Ground State and Floquet Realization of Anyons in a One-Dimensional Optical Lattice

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Abstract

We analyze the fractional statistics of anyons within the realm of a one dimensional (1D) lattice model. To this end we consider the Anyon-Hubbard Hamiltonian, where the hopping dynamics of correlated anyons can be mapped to an occupation-dependent hopping Bose-Hubbard model using the fractional Jordan-Wigner transformation. By calculating the two-point correlation function of either anyonic or bosonic creation and annihilation operators, we investigate the quasi-momentum distributions of anyons and bosons interpolating between Bose-Einstein and Fermi-Dirac statistics. To this end we apply a modified Gutzwiller mean-field approach, which goes beyond a classical one by including the influence of the fractional phase of anyons within the many-body wavefunction. Numerically, we use the density-matrix renormalization group in the language of matrix product states. The results show that shift and asymmetry of the quasi-momentum distribution of bosons strongly depend on the particle number density, whereas for anyons it mainly originates from its own nonlocal string property.
We propose a simple scheme for realizing the physics of 1D anyons with ultracold bosonic atoms in an optical lattice. It relies on lattice-shaking-induced resonant tunneling against the energy offsets created by the combination of both a potential tilt and on-site repulsion. In contrast to former proposals based on internal atomic degrees of freedom, no lasers additional to those already used for the creation of the optical lattice are required.

Anyon-Hubbard Model

- Anyon-Hubbard Model in 1D lattice:

$$\hat{H}^a = -J\sum_{j=1}^{L}(\hat{a}_j^\dagger \hat{a}_{j+1} + \text{h.c.}) + \frac{U}{2}\sum_{j=1}^{L}\hat{n}_j(\hat{n}_j - 1)\,,$$

where the operators $\hat{a}_j^\dagger$ and $\hat{a}_j$ obey the generalized commutation relations [1,2]:

$$\hat{a}_j\hat{a}_k^\dagger - e^{-\mathrm{i}\theta\mathrm{sgn}(j-k)}\hat{a}_k^\dagger\hat{a}_j = \delta_{jk}\,, \quad \hat{a}_j\hat{a}_k - e^{\mathrm{i}\theta\mathrm{sgn}(j-k)}\hat{a}_k\hat{a}_j = 0\,.$$

- Occupation-dependent hopping Bose-Hubbard Model [1]:

$$\hat{H}^b = -J\sum_{j=1}^{L}(\hat{b}_j^\dagger \hat{b}_{j+1}e^{\mathrm{i}\theta\hat{n}_j} + \text{h.c.}) + \frac{U}{2}\sum_{j=1}^{L}\hat{n}_j(\hat{n}_j - 1),$$

by using fractional version of the Jordan-Wigner transformation: $\hat{a}_j = \hat{b}_j \exp\left(\mathrm{i}\theta\sum_{i=1}^{j-1}\hat{n}_i\right)$.

Quasi-Momentum Distribution of Bosons

- Correlation function of bosons:

$$\langle \hat{b}_i^\dagger \hat{b}_j\rangle = \delta_{ij}n_0 + (1-\delta_{ij})[A + Be^{\mathrm{i}(i-j)\theta}]\,,$$

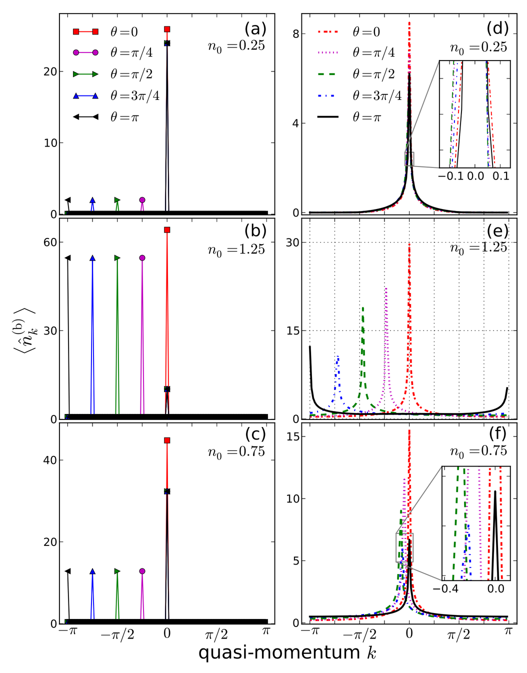
where $A \equiv F_1^2(F_0^2 + \sqrt{2}F_0F_2)$, $B \equiv F_1^2(\sqrt{2}F_0F_2 + 2F_2^2)$.

- Quasi-momentum distribution of bosons:

$$\langle \hat{n}_k^{(\mathrm{b})}\rangle = n_0 - (A+B) + AL\delta_{k,0} + B\frac{1-\cos[(k+\theta)L]}{L[1-\cos(k+\theta)]}\,.$$

- Thermodynamic limit:

$$\langle \hat{n}_k^{(\mathrm{b})}\rangle \xrightarrow{L\to\infty} n_0 - (A+B) + A\delta(k) + B\delta(k+\theta)\,.$$



Quasi-Momentum Distribution of Anyons

- Correlation function of anyons:

$$\langle \hat{a}_i^\dagger \hat{a}_j\rangle \xrightarrow{i<j} [A + Be^{\mathrm{i}(i-j+1)\theta}]\prod_{i<l<j} w\,,$$

$$\langle \hat{a}_i^\dagger \hat{a}_j\rangle \xrightarrow{i>j} [A + Be^{\mathrm{i}(i-j-1)\theta}]\prod_{j<l<i} w^*\,.$$

- Quasi-momentum distribution of anyons:

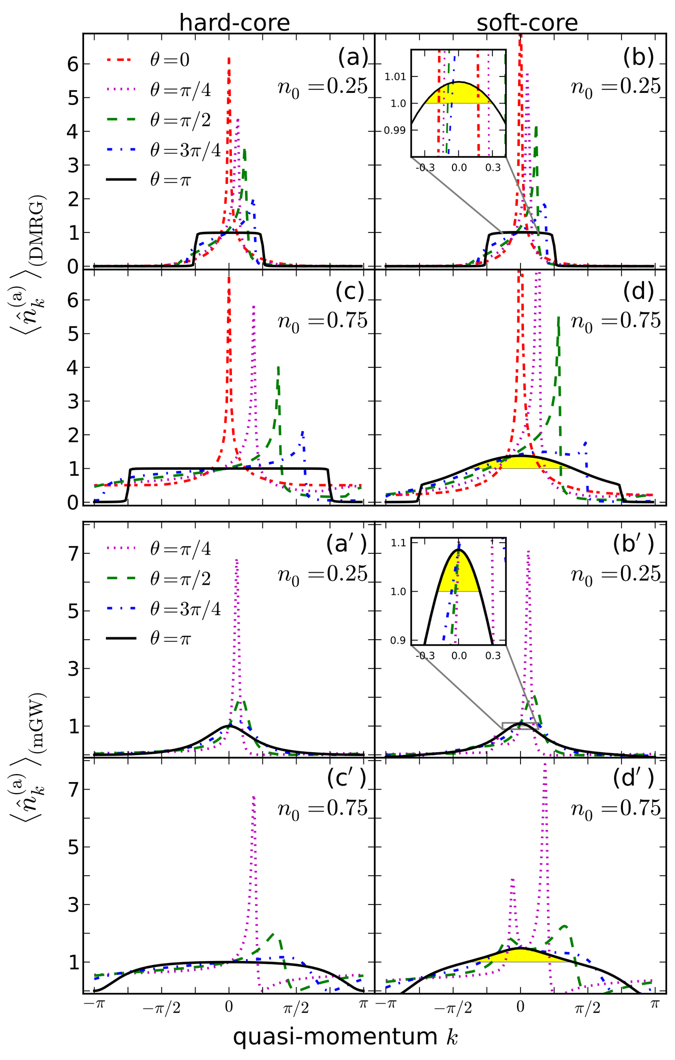
$$\langle \hat{n}_k^{(\mathrm{a})}\rangle = n_0 + \frac{A}{L}\left[e^{-\mathrm{i}k}\frac{(L-1)-Lu+u^L}{(1-u)^2} + \text{c.c.}\right] + \frac{B}{L}\left[e^{-\mathrm{i}k}\frac{(L-1)-Lv+v^L}{(1-v)^2} + \text{c.c.}\right]\,,$$

where $u = we^{-\mathrm{i}k}$ and $v = we^{-\mathrm{i}(k+\theta)}$.

- Thermodynamic limit:

$$\langle \hat{n}_k^{(\mathrm{a})}\rangle \xrightarrow[|w|<1]{L\to\infty} n_0 + A\frac{2\cos k - 2W\cos\chi}{1 - 2W\cos(k-\chi) + W^2} + B\frac{2\cos k - 2W\cos(\theta-\chi)}{1 - 2W\cos[(k+\theta)-\chi] + W^2}\,,$$

where $W = |w|$ and $\chi = \arg(w)$.



Hard-Core Limit

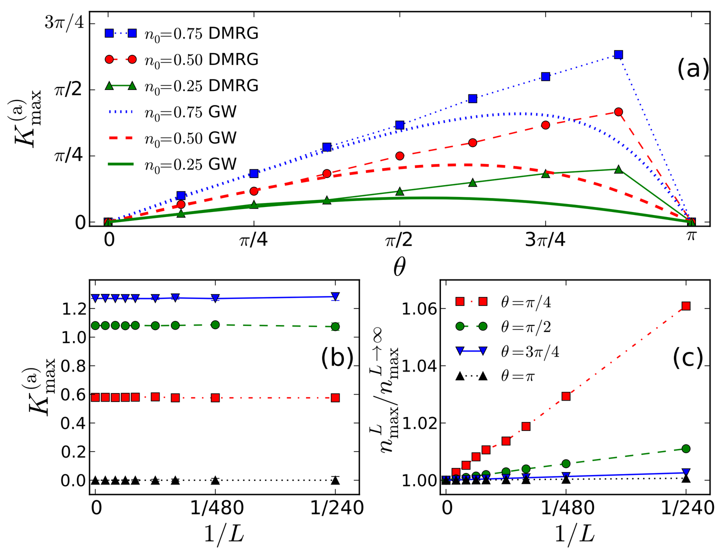
- Quasi-momentum distribution has maximum:

$$K_{\max}^{(\mathrm{a})} = \begin{cases} \pi - \mathcal{K},\ n_0 \geq 2(\sqrt{2}-1) \text{ and } \vartheta_1 \leq \theta \leq \vartheta_2, \\ \mathcal{K}, \text{ Otherwise,} \end{cases}$$

where we have introduced the abbreviations

$$\vartheta_1 = \frac{\pi}{4} + \arcsin\Big(\frac{2-n_0}{\sqrt{2}n_0}\Big), \quad \vartheta_2 = \frac{3\pi}{2} - \vartheta_1\,,$$

$$\mathcal{K} = \arcsin\frac{\sin\theta(2n_0 - n_0^2) + n_0^2\sin\theta\cos\theta}{(1-\cos\theta)(1-n_0)^2 + (1+\cos\theta)}\,.$$



Floquet Realization of 1D Anyons

- Raman-assisted scheme [1]
- Raman-assisted scheme without the need of Feshbach resonances [3]
- Optical lattice

$$\hat{H}(t) = \sum_l\left\{-J'\left[\hat{b}_l^\dagger\hat{b}_{l-1} + \text{h.c.}\right] + \frac{U'}{2}\hat{n}_l(\hat{n}_l - 1) + V_l\hat{n}_l + [\Delta + F(t)]l\hat{n}_l\right\}$$

will static tilt and time-periodic inertial force

$$F(t) = F(t+T) = -\hbar\dot{\chi}(t)\,.$$

- Resonance and high-frequency conditions for "photon" assisted tunneling

$$\Delta = \hbar\omega, \quad U' = 2\hbar\omega + U\,,$$

$$J', \delta, U, |V_l - V_{l-1}| \ll \hbar\omega\,.$$

- Dynamics is described by time-independent effective Hamiltonian:

$$\hat{H}_{\text{eff}} = -\sum_l\left[\hat{b}_l^\dagger\hat{b}_{l-1}J_{\text{eff}}(\hat{N}_{l,l-1}) + \text{h.c.}\right] + \sum_l\left[\frac{U}{2}\hat{n}_l(\hat{n}_l - 1) + V_l\hat{n}_l\right]$$

with number-dependent complex effective tunneling matrix element for N-"photon" process

$$J_{\text{eff}}(N) = \frac{J'}{T}\int_0^T \mathrm{d}t\exp\Big(\mathrm{i}\omega tN - \mathrm{i}\chi(t)\Big)\,.$$

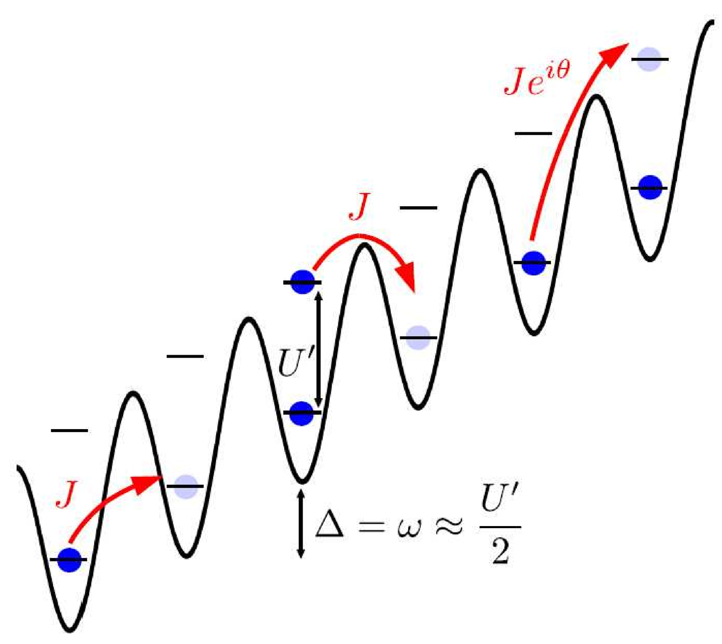
- Effective tunneling matrix elements in the low density limit:

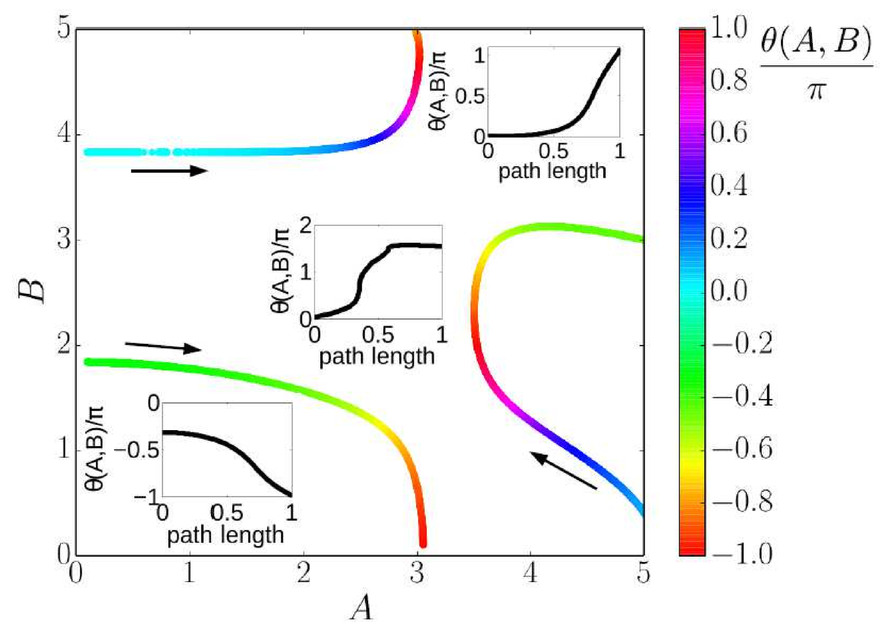
$$J_{\text{eff}}(1) = J_{\text{eff}}(-1) = Je^{\mathrm{i}\varphi_0}\,,$$

$$J_{\text{eff}}(3) = Je^{\mathrm{i}(\theta+\varphi_0)}\,,$$

which are achieved by the driving function

$$\chi(t) = A\cos(\omega t) + B\cos(2\omega t)\,.$$





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