (1)$$

- Our mappings aim at relating exactly in a non-trivial way the dynamics of a quantum gas evolving under 2 completely different experimental conditions, A and B. Any particular mapping is defined by a function $\lambda(t)$.



- Suppose that a first quantum field $\hat{\psi}_B$ evolves following Eq. (1). Construct a second quantum field $\hat{\psi}_A$ such that [1, 2]:

$$\hat{\psi}_B(\mathrm{r},t_B) = \lambda^{D/2} e^{-i\frac{M_n}{2\hbar\lambda}\frac{d\lambda}{dt}\mathrm{r}^2}\,\hat{\psi}_A(\lambda\mathrm{r},t_A),\quad t_A(t_B) = \int_0^{t_B}\lambda(t')^2 dt'. \quad (2)$$

Then $\hat{\psi}_A$ also evolves according to Eq. (1) with transformation formulas:

- $U_A(\mathrm{r},\mathrm{r}',t_A) \to U_B(\mathrm{r},\mathrm{r}',t_B) = \lambda^{2-s}U(\mathrm{r},\mathrm{r}',t_A)$;
- $V_A(\mathrm{r},t_A) \to V_B(\mathrm{r},t_B) = \lambda^2\left[V(\lambda\mathrm{r},t_A) + \frac{1}{2}M_n f(t_B)\mathrm{r}^2\right]$,

where $f(t) = \lambda\left(\frac{1}{\lambda^2}\frac{d}{dt}\right)^2\lambda$. For a contact interaction, $s = D$; and for a dipole-dipole interaction, $s = 3$. No restriction on initial states is required.

Driving without heating: setup

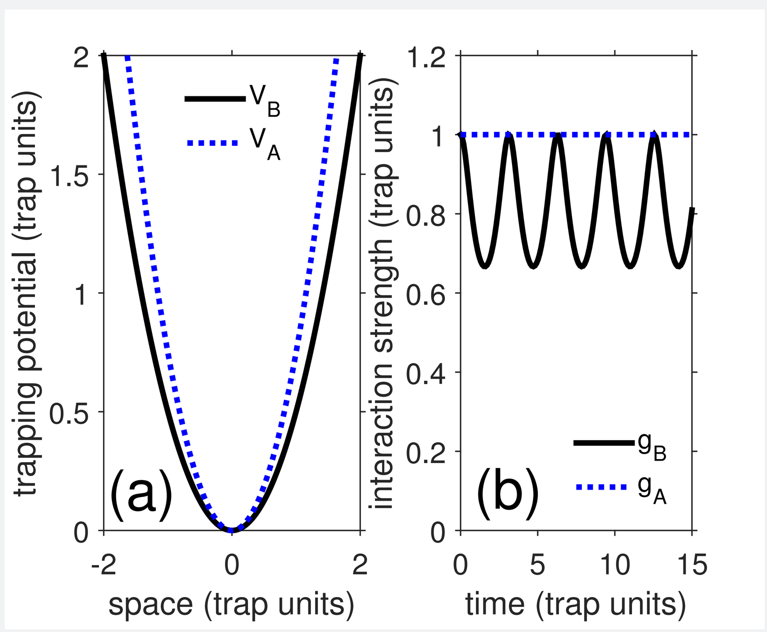
- Consider two experiments:
  - A (**static evolution**): the trap and interaction strengths are constant.
  - B (**Floquet evolution**): the interaction strength is modulated with driving frequency $2\omega_B$ while the trap remains constant.
- The pair of trapping potentials (static) and interaction strengths are:

$$\{g_A,\ V_A = \frac{M\omega_A^2 r^2}{2}\} \leftrightarrow \{g_B = g_A\lambda(t)^{2-D},\ V_B = \frac{M\omega_B^2 r^2}{2}\}. \quad (3)$$

- The free parameter is:

$$\lambda(t_B) = \frac{\omega_B}{\sqrt{\omega_B^2\cos^2(\omega_B t_B) + \omega_A^2\sin^2(\omega_B t_B)}}. \quad (4)$$

For a sketch of interaction strengths and traps for $D = 1$, see this picture→



- **Why the absence of heating?**

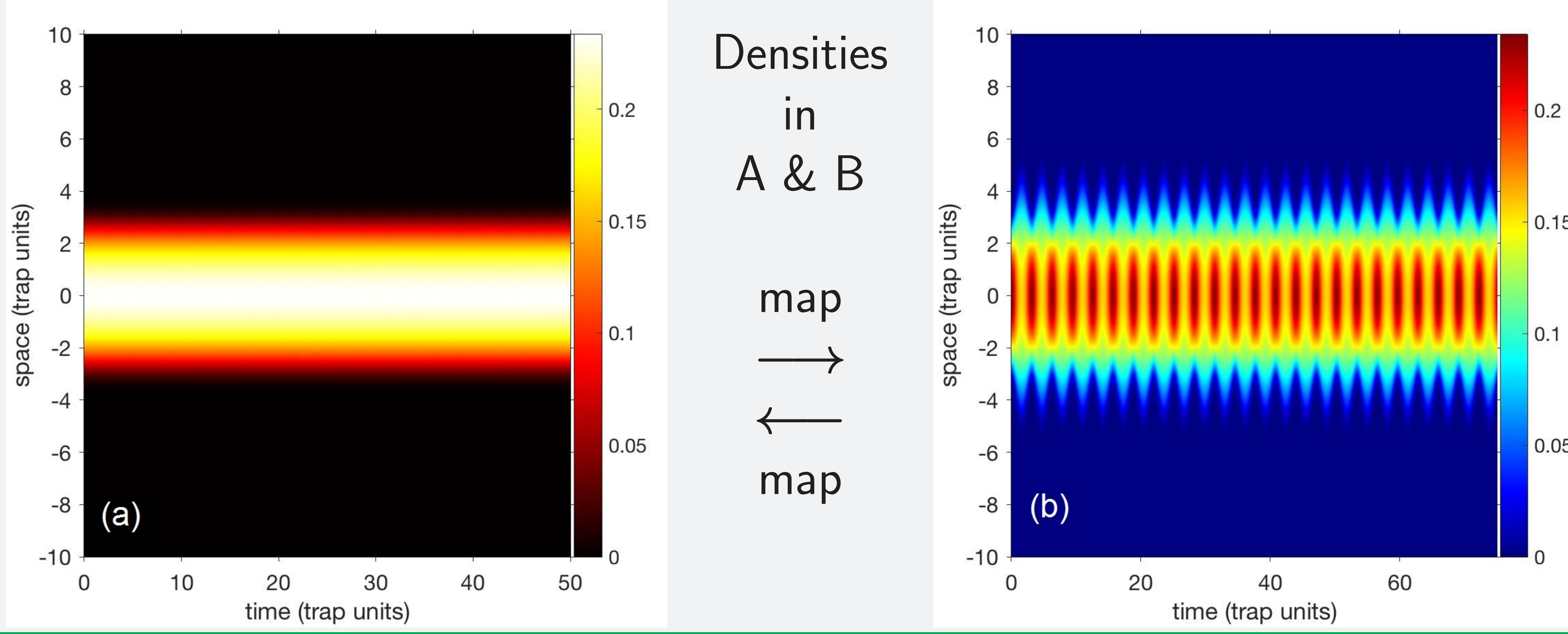
→ The fields coincide after each period:

$$\hat{\psi}_A\left(\mathrm{r},t_A\left(\frac{n\pi}{\omega_B}\right)\right) = \hat{\psi}_B\left(\mathrm{r},\frac{n\pi}{\omega_B}\right).$$

No secular heating in A ⟹ No secular heating in B [3].

→ The driving is a certain periodic but anharmonic modulation of the gas's two-body interaction, at a *particular frequency*, which makes it possible to map the Floquet experiment onto a static evolution with no secular heating.

Mean-field example of the mapping: numerical result

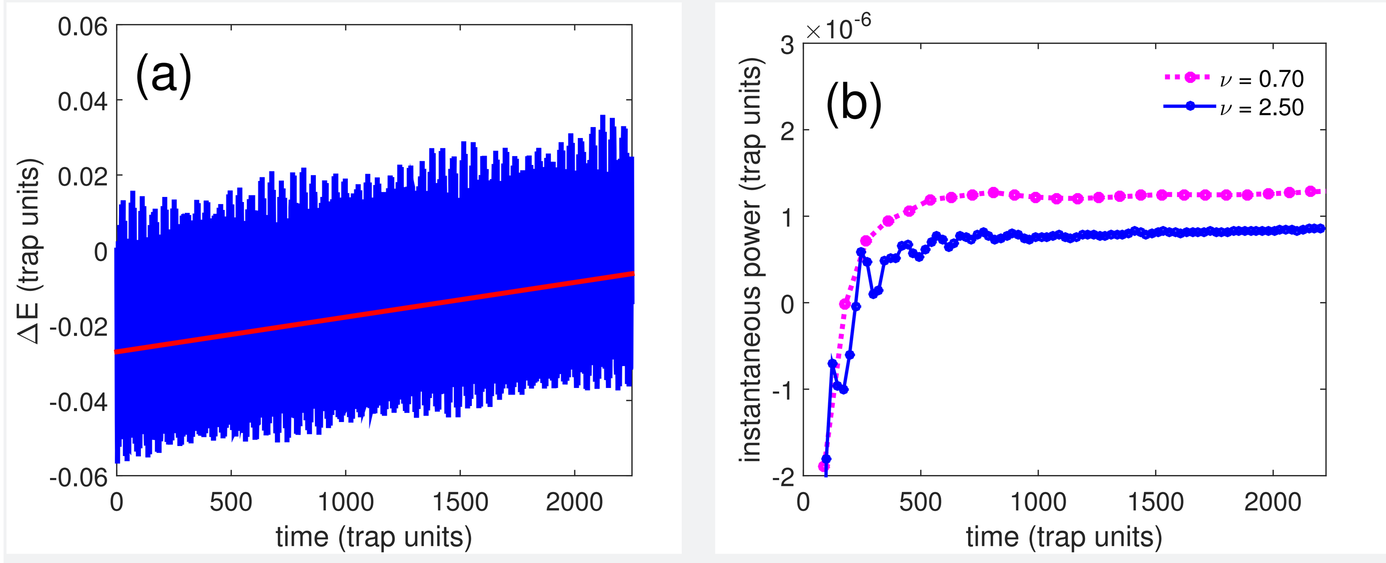


Broader class of periodically driven systems

- **Setup**
- Consider an experimental setting with:

$$V_B = \frac{M\omega_B^2 x^2}{2},\quad g_B(t) = g_A\frac{\omega_B}{\sqrt{\omega_B^2\cos^2(\frac{1}{2}\omega t) + \omega_A^2\sin^2(\frac{1}{2}\omega t)}}. \quad (5)$$

- Evolve the system at constant g_A over many time periods $T = 2\pi/\omega$ and run it for different strengths g_A and frequencies ω from $0.1\omega_B$ to $10\omega_B$.
- **Expectations:** $\omega = 2\omega_B$ (no heating), $\omega \neq 2\omega_B$ (heating)
- **Numerical estimation of the heating rate**



Left: energy growth ΔE along with the fitting model (red line) $\Delta E \to \mathcal{E} + \beta_\tau t$

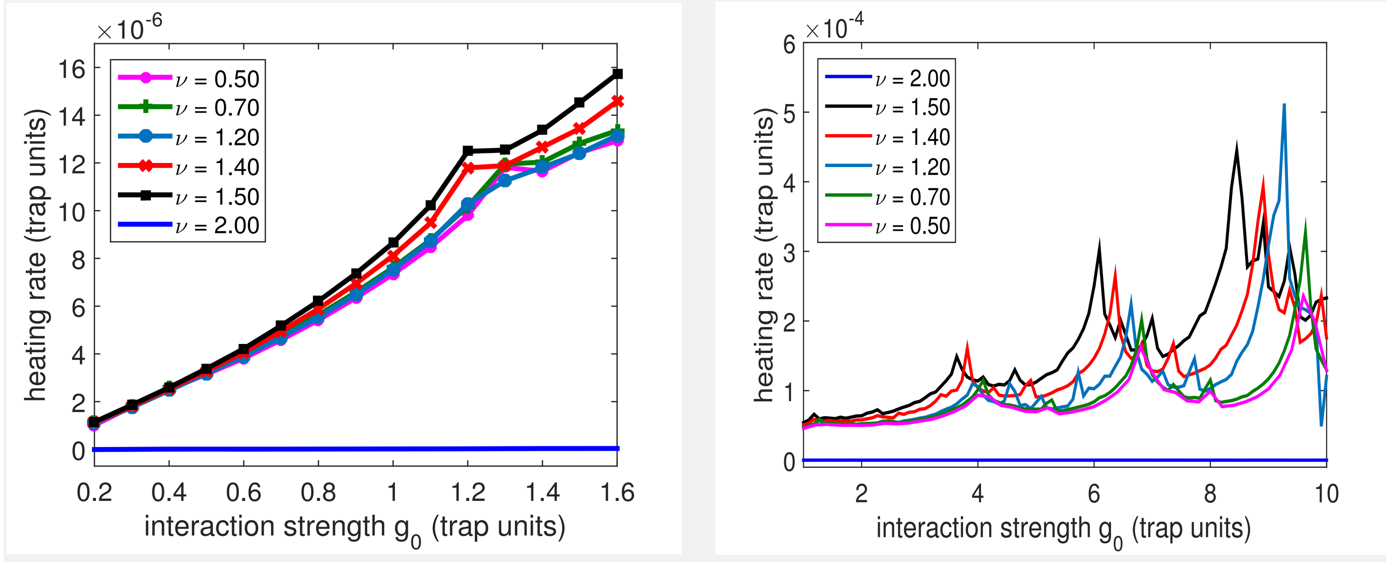
Right: secular growth rate β_τ

The energy growth is $\Delta E = E_{GP}(t) - E_{GP}(0)$ with

$$E_{\mathrm{GP}}(t) = \int dx\left[\frac{\hbar^2}{2M}\left|\frac{\partial\psi}{\partial x}\right|^2 + V(x)|\psi|^2 + \frac{g(t)}{2}|\psi|^4\right]. \quad (6)$$

It is taken at times $t_n = nT$. The **heating rate** is obtained as $\Gamma := \beta_\infty$.

Effect of interaction on the heating

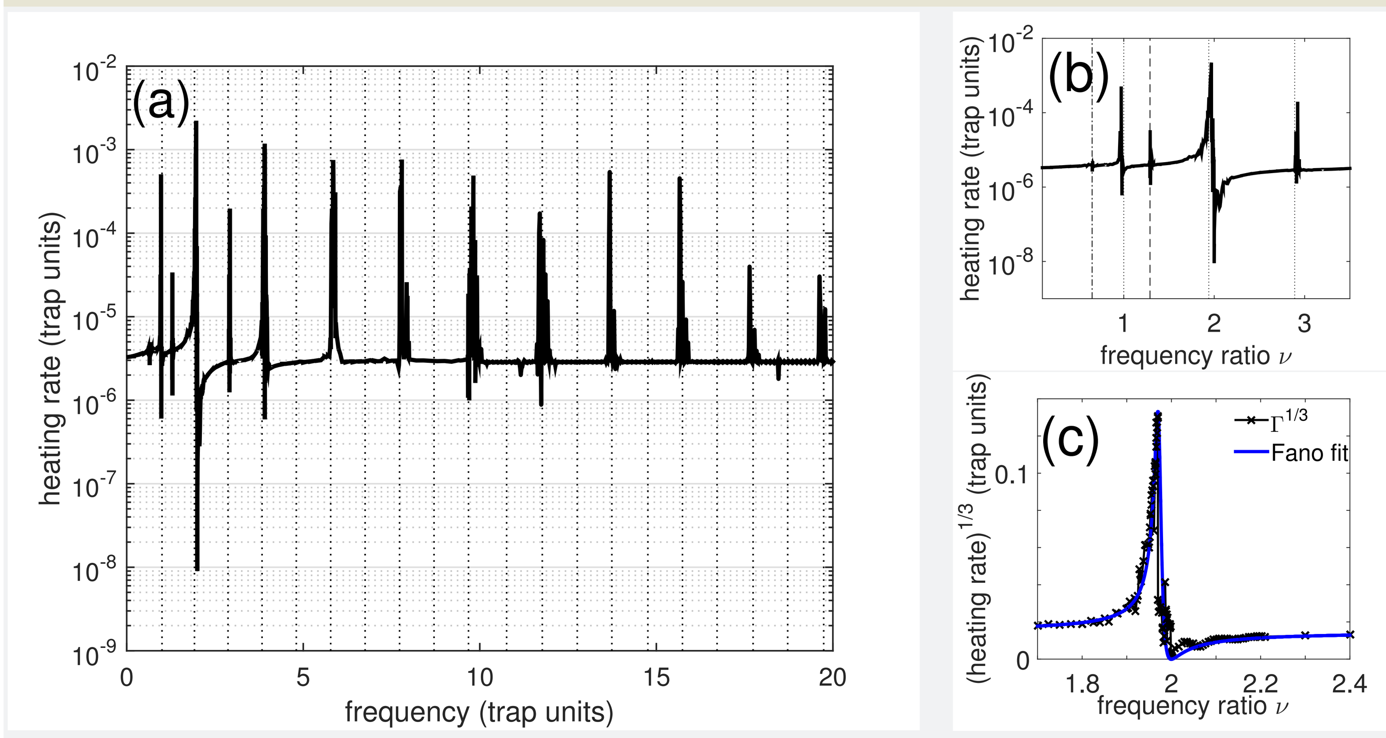


Left: heating (for frequencies up to the heating trough and) for smaller g_A

Right: heating rate for larger interaction strengths

- $\omega = 2\omega_B$ (**no heating**: $\Gamma = 0$ due to the mapping)
- $\omega \neq 2\omega_B$ (**heating increases** with interaction strength g_A)

Effect of frequency driving on the heating



(a) Semilog plot of Γ vs driving frequency $\nu\omega$

(b) Zoom into smaller $\nu\omega$ where subharmonics are excited

(c) Fano resonance peak fitting of $\Gamma^{\frac{1}{3}}$ around the heating zero

- **Heating spikes** at BdG excitation frequencies of the background gas.
- **Heating trough** at $\omega = 2\omega_B$
- **Fano resonance** around the heating trough, with Fano function:

$$\Gamma^{1/3} = 1.55\sigma\frac{(\nu-2)^2}{\sigma^2 + (\nu - 2 - \delta)^2}, \quad (7)$$

where $\sigma = 1/105$ (shape factor) and $\delta = -0.027$ (asymmetry parameter).

References & Acknowledgments

[1] E. Wamba, A. Pelster, and J. R. Anglin, Phys. Rev. A **94**, 043628 (2016).
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