Evolution of the BEC in Gravitational Cavity: Comparing Soft and Hard-Wall Boundary Condition

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Motivation

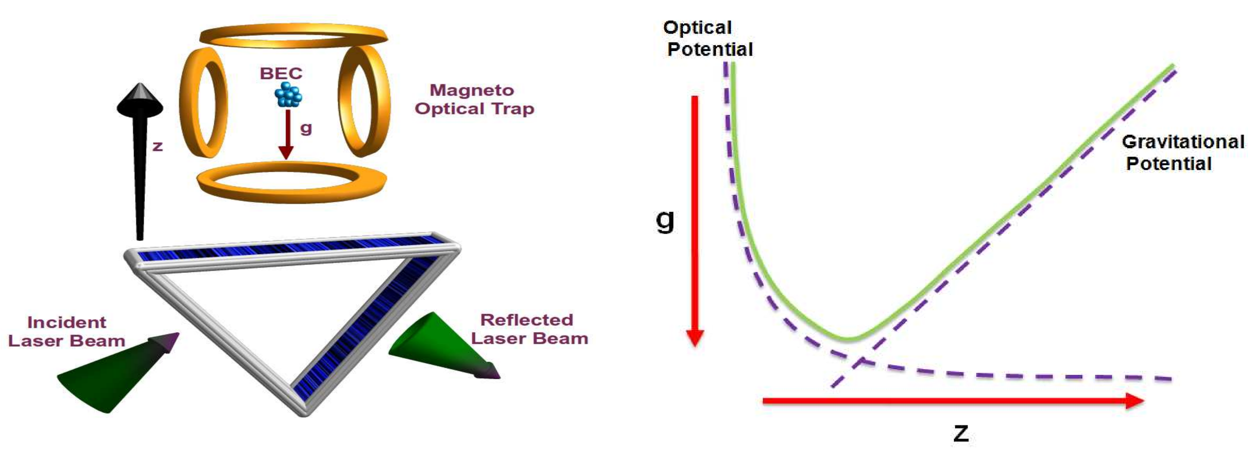
Within a one-dimensional gravitational cavity [1–4] the effect of gravity is compensated by an exponentially decaying potential, which is created by the total internal reflection of an incident laser beam from the surface of a dielectric serving as a mirror for the atoms. We describe a weakly interacting Bose-Einstein condensates (BEC) in such a one-dimensional gravitational cavity with modified Gaussian trial wave functions, where both its width and its height are considered as variational parameters. For larger interaction strengths, we model our system with modified Thomas-Fermi ansatz. In particular, we determine the variational results for the BEC equilibrium configuration when the surface is modeled by a soft or a hard wall boundary condition and compare our theoretical findings with numerics. Furthermore, we analyze how the BEC cloud expands ballistically due to gravity after switching off the evanescent laser field.

Gross-Pitaevskii (GP) Equation

- The dynamics of a one dimensional BEC at zero temperature is determined by the time dependent GP Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(z,t)=\left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+V(z)+G\|\Psi(z,t)\|^2\right\}\Psi(z,t)$$

- The last term represents the two-particle interaction of BEC atoms, where its strength $G=2Na\hbar\omega_r$ is related to the s-wave scattering length a and potential energy is $V(z)=V_0e^{-\kappa z}+mgz$.



Dimensionless parameters

- Dimensionless 1D GP Equation

$$i\frac{\partial}{\partial\tau}\tilde{\Psi}(\tilde{z},\tau)=\left\{-\frac{\tilde{k}}{2}\frac{\partial^2}{\partial\tilde{z}^2}+\tilde{z}+\tilde{V}_0e^{-\tilde{z}}+\tilde{G}\|\tilde{\Psi}(\tilde{z},\tau)\|^2\right\}\Psi(\tilde{z},\tau)$$

- The number of Cs atoms $N=10^6$, and the s-wave scattering length is $a=440\ a_0$ here a_0 is Bohr radius .
- The inverse decay length amounts to $\kappa=6.67\times10^6\ \mathrm{m}^{-1}$ as the EW is produced by a far-detuned laser with wavelength $\lambda=852$ nm and the axial harmonic frequency amounts to $\omega_z=2\pi\times1.2$ kHz
- Dimensionless time $\tau=\omega t$, $\tilde{A}(\tau)=\kappa A(t)$ as a dimensionless width of the BEC, $\tilde{z}_0(\tau)=\kappa z_0(t)$ as a dimensionless mean height of the BEC from the optical mirror. We measure energies in units of the gravitational energy $mg\kappa$ and get $\tilde{\omega}=\hbar\kappa/gm\omega$ as a dimensionless frequency, $\tilde{V}_0=\kappa V_0/gm$ as a dimensionless strength of the evanescent field, and $\tilde{k}=\hbar^2\kappa^3/gm^2$ as a dimensionless kinetic energy. The dimensionless two-particle interaction then turns out to be $\tilde{G}=N\tilde{\omega}_r\tilde{a}$ with $\tilde{a}=a\kappa$ being a dimensionless S-wave scattering length.
  - dimensionless s-wave scattering length is given by $\tilde{a}=0.15$
  - The dimensionless optical decaying strength is $\tilde{V}_0=4.07\times10^7$,
  - The dimensionless kinetic energy amounts to $\tilde{k}=6.83$,
  - The dimensionless radial frequency amounts to be $\tilde{\omega}_r=24.377$,
  - The resulting dimensionless two-particle interaction is $\tilde{G}=3.7N$.
  - For simplicity, we will drop the tilde

Modified Gaussian trial function

- We consider the one-dimensional modified Gaussian trial function

$$\psi(z)=\frac{\exp\left(-\frac{z^2}{2A^2}\right)\sinh\left(\frac{\gamma z}{A}\right)}{\sqrt{\frac{1}{4}\sqrt{\pi}A\left(e^{\gamma^2}-1\right)}}\propto\exp\left(\frac{-(z-z_0)^2}{2A^2}\right)-\exp\left(\frac{-(z+z_0)^2}{2A^2}\right)$$

- Which respects the hard-wall boundary condition due to the mirror principle [5]
- Here A and γ are variational parameters, $\gamma\approx z_0/A$ depends on the mean position of the BEC, here A is the width of the BEC.
- Increasing V_0 yields better variational results

Comparing analytical and numerical results

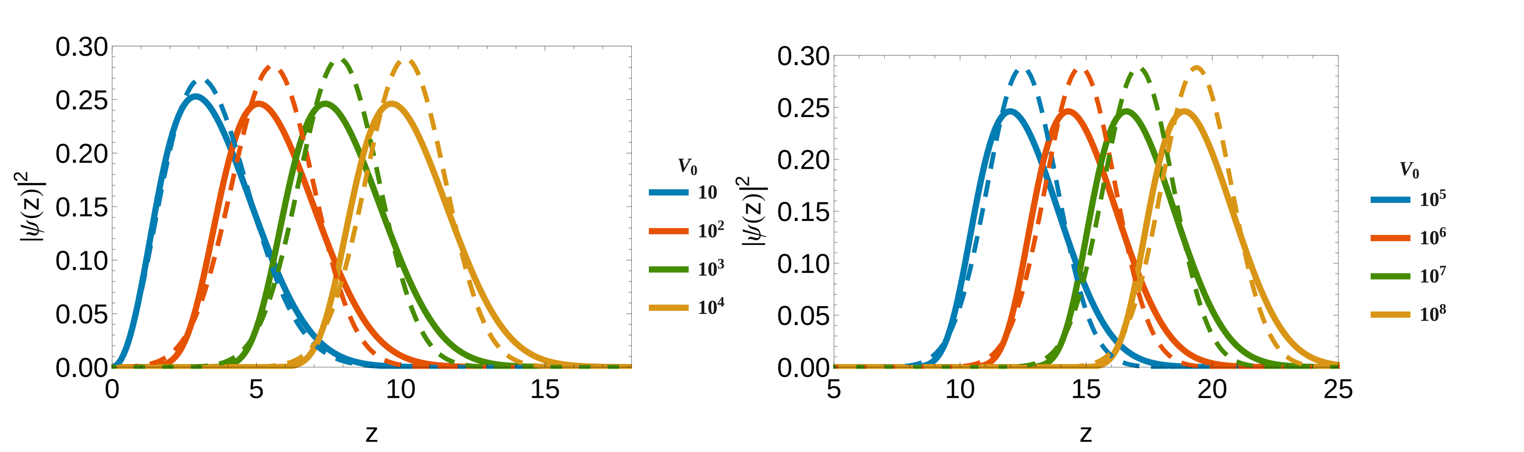


Fig. 1: For $G=10$, here we plot numerical and theoretical results for the density profile of the BEC for different values of V_0. Here solid lines represents numerical and dashed lines describes modified Gaussian ansatz.

Thomas-Fermi regime

- For larger value of G, we model the BEC with modified Thomas-Fermi ansatz

$$\psi(z)=\sqrt{\left(\frac{\mu}{G}\right)\left(1-\frac{z}{\mu}-\frac{V_0}{\mu}e^{-z}\right)}-\sqrt{\left(\frac{\mu}{G}\right)\left(1+\frac{z}{\mu}-\frac{V_0}{\mu}e^{z}\right)}$$

- where $\mu=1+\log(V_0/\sqrt{2G})+\sqrt{2G}$.
- The function is set to zero for $z<(1/2)\|\log(V_0/\sqrt{2G})\|+(1/2)\log(V_0/\sqrt{2G})$ and $z>\mu$.
- From soft-wall to hard-wall: comparison of theoretical and numerical density profiles

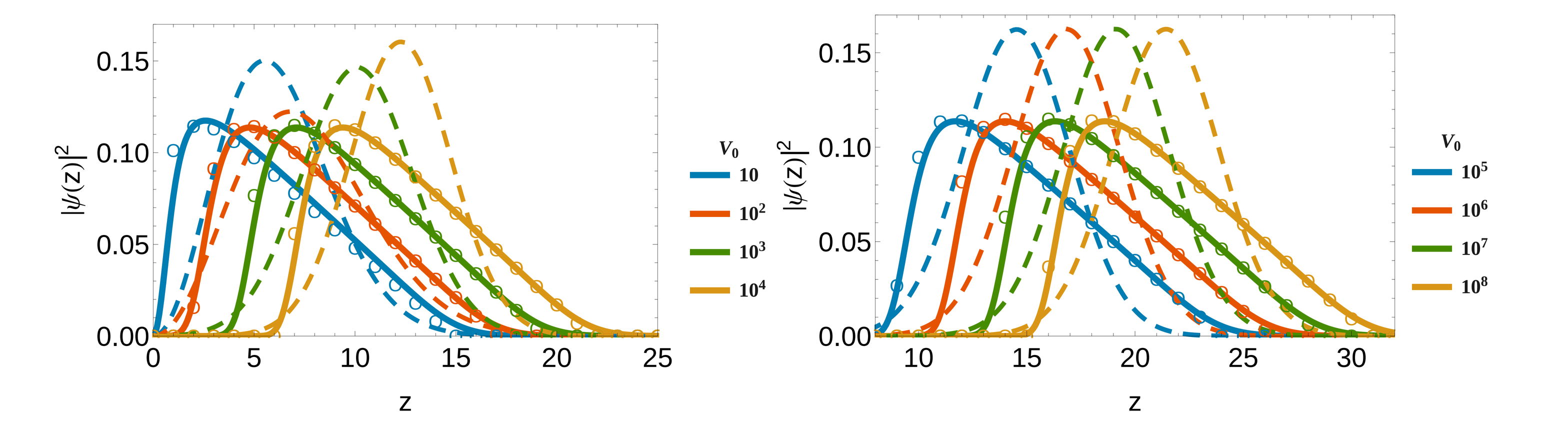


Fig. 2: Here solid lines represents numerical, dashes lines describes the modified Gaussian ansatz and cilrcles stands for modified Thomas-Fermi ansatz for the $G=100$.

- For higher G values, we only use the Thomas-Fermi ansatz to describe our system theoretically.

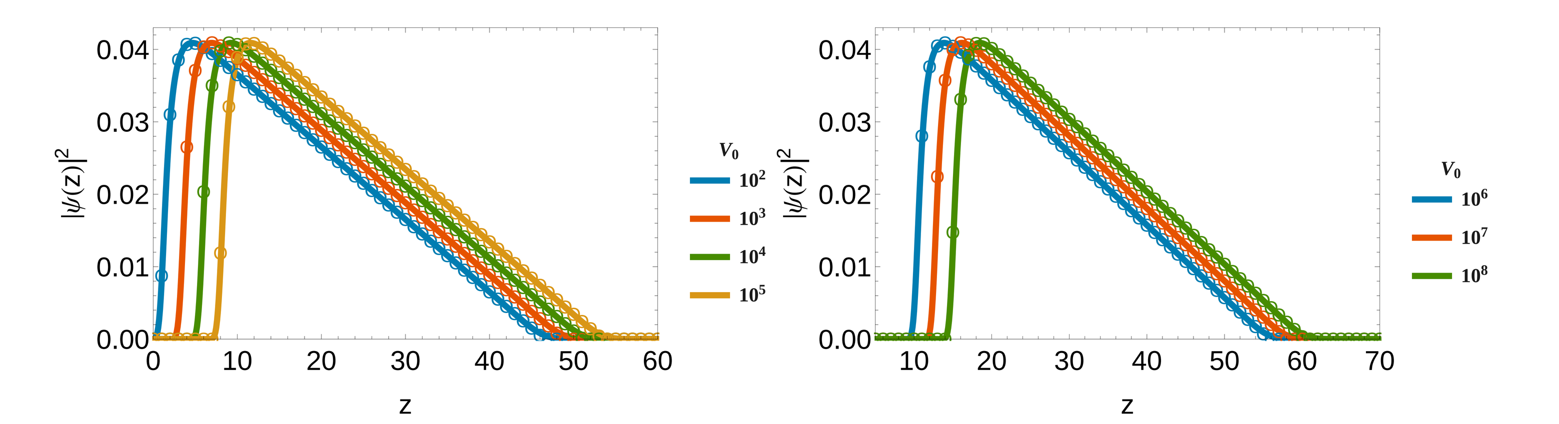


Fig. 3: In these graphs, we compare the BEC density profile for numerical and Thomas-Fermi ansatz for $G=10^3$.

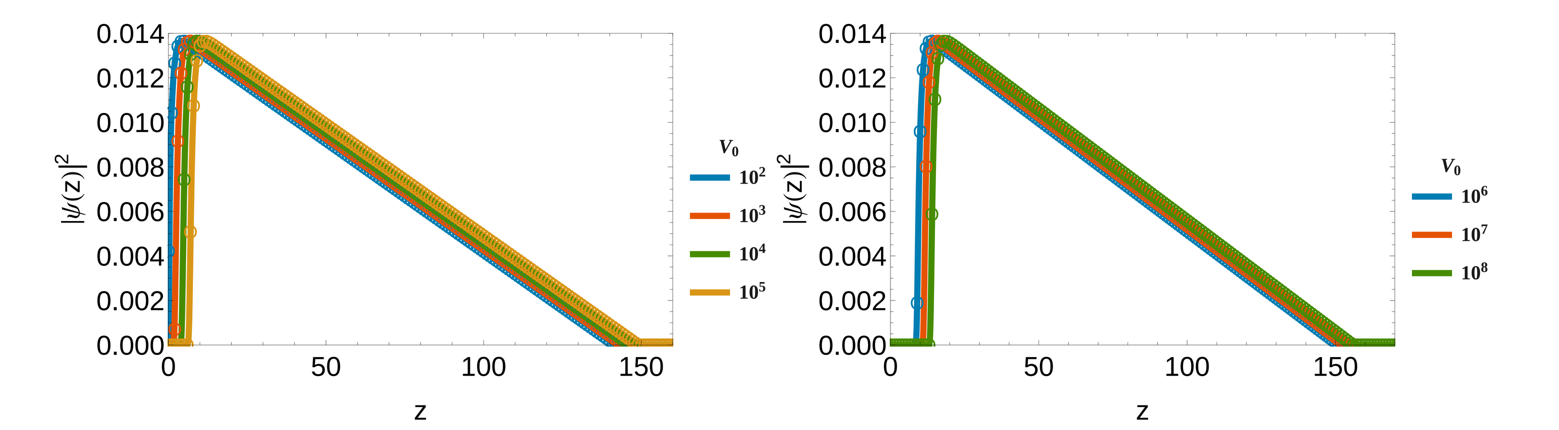


Fig. 4: In these graphs, we compare the BEC density profile for numerical and Thomas-Fermi ansatz for $G=10^4$.

Time-of-flight expansion

- We determine time of flight of the BEC in GOST trap by putting $V_0=0$

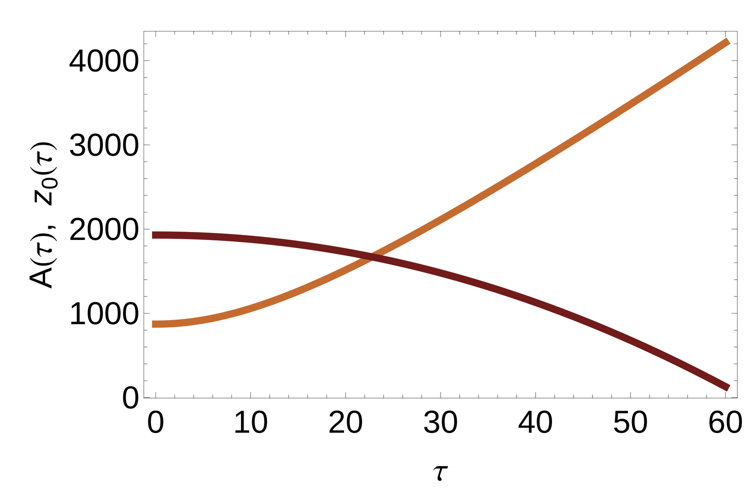


Fig. 5: Time evolution of BEC in gravitational cavity with dimensionless time τ for an inverse decay length $\kappa=6.67\times10^6\ \mathrm{m}^{-1}$.

References and Acknowledgment

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[6] We gratefully acknowledge support from the German Academic Exchange Service (DAAD).