Non-Equilibrium Phase Transition of Ultracold Bosons in an Optical Lattice Coupled to a BEC-Reservoir

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Introduction & Motivation

- Dynamics of driven, open quantum system governed by master equation [1]:

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\hat{\varrho}(t) = \left[\hat{H}, \hat{\varrho}(t)\right] + i\hbar\mathcal{L}\,\hat{\varrho}(t)$$

Here, Hamiltonian of Bose-Hubbard type [2]:

$$\hat{H} = -J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\,\hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

$$\hat{n}_i = \hat{a}_i^\dagger\,\hat{a}_i$$

and dissipator in Lindblad-form:

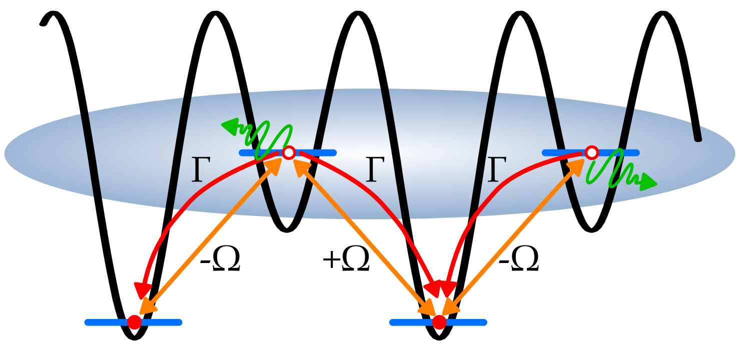
$$\mathcal{L}\,\hat{\varrho} = \kappa\sum_{<i,j>}\left(2\,\hat{c}_{ij}\,\hat{\varrho}\,\hat{c}_{ij}^\dagger - \hat{c}_{ij}^\dagger\,\hat{c}_{ij}\,\hat{\varrho} - \hat{\varrho}\,\hat{c}_{ij}^\dagger\,\hat{c}_{ij}\right)$$

with quantum jump operators [3]:

$$\hat{c}_{ij} = \left(\hat{a}_i^\dagger + \hat{a}_j^\dagger\right)\left(\hat{a}_i - \hat{a}_j\right)$$

bosonic creation/annihilation operator at lattice site i

- Physical realisation [3]:
  - optical lattice
  - BEC-reservoir
  - Raman laser
  - upper and lower band
  - Bogoliubov excitation
  - dissipative decay



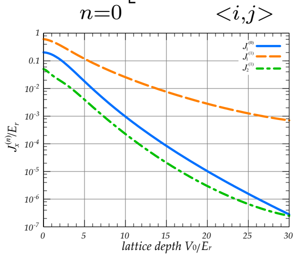
The Model

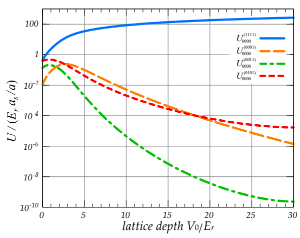
- Cubic lattice with sublattice:

$$V_{\text{lat}}(x) = V_0\sum_{i=1}^{3}\sin^2(k_0\,x_i) + \frac{1}{4}V_1\sum_{i=1}^{3}\sin^2(2k_0\,x_i)$$

- Two-band Bose-Hubbard-model:

$$\hat{H} = \sum_{n=0}^{1}\left[-J^{(n)}\sum_{<i,j>}\hat{a}_i^{(n)\dagger}\,\hat{a}_j^{(n)} + \frac{1}{2}U^{(n)}\sum_i \hat{n}_i^{(n)}\left(\hat{n}_i - 1\right)\right]$$





- Raman laser: $\hat{H}_{\text{AL}} \sim \sum_i (-1)^i\hat{a}_i^{(1)\dagger}\left(\hat{a}_i^{(0)} - \hat{a}_{i+1}^{(0)}\right) + \text{h.c.}$
- Coupling of BEC-reservoir to lattice via density-density interaction in Bogoliubov approximation:

$$\hat{H}_{ab} = \sum_k\sum_{<i,j>}\mathcal{G}_{i,j;k}^{(1,0)}\,\hat{a}_i^{(1)\dagger}\,\hat{a}_j^{(0)}\,\hat{B}_k + \text{h.c.},$$

with matrix element: Wannier function

$$\mathcal{G}_{i,j;k}^{(1,0)} = \mathcal{G}_0\int w^{(1)*}\left(x - x_i^{(1)}\right)w^{(0)}\left(x - x_j^{(0)}\right)\mathrm{e}^{ikx}\mathrm{d}x$$

operator $\hat{B}_k^\dagger$ creating a Bogoliubov excitation with momentum k.

- Derivation of master equation via Born-Markov and rotating wave approximation [1]
- Nonzero coupling κ for 1D lattice only
- Adiabatic elimination of upper band due to dissipation

[1] Breuer & Petruccione: *The theory of open quantum systems*, Clarendon Press, 2007
[2] Fisher, Weichman, Grinstein, Fisher, PRB **40** (1) 546 (1989)
[3] Diehl, Micheli, Kantian, Kraus, Büchler, Zoller, Nature **4** (11) 878 (2008)
[4] Diehl, Tomadin, Micheli, Fazio, Zoller, PRL **105** (1) 015702 (2010)

Nonequilibrium QPT

- Consider dynamics of moments $\mathcal{M}_{\alpha,\beta} = \text{Tr}\left\{\hat{a}^{\dagger\alpha}\,\hat{a}^\beta\,\hat{\varrho}\right\}$ using master equation in mean-field approximation
- Consideration of stationary states introduces new parameter $\tilde{\mu}$

$$\begin{aligned}0 = &-i\tilde{U}(\beta-\alpha)\mathcal{M}_{\alpha+1,\beta+1} - 2\alpha\,\Psi\,\mathcal{M}_{\alpha+1,\beta} - 2\beta\,\Psi\,\mathcal{M}_{\alpha,\beta+1}\\ &-\Big[\frac{1}{2}i\tilde{U}\big(\beta(\beta-1) - \alpha(\alpha-1)\big) - i\tilde{\mu}(\beta-\alpha) + \alpha + \beta\\ &+(\alpha-\beta)^2\Big]\mathcal{M}_{\alpha,\beta} + 2\alpha\beta\mathcal{M}_{1,1}\,\mathcal{M}_{\alpha-1,\beta-1}\\ &+2\alpha\left[\mathcal{M}_{2,1} - \left(\beta - 1 + \frac{i}{2}\tilde{J}\right)\Psi\right]\mathcal{M}_{\alpha-1,\beta}\\ &+2\beta\left[\mathcal{M}_{1,2} - \left(\alpha - 1 - \frac{i}{2}\tilde{J}\right)\Psi\right]\mathcal{M}_{\alpha,\beta-1}\\ &+\alpha(\alpha-1)\mathcal{M}_{2,0}\,\mathcal{M}_{\alpha-2,\beta} + \beta(\beta-1)\mathcal{M}_{0,2}\,\mathcal{M}_{\alpha,\beta-2}\end{aligned}$$

$$\tilde{U} = U/\hbar\kappa z \qquad \tilde{J} = J/\hbar\kappa$$

with real order parameter $\Psi = \mathcal{M}_{0,1} = \text{Tr}\left\{\hat{a}\,\hat{\varrho}\right\}$

- Limit $\kappa \to 0$ can exactly be solved for phase boundary and yields Mott-Superfluid phase-transition [2]
- Moments can be expanded in a series in Ψ and $n = \mathcal{M}_{1,1}$: particle density

$$\mathcal{M}_{\alpha,\beta} = \Psi^{|\beta-\alpha|}n^{\min(\alpha,\beta)}\sum_{\ell,k=0}^{\infty}\Psi^{2\ell}n^k M_{\alpha,\beta}^{(\ell,k)}$$

- Approximate solution for Ψ near phase boundary and small n
- Fix $\tilde{\mu}$ to get a real Ψ^2:

$$\Rightarrow \tilde{\mu} = \tilde{U}\,n \quad (\tilde{J} = 0)$$

$$\Psi^2 \approx \left[1 - \left(\tilde{U}/\tilde{U}_c\right)^2\right]n$$

- On phase boundary, i.e. $\Psi \to 0$:

$$\tilde{U} = \tilde{U}_c = 2\sqrt{2}$$

- These results are in agreement with [4]