Dimensionally Induced 1D-3D Phase Transition of the Weakly Interacting Ultracold Bose Gas

Bernhard Irsigler[1,2] and Axel Pelster[3]

[1] Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität Frankfurt am Main, Germany
[2] Institut für Theoretische Physik, Freie Universität Berlin, Germany
[3] Fachbereich Physik und Forschungszentrum OPTIMAS, Technische Universität Kaiserslautern, Germany

Abstract

We investigate the dimensionally induced phase transition from the normal to the Bose-Einstein-condensed phase for a weakly interacting Bose gas in an optical lattice [1]. To this end we make use of the Hartree-Fock-Bogoliubov-Popov theory, where we include numerically exact hopping energies and effective interaction strengths. At first we determine the critical chemical potential, where we find a much better agreement with recent experimental data than a pure Hartree-Fock treatment. This finding is in agreement with the dominant role of quantum fluctuations in lower dimensions, as they are explicitly included in our theory. Furthermore, we determine for the 1D-3D-transition the power-law exponent of the critical temperature for two different non-interacting Bose gas models yielding the same value of 1/2, which indicates that they belong to the same universality class. For the weakly interacting Bose gas we find for both models that this exponent is robust with respect to finite interaction strengths.

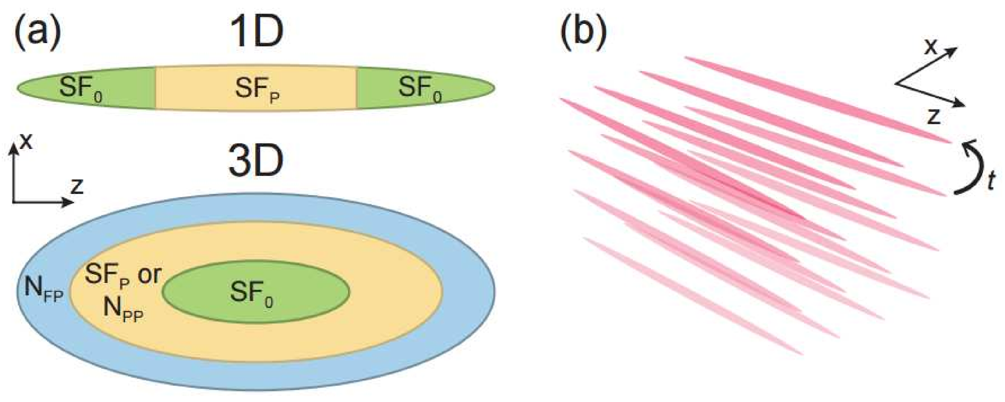
Motivation

Mermin-Wagner theorem: No Bose-Einstein condensation in one- and two dimensional, homogeneous systems at $T \neq 0$ [2]

- continuous change of dimensionality of Bose system with optical lattices [3]
- experiment proposed by Ref. [4,5] featuring a 1D Luttinger liquid theory
- mismatch between theory and experiment for Luttinger liquid (1D) and Hartree-Fock (3D) theory

Investigate the phase transition to the Bose-Einstein condensate in the 1D-3D crossover.

Recent experiment of 1D-3D crossover for imbalanced Fermion system showed intriguing phase separation and ordering [6].

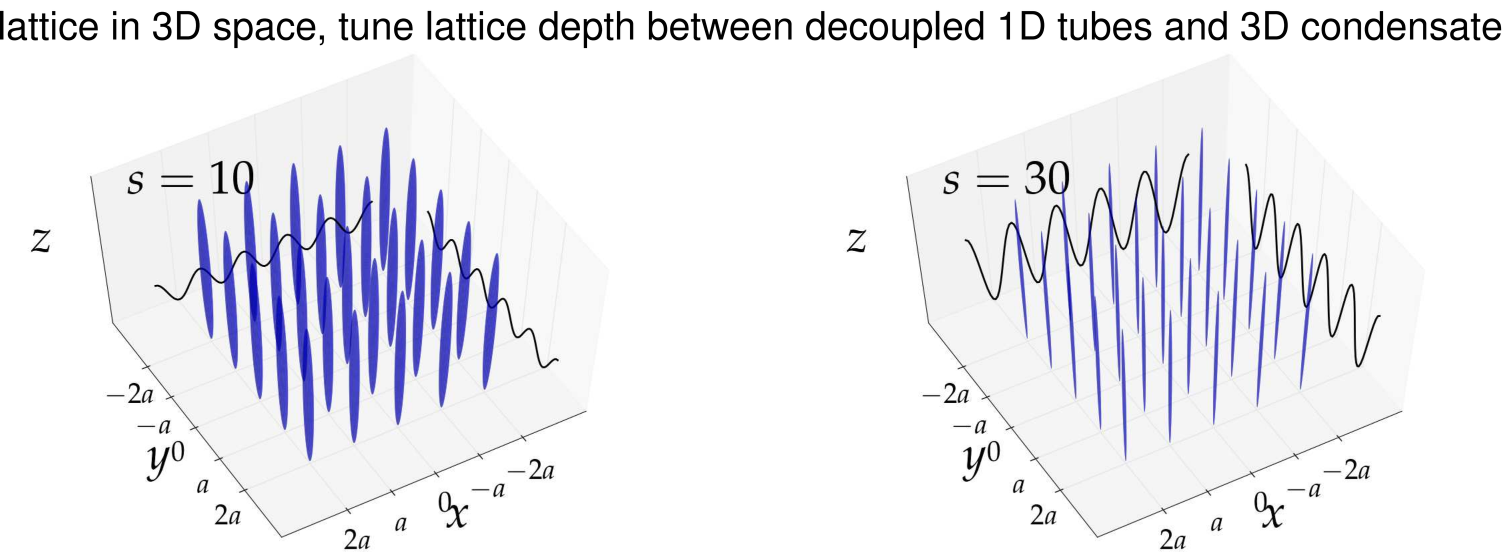


Mermin-Wagner theorem [7]

- Bogoliubov inequality $\langle\{\hat{A},\hat{A}^\dagger\}\rangle\langle[\hat{B}^\dagger,[\hat{H},\hat{B}]]\rangle \geq 2k_BT|\langle[\hat{A},\hat{B}]\rangle|^2$
- set $\hat{A}=\hat{b}^\dagger_{\boldsymbol{k}}$, $\hat{B}=\sum_{\boldsymbol{k}}\hat{b}^\dagger_{\boldsymbol{k}}\hat{b}_{\boldsymbol{k}+\boldsymbol{p}}$ with $[\hat{b}_{\boldsymbol{k}},\hat{b}^\dagger_{\boldsymbol{p}}]=\delta_{\boldsymbol{k},\boldsymbol{p}}$ and $\epsilon_{\boldsymbol{k}}=\frac{\hbar^2\boldsymbol{k}^2}{2M}$
- Hamiltonian incorporating two-body interactions $\hat{H}=\sum_{\boldsymbol{k}}\epsilon_{\boldsymbol{k}}\hat{b}^\dagger_{\boldsymbol{k}}\hat{b}_{\boldsymbol{k}}+\frac{1}{2}\sum_{\boldsymbol{k},\boldsymbol{p},\boldsymbol{q}}V_{\boldsymbol{k}}\hat{b}^\dagger_{\boldsymbol{p}+\boldsymbol{k}}\hat{b}^\dagger_{\boldsymbol{q}-\boldsymbol{k}}\hat{b}_{\boldsymbol{p}}\hat{b}_{\boldsymbol{q}}$
- lower boundary for number of particles $n_{\boldsymbol{k}} \geq \frac{k_BTM|\langle\hat{b}_0\rangle|^2}{N\hbar^2\boldsymbol{k}^2}-\frac{1}{2}$ with $n_{\boldsymbol{k}}=\langle\hat{b}^\dagger_{\boldsymbol{k}}\hat{b}_{\boldsymbol{k}}\rangle$
- order parameter must vanish in low dimensions ($D=1,2$) at $T\neq 0$ due to infrared divergence

Hybrid model

- 2D lattice in 3D space, tune lattice depth between decoupled 1D tubes and 3D condensate

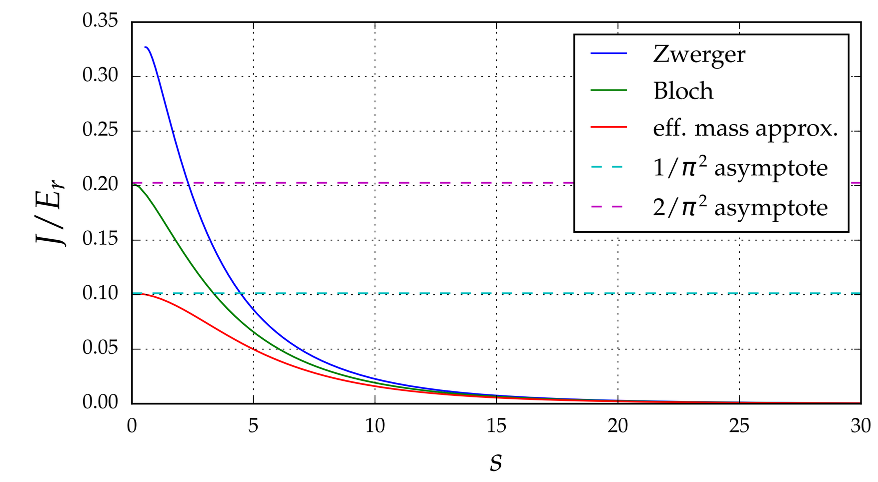


- lattice depth V_0 proportional to laser intensity of laser pairs, dimensionless lattice depth $s=V_0/E_r$ with recoil energy $E_r=\pi^2\hbar^2/(2a^2M)$
- Bloch dispersion for particle in 3D with 2D lattice: $\epsilon_{\boldsymbol{k}}=4J-2J\cos(k_xa)-2J\cos(k_ya)+\frac{\hbar^2k_z^2}{2M}$
- quadratic approximation of lattice dimension with effective mass M^*: $\frac{\partial^2\epsilon_{\boldsymbol{k}}}{\partial k_x^2}=\frac{\hbar^2}{M^*}=2Ja^2$

Hartree-Fock-Bogoliubov-Popov theory [8-10]

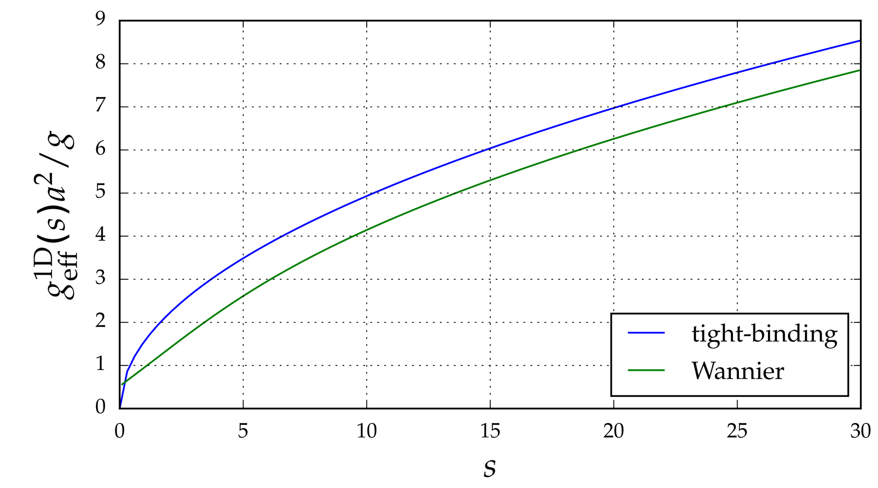
- Bogoliubov dispersion $E_{\boldsymbol{k}}=\sqrt{\varepsilon_{\boldsymbol{k}}^2+2gn_0\varepsilon_{\boldsymbol{k}}}$ with $\varepsilon_{\boldsymbol{k}}=\epsilon_{\boldsymbol{k}}-\mu+gn_0+2g\tilde{n}$ and $n=n_0+\tilde{n}$
- free energy $\mathcal{F}=V(-\mu n_0+\frac{g}{2}n_0^2-g\tilde{n}^2)+\frac{1}{2}\sum_{\boldsymbol{k}}(E_{\boldsymbol{k}}-\varepsilon_{\boldsymbol{k}}-gn_0)-\frac{1}{\beta}\sum_{\boldsymbol{k}}\ln\left(1-e^{-\beta E_{\boldsymbol{k}}}\right)$ with quantum and thermal fluctuations
- extremization $\frac{\partial\mathcal{F}}{\partial n_0}=0 \Rightarrow \mu=gn_0+\frac{2g}{V}\sum_{\boldsymbol{k}}\left[\frac{\varepsilon_{\boldsymbol{k}}+\frac{1}{2}gn_0}{E_{\boldsymbol{k}}}\left(\frac{1}{e^{\beta E_{\boldsymbol{k}}}-1}+\frac{1}{2}\right)-\frac{1}{2}\right]$
- resolve mismatch from Hugenholtz-Pines theorem with second-order Beliaev theory
- particle density $-\frac{1}{V}\frac{\partial\mathcal{F}}{\partial\mu}=n=n_0+\frac{1}{V}\sum_{\boldsymbol{k}}\left[\frac{\varepsilon_{\boldsymbol{k}}+gn_0}{E_{\boldsymbol{k}}}\left(\frac{1}{e^{\beta E_{\boldsymbol{k}}}-1}+\frac{1}{2}\right)-\frac{1}{2}\right]$

Hopping energy



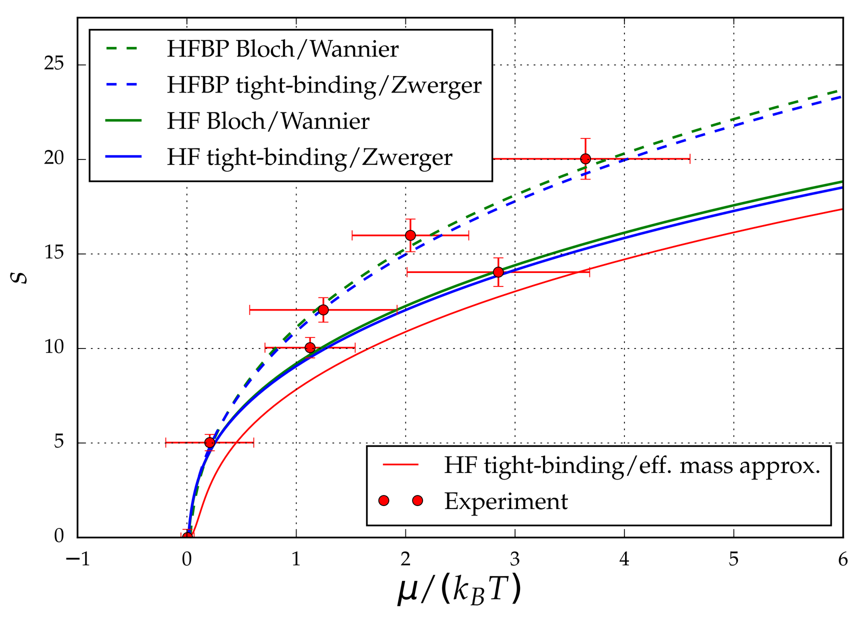
- tight-binding approximation by Zwerger [11] $J(V_0)=\frac{4}{\sqrt{\pi}}E_r\left(\frac{V_0}{E_r}\right)^{3/4}e^{-2\sqrt{V_0/E_r}}$
- numerically computed band energies ϵ_k by solving one-dimensional Schrödinger equation with Bloch theorem $J=\frac{a}{\pi}\int_{\mathrm{BZ}}\mathrm{d}k\,\epsilon_k e^{ika}$
- approximate ϵ_k quadratically with effective mass M^*

Effective interaction strength



- tight-binding approximation $g^{\mathrm{1D}}_{\mathrm{eff}}(s)\approx 4a_sE_r\sqrt{s}$
- Bose-Hubbard parameter $U=g\int\mathrm{d}\boldsymbol{r}|w(\boldsymbol{r})|^4$, $g=4\pi\hbar^2a_s/M$
- for 2D lattice $g^{\mathrm{1D}}_{\mathrm{eff}}(s)=g\left[\int\mathrm{d}x|w(x)|^4\right]^2$
- effective interaction strength decreases with decreasing lattice depth through coherence with neighbouring sites

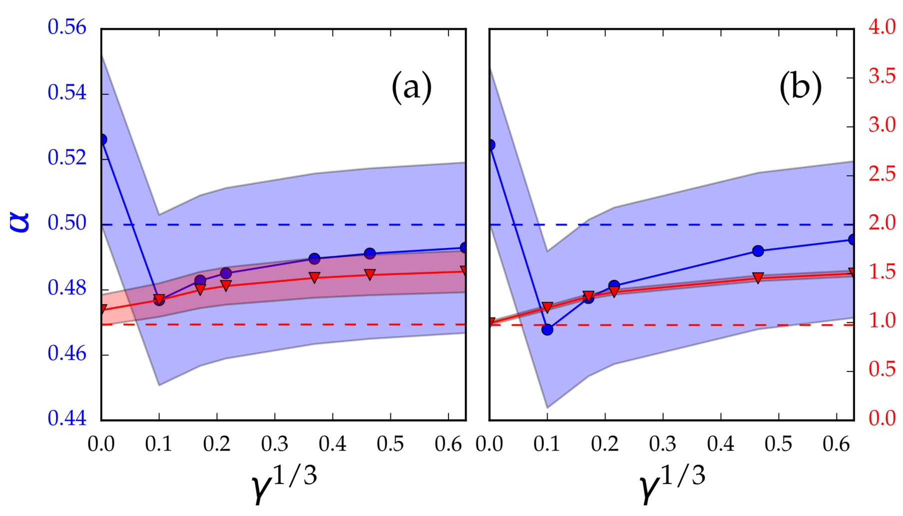
Critical chemical potential



- critical chemical potential separates phases of decoupled 1D tubes and 3D condensate
- Hartree-Fock (HF) $\mu_c=2g_{\mathrm{eff}}n_c$
- Hartree-Fock-Bogoliubov-Popov (HFBP) $\mu_c=2g_{\mathrm{eff}}n_c-gn_0$
- experimental data of Ref. [3]
- effective mass approximation biggest deviation
- improvement through numerically exact Wannier functions
- very good agreement of HFBP theory with experiment

Quantum fluctuations need to be considered in the dimensional crossover.

Critical temperature



- power-law of critical temperature for 1D regime $k_BT_c/E_r=K\,(J/E_r)^\alpha$
- non-interacting exponent $\alpha=1/2$
- α of hybrid model (a) robust against weak interactions, gas parameter: $\gamma=na_s^3$, a_s s-wave scattering length
- introduce 3D lattice model (b): $\epsilon_{\boldsymbol{k}}=2\sum_i J_i[1-\cos(k_ia)]$, $i=x,y,z$
- find same robust exponent for both systems

Same universality class is empirically confirmed.

References

1. B. Irsigler and A. Pelster, arXiv:1612.08920 (2016)
2. N.D. Mermin and H. Wagner, *Phys. Rev. Lett.* **17**, 1133 (1966)
3. A. Vogler, R. Labouvie, G. Barontini, S. Eggert, V. Guarrera, and H. Ott, *Phys. Rev. Lett.* **113**, 215301 (2014)
4. A. F. Ho, M. A. Cazalilla, and T. Giamarchi, *Phys. Rev. Lett.* **92**, 130405 (2004),
5. M. A. Cazalilla, A. F. Ho, and T. Giamarchi, *New J. Phys.* **8**, 158 (2006)
6. M. C. Revelle, J.A. Fry, B. A. Olsen, and R. G. Hulet, *Phys. Rev. Lett.* **117**, 235301 (2016)
7. M. Ueda, *Fundamentals and New Frontiers of Bose-Einstein Condensation*, World Scientific (2010)
8. H. Shi and A. Griffin, *Phys. Rep.* **304**, 1 (1998)
9. J. O. Andersen, *Rev. Mod. Phys.* **76**, 599 (2004)
10. A. Griffin, T. Nikuni, and E. Zaremba, *Bose-Condensed Gases at Finite Temperatures*, Cambridge University Press (2009)
11. W. Zwerger, *J. Opt. B.* **5**, 9 (2003)

Acknowledgements

The authors gratefully thank Antun Balaž, Sebastian Eggert, Denis Morath, Herwig Ott, and Dominik Straßel for fruitful discussions. Furthermore, we acknowledge support of binational projects funded by the German Academic Exchange Service (DAAD) and the Brazilian Federal Agency for Support and Evaluation of Graduate Education (CAPES), as well as the Ministry of Education, Science, and Technological Development of the Republic of Serbia. Finally, we thank the German Research Foundation (DFG) via the Collaborative Research Centers SFB/TR 49 and SFB/TR 185 for financial support.