Dipolar Fermi Gases in the Hydrodynamic Regime

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Abstract

Recent experimental progress in trapping and cooling heteronuclear molecules down to their rovibrational ground state [1–5] has brought fermionic systems interacting through the long-range and anisotropic electric dipole-dipole interaction within experimental reach. In order to describe such a polarized dipolar Fermi gas in the hydrodynamic regime, where collisions assure local equilibrium, we work out a variational time-dependent Hartree-Fock approach. We then determine statical as well as dynamical properties of such a system as, for instance, the equilibrium aspect ratios, the frequencies of the low-lying excitations and the time-of-flight expansion.

Time-dependent variational approach [6,7]

- **One-particle (1-p) Hamiltonian**

$$H=\sum_{i=1}^{N}\left[-\frac{\hbar^2\triangle_i}{2M}+U_{\rm trap}(\mathbf{x}_i)\right]+\frac{1}{2}\sum_{i\neq j}^{N}V_{\rm int}(\mathbf{x}_i-\mathbf{x}_j)$$

- **Trap and interaction potentials**

$$U_{\rm trap}(\mathbf{x})=\frac{M}{2}\sum_i\omega_i^2x_i^2;\quad V_{\rm dd}(\mathbf{x})=\frac{C_{\rm dd}}{4\pi|\mathbf{x}|^3}\left[1-3\frac{z^2}{|\mathbf{x}|^2}\right]$$

- **Action**

$$A=\int_{t_1}^{t_2}{\rm d}t\langle\Psi|i\hbar\frac{\partial}{\partial t}-H|\Psi\rangle \text{ Slater determinant}$$

Factorization of 1-p orbitals [8] $\Rightarrow$ Time-even Slater determinant

$$\psi_i(x,t)=e^{iM\chi(x,t)/\hbar}|\psi_i(x,t)|\Rightarrow\Psi_0(x_1,\cdots,x_N,t)={\rm SD}\left[|\psi(x,t)|\right]$$

- **Time-even 1-p density matrix**

$$\rho_0(x,x';t)=\prod_{i=2}^{N}\int{\rm d}^3x_i\Psi_0^*(x',x_2,\cdots,x_N,t)\Psi_0(x,x_2,\cdots,x_N,t)$$

- **Action becomes**

$$A=\int_{t_1}^{t_2}{\rm d}t\left\{\int{\rm d}^3x\left[-\hbar\dot{\chi}(x,t)\rho_0(x;t)-\frac{\hbar^2}{2M}\rho_0(x;t)\left(\nabla\chi(x,t)\right)^2\right]-\langle\Psi_0|H|\Psi_0\rangle\right\}$$

- **Wigner transformation**

$$\nu_0(\mathbf{x},\mathbf{k};t)=\int{\rm d}^3s\,\rho_0\left(\mathbf{x}+\frac{\mathbf{s}}{2},\mathbf{x}-\frac{\mathbf{s}}{2};t\right)e^{-i\mathbf{k}\cdot\mathbf{s}}$$

- **Mean-field energy contributions**

$$E_{\rm trap}=\int\frac{{\rm d}^3x{\rm d}^3k}{(2\pi)^3}\nu_0(\mathbf{x},\mathbf{k};t);\quad U_{\rm trap}(\mathbf{x})=\int\frac{{\rm d}^3x{\rm d}^3k}{(2\pi)^3}\nu_0(\mathbf{x},\mathbf{k};t)\frac{\hbar^2\mathbf{k}^2}{2M}$$

$$E_{\rm dd}^{\rm Dir}=\frac{1}{2}\int\frac{{\rm d}^3x{\rm d}^3k}{(2\pi)^3}\frac{{\rm d}^3x'{\rm d}^3k'}{(2\pi)^3}\nu_0(\mathbf{x},\mathbf{k};t)\,V_{\rm dd}(\mathbf{x}-\mathbf{x}')\,\nu_0(\mathbf{x}',\mathbf{k}';t)$$

$$E_{\rm dd}^{\rm Ex}=-\frac{1}{2}\int\frac{{\rm d}^3R{\rm d}^3k}{(2\pi)^3}\frac{{\rm d}^3s{\rm d}^3k'}{(2\pi\hbar)^3}\nu_0(\mathbf{R},\mathbf{k};t)\,V_{\rm dd}(\mathbf{s})\,\nu_0(\mathbf{R},\mathbf{k}';t)\,e^{i\mathbf{s}\cdot(\mathbf{k}-\mathbf{k}')}$$

- **Variational ansatz**

$$\chi(x,t)=\frac{1}{2}\sum_i\alpha_i(t)x_i^2;\quad\nu_0(\mathbf{x},\mathbf{k};t)=\Theta\left(1-\sum_i\frac{x_i^2}{R_i(t)^2}-\sum_i\frac{k_i^2}{K_i(t)^2}\right)$$

- **Equations of motion ($\tilde{\bullet}$ represents $\bullet$ in units of the free gas)**

$$\frac{1}{\omega_i^2}\frac{d^2\tilde{R}_i}{dt^2}=-\tilde{R}_i+\sum_j\frac{\tilde{K}_j^2}{3\tilde{R}_i}-\epsilon_{\rm dd}Q_i\left(\tilde{\mathbf{R}},\tilde{K}_x,\tilde{K}_z\right)$$

$$\tilde{K}_z^2-\tilde{K}_x^2=\frac{3c_{\rm d}}{\overline{\tilde{R}}^3}\left[-1+\frac{\left(2\tilde{K}_x^2+\tilde{K}_z^2\right)f_s\left(\tilde{K}_z/\tilde{K}_x\right)}{2\left(\tilde{K}_x^2-\tilde{K}_z^2\right)}\right]$$

with the auxiliary functions

$$Q_x(\mathbf{r},\mathbf{k})=\frac{c_{\rm d}}{x^2yz}\left[f\left(\frac{x}{z},\frac{y}{z}\right)-\frac{x}{z}f_1\left(\frac{x}{z},\frac{y}{z}\right)-f\left(\frac{k_z}{k_x},\frac{k_z}{k_y}\right)\right]$$

$$Q_y(\mathbf{r},\mathbf{k})=\frac{c_{\rm d}}{xy^2z}\left[f\left(\frac{x}{z},\frac{y}{z}\right)-\frac{y}{z}f_2\left(\frac{x}{z},\frac{y}{z}\right)-f\left(\frac{k_z}{k_x},\frac{k_z}{k_x}\right)\right]$$

$$Q_z(\mathbf{r},\mathbf{k})=\frac{c_{\rm d}}{xyz^2}\left[f\left(\frac{x}{z},\frac{y}{z}\right)+\frac{x}{z}f_1\left(\frac{x}{z},\frac{y}{z}\right)+\frac{y}{z}f_2\left(\frac{x}{z},\frac{y}{z}\right)-f\left(\frac{k_z}{k_x},\frac{k_z}{k_x}\right)\right]$$

and

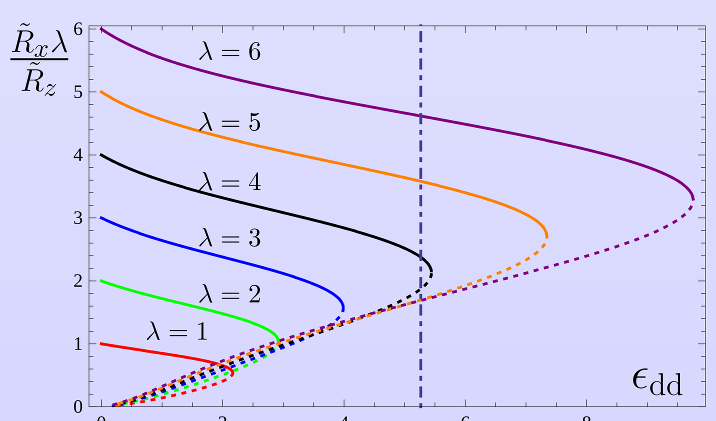
$$f(x,y)=1+3xy\frac{E(\varphi,q)-F(\varphi,q)}{(1-y^2)\sqrt{1-x^2}},\quad\epsilon_{\rm dd}=\frac{C_{\rm dd}}{4\pi}\left(\frac{M^3\overline{\omega}}{\hbar^5}\right)^{\frac{1}{2}}N^{\frac{1}{6}}$$

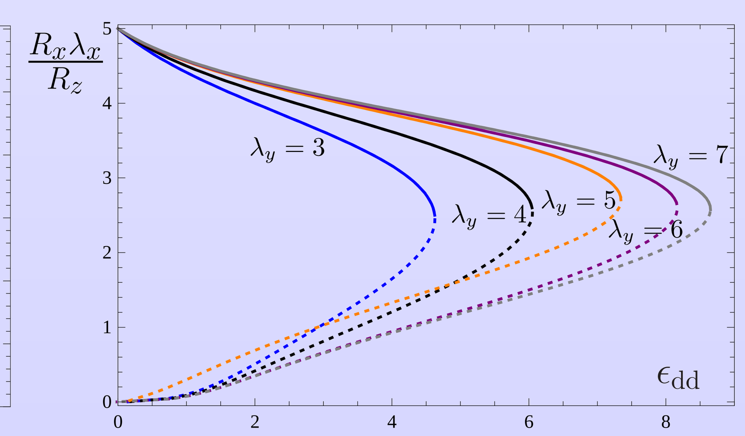
$F(\varphi,q)$ and $E(\varphi,q)$ are elliptic integrals with $\varphi=\arcsin\sqrt{1-x^2}$ and $q^2=(1-y^2)/(1-x^2)$; $f_1(x,y)=\partial_xf(x,y)$, $f_2(x,y)=\partial_yf(x,y)$
★ $f(x,x)=f_s(x)$ positive (negative) for $x<1$ ($x>1$) [9,10]

Equilibrium properties [11,12,6,7]

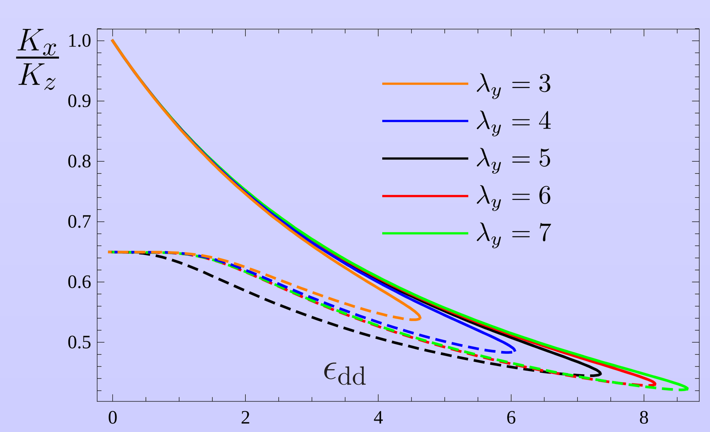
- **Equilibrium conditions** $\frac{d^2\tilde{R}_i}{dt^2}=0$

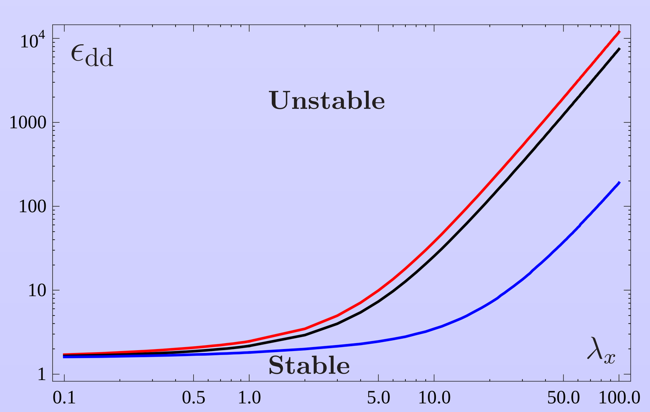
- **Aspect ratio in real space**





- **Aspect ratio in momentum space and stability diagram**





Low-Lying excitations [13,14,6,7]

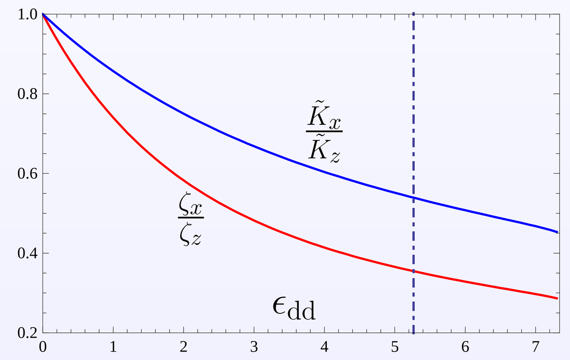
- **Linerization of the equations of motion**

$$\tilde{R}_i=\tilde{R}_i(0)+\eta_ie^{i\Omega t};\qquad\tilde{K}_i=\tilde{K}_i(0)+\zeta_ie^{i\Omega t}$$

equilibrium, oscillation amplitude, oscillation frequency

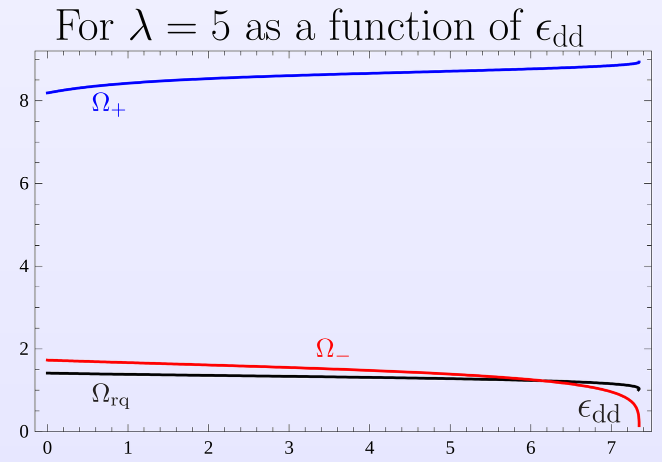
- **Anisotropic momentum oscillations**

$$\frac{\zeta_x}{\zeta_z}=\frac{\tilde{K}_x}{\tilde{K}_z}\frac{\tilde{K}_x^2+\tilde{K}_z^2-\epsilon_{\rm dd}\tilde{K}_z\partial C/\partial\tilde{K}_z}{2\tilde{K}_z^2-\epsilon_{\rm dd}\tilde{K}_z\partial C/\partial\tilde{K}_z}$$

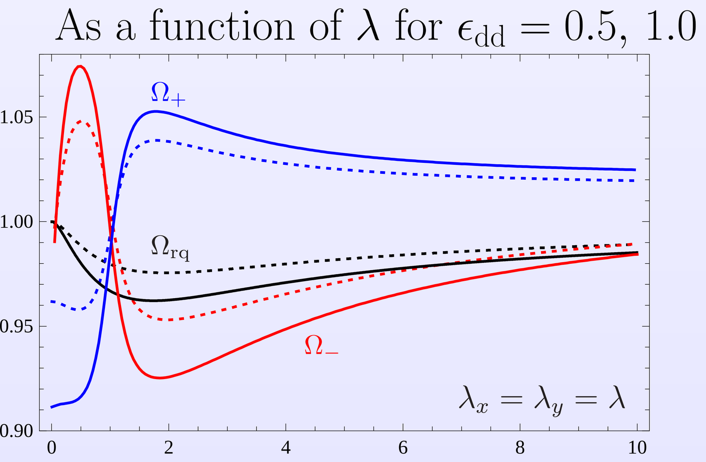


- **Monopole (Ω_+), quadrupole (Ω_-) and radial quadrupole ($\Omega_{\rm rq}$) oscillation frequencies**

For $\lambda=5$ as a function of $\epsilon_{\rm dd}$

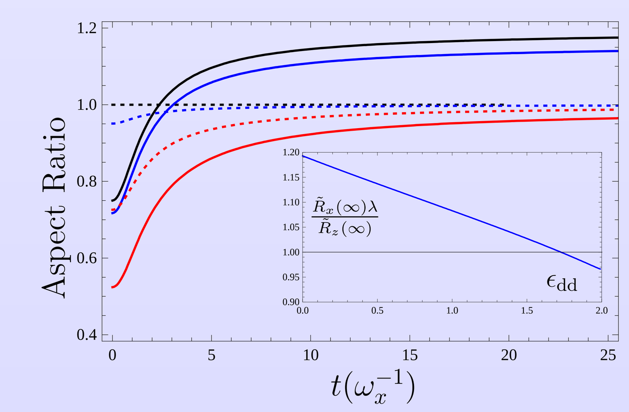


As a function of λ for $\epsilon_{\rm dd}=0.5,\ 1.0$



Time-of-flight pictures [6]

The equations of motion are solved while setting the harmonic restoring force to zero.
On the right: Real-space (continuous) and momentum-space (dashed) for $\epsilon_{\rm dd}=0$, 0.3, 1.8 and $\lambda=.75$



Perspectives

- Interpolation between hydrodynamic [6,7] and collisionless regimes [14]
- Spinorial degrees of freedom
- Crystal phases in harmonic traps and optical lattices

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