Bose-Einstein Condensates in Compact Astrophysical Objects

Christine Gruber[1] and Axel Pelster[2,3]

[1]Department of Physics, Freie Universität Berlin, Fabeckstraße 60, 14195 Berlin, Germany
[2]Hanse-Wissenschaftskolleg, Lehmkuhlenbusch 4, 27753 Delmenhorst, Germany
[3]Department of Physics, Technische Universität Kaiserslautern, Erwin-Schrödinger-Straße, 67663 Kaiserslautern, Germany

Abstract

We work out in detail two possible applications of the theory of Bose-Einstein condensates (BECs) in astrophysical contexts, one being white dwarfs (WDs) and neutron stars (NSs), due to the formation of Cooper-like pairs of fermions, the other being BECs of dark matter (DM). There is a general consensus that the conditions in these astrophysical environments allow for BECs, and thus models featuring Bose-Einstein condensates are viable candidates to determine the physical properties of these astrophysical objects, see e.g. [1–5]. In the case of dark matter, it is even possible to solve some as yet unexplained phenomena as the observed galactic rotation curves [6–10].

Introduction

The framework of the calculations is that of a system of self-gravitating, uncharged particles subject to both contact interaction and gravitational interaction. The equation of state of the particle ensemble is found from the theory of Bose-Einstein condensation, and then inserted into an appropriate theory of gravity. We can identify three regimes of the physical properties of the system: non-relativistic particles with Newtonian or Einsteinian gravity [11–14], and relativistic particles in a general relativistic framework [15,16]. The three regimes are compared by investigating possible observable quantities as, for instance, the respective bounds for the maximum masses, their density profiles, or the specific heat of the compact objects.

Possible approaches to the description of self-gravitating bosons

Two crucial assumptions have to be done at the beginning of the analysis: whether the particles are assumed to have non-relativistic or relativistic dispersion relations, and whether the gravitational setting is to be Newtonian or general relativistic. Thus a priori we have three different schemes to describe systems of self-gravitating bosons which are distinguished by different initial assumptions:

1. Non-relativistic particles in non-relativistic gravity [11]

Particles with mass m are described by the Gross-Pitaevskii (GP) equation with gravitational and contact interaction,

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + \int d^3r' \left(g\delta(\mathbf{r}-\mathbf{r}') - \frac{G\,m^2}{|\mathbf{r}-\mathbf{r}'|}\right)|\psi(\mathbf{r}',t)|^2\right]\psi(\mathbf{r},t)\,, \tag{1}$$

with the interaction strength $g = 4\pi\hbar^2 a/m$ depending on the s-wave scattering length a, embedded into Newtonian gravity.

2. Non-relativistic particles in General Relativity [11]

Description of particles as before with Schrödinger type equation with contact interaction, but now gravity is formulated within the framework of General Relativity (GR), using a spherically symmetric metric ansatz

$$ds^2 = -e^{-\nu(r)}dt^2 + e^{\mu(r)}dr^2 + r^2 d\Omega^2 \tag{2}$$

and a perfect-fluid energy-momentum tensor $T_{\mu\mu}$, leading to the Tolman-Oppenheimer-Volkoff (TOV) equations to describe the system,

$$\frac{dP(r)}{dr} = -\frac{G\left[\rho + \frac{P(r)}{c^2}\right]\left[\frac{4\pi P(r) r^3}{c^2} + M(r)\right]}{r^2\left[1 - \frac{2GM(r)}{rc^2}\right]}\,, \tag{3}$$

together with the mass conservation condition, $\frac{dM(r)}{dr} = 4\pi\rho r^2$, where $\rho = m^2|\psi|^2$.

3. Relativistic particles in General Relativity [15, 16]

In the case of a relativistic dispersion relation, it is necessary to use the Klein-Gordon equation for bosons,

$$\left(\Box_{\mathrm{LB}} + \frac{m^2c^2}{\hbar^2} - \lambda|\phi|^2\right)\phi = 0\,, \tag{4}$$

derived from the action of a bosonic field ϕ, where $\lambda = \hbar^2 g/2m$. This equation together with the Einstein equations under the assumption of spherical symmetry and the energy-momentum tensor

$$T^{\mu}_{\nu} = \frac{1}{2}g^{\mu\sigma}\left(\phi^*_{;\sigma}\phi_{;\nu} + \phi_{;\sigma}\phi^*_{;\nu}\right) - \frac{1}{2}\delta^{\mu}_{\nu}\left[g^{\kappa\sigma}\phi^*_{;\kappa}\phi_{;\sigma} + m^2|\phi|^2 + \frac{1}{2}\lambda|\phi|^4\right] \tag{5}$$

provide the framework to describe relativistic particles in a general relativistic situation.

A comparison of these three regimes investigated in the literature is achieved by calculating quantities that predict the large-scale behavior of the condensate. In the case of compact objects like white dwarfs or neutron stars it is possible to calculate the Chandrasekhar/TOV-limit, respectively, compared for the three methods in Fig. 1 as a function of the s-wave scattering length of the condensate particles (blue=1, red=2, green=3).

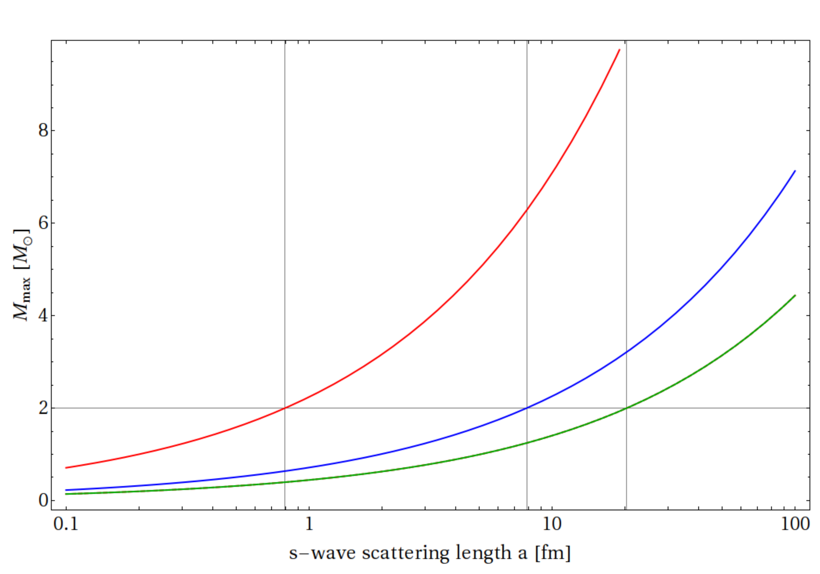

Fig. 1: TOV-limits for different treatments in the case of a neutron star with $m = 2m_n$.

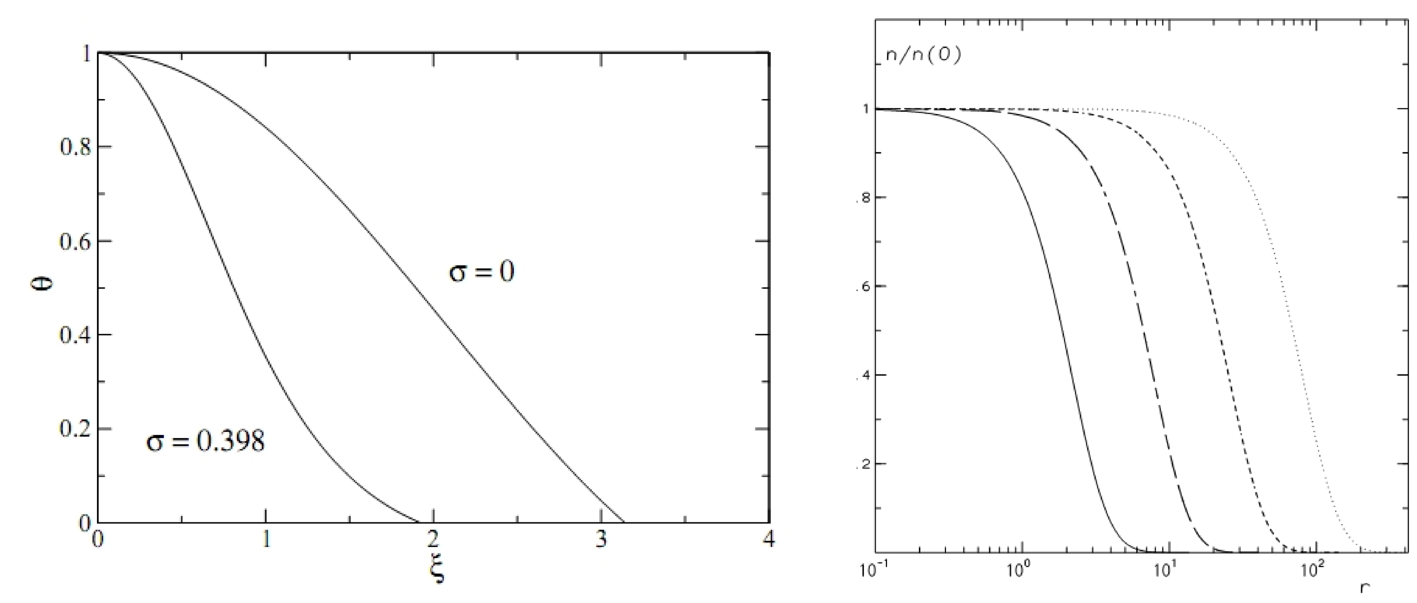

Fig. 2: Density profiles for a neutron star with $\sigma = 2\pi\hbar^2 a\rho_c/m^3c^2$ (left) and a generic Bose star (right).

Fig. 2 shows another prediction obtained from the theories mentioned beforehand - the dependence of the dimensionless density of the Bose-Einstein condensate $\theta = \rho/\rho_c$ as a function of the dimensionless radial coordinate $\xi = r/L$, with $L = \sqrt{\hbar^2 a/Gm^3}$, for the case of a neutron star (left, from method 2, [11]) and a general Bose star (right, from method 3, [16]).

Choosing the right treatment

In order to determine the correct treatment of the problem in question, we have to estimate the typical particle velocities and typical length scales. The question of gravity is determined by the relation of the typical radius of a compact object with its Schwarzschild radius, defined as

$$R_S = \frac{2GM}{c^2} = \frac{2G}{c^2}\cdot\frac{4\pi}{3}R^3\rho_c\,. \tag{6}$$

In the question of particle dispersion, the temperature inside the compact object is of relevance - by identifying the temperature with the kinetic energy of a particle, it is possible to determine the average speed of particles as

$$k_B T = \frac{1}{2}m\langle v\rangle^2 \Rightarrow \langle v\rangle = \sqrt{\frac{2k_BT}{m}}\,. \tag{7}$$

In the table below an overview over the possible astrophysical scenarios is given, and the required treatment identified.

Scenario	$\langle v\rangle$ [m/s]	R_{typ}/R_S	Suggested Framework
Neutron Stars	$2.87\cdot10^5$	2.7	GP + GR
White Dwarfs	$2.03\cdot10^5$	$2.1\cdot10^3$	GP + Newton gravity
DM Halos: lower bound	50	$3.2\cdot10^5$	GP + Newton gravity
DM Halos: upper bound	$5\cdot10^7$	$3.2\cdot10^5$	KG + Newton gravity

We see that the non-relativistic Gross-Pitaevskii equation is appropriate to describe the conditions in the interiors of NSs or WDs, in connection with two different gravitational settings. Due to the unknown nature of DM, a wide range of conditions is possible - here we compare DM particles with very large mass in low temperature environments with very light particles in high temperature settings.

Existing works and what can be done further

There have been done plenty of calculations in the field of Bose-Einstein condensates in astrophysical contexts, in the case of generic boson stars as well as in the case of neutron stars, and all kinds of scenarios have been considered, analytically as well as within the framework of numerical simulations. However, all investigations have been carried out at zero temperature, and neglecting thermal fluctuations around the Bose- Einstein condensated ground state. Our estimations of temperatures and conditions in the astrophysical settings in question have shown that this assumption of zero temperature is in reality not justified, and thus thermal fluctuations could change predictions of measurable quantities, such as shown in the figures, considerably. It is expected that thermal fluctuations would destabilize compact astrophysical objects, resulting in lower TOV-limits or objects with larger spatial extension and lower densities.
As for Bose-Einstein condensed dark matter, there is a lot more to be investigated apart from including fluctuations into the calculations. As can be seen in Fig. 3, taken from [17], the curves for the tangential velocity v_{tg} of a test particle in the halo of a galaxy can be well explained by the theory of BEC matter (solid curve), a feature that generic baryonic matter has failed to explain so far (dashed curve). By fitting BEC-DM models to observational data, it is in principle possible to constrain the mass and properties of the dark matter particles, and one could contribute thus to inferring more about the nature of these so far completely unknown kind of matter.

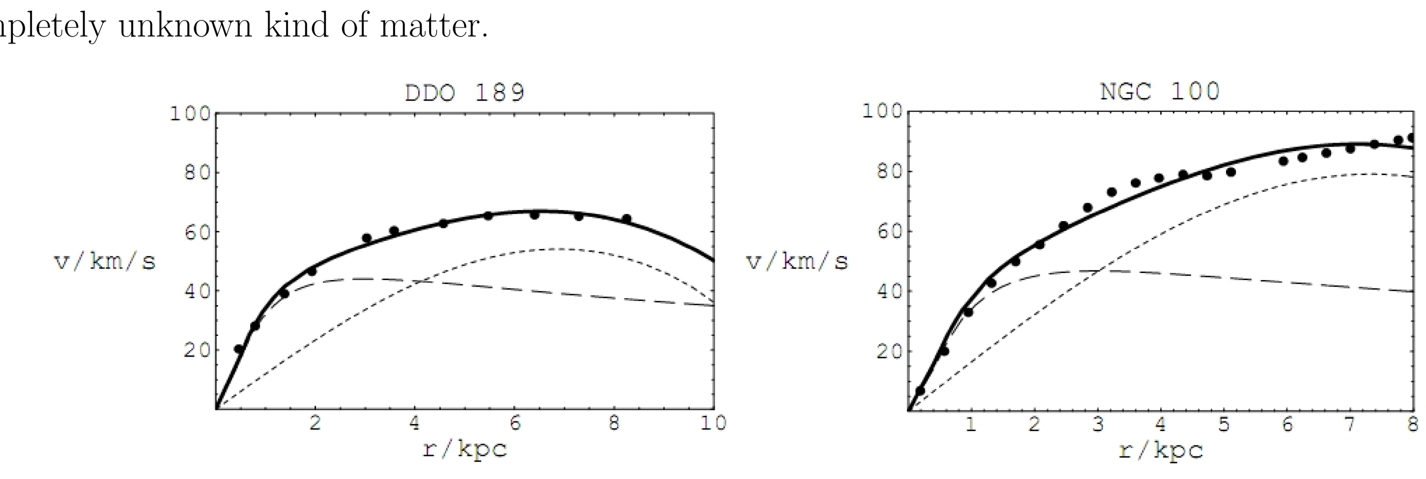

Fig. 3: Rotation curves of DM halos, obtained by theory (solid curve: v_{tg} of test particle in BEC DM background; dotted curve: v_{tg} of BEC particle, dashed curve: v_{tg} due to baryonic matter in galaxy), and confronted with data points [17].

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