Numerical Solutions of Gross-Pitaevskii equation for a disordered Bose condensed gas

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Abstract

We present a numerical study of a Bose-condensed gas in a harmonic trapping potential and a Gaussian-distributed disorder potential in one dimension at zero temperature. The underlying Gross-Pitaevskii equation for the condensate wave function represents a nonlinear, partial differential equation and is difficult to solve exactly. Using a computer program [1] that solves the time-independent Gross-Pitaevskii equation in one space dimension in a harmonic trap using the imaginarytime propagation, we are able to obtain its numerical solution for each realization of the disorder potential. Performing disorder ensemble averages we have access to both the condensate density and to the density of disconnected local minicondensates in the respective minima of the disorder potential [2]. Our study is performed for different values of the disorder strength and the correlation length of the disorder, so that we can study the influence of both of them on the numerical solutions. For small disorder strengths we reproduce the seminal results of Huang and Meng for a Bogoliubov theory of dirty bosons [3,4].

Problem

- **Gross-Pitaevskii equation for the ground state**

$$\left[-\frac{\hbar^2\Delta}{2m}-\mu+U(\mathbf{x})+V(\mathbf{x})+\frac{g}{2}\psi^*(\mathbf{x})\psi(\mathbf{x})\right]\psi(\mathbf{x})=0\,.$$

  - Disorder potential $U(x)$, disorder ensemble average $\langle\bullet\rangle$

$$\langle U(\mathbf{x})\rangle=0 \quad\text{and}\quad \langle U(\mathbf{x})U(\mathbf{x}')\rangle=R(\mathbf{x}-\mathbf{x}')$$

  - Trap $V(\mathbf{x})$

- **Condensate depletion**
  - Particle density $n=\langle\psi(\mathbf{x})^2\rangle$
  - Condensate density $n_0=\langle\psi(\mathbf{x})\rangle^2$
  - Depletion $q=n-n_0$

Model

- **One dimension**
- **Gaussian correlation function** $R(x)=\frac{\varepsilon^2}{\sqrt{2\pi}\lambda}\exp\left\{-\frac{x^2}{2\lambda^2}\right\}$
- **Harmonic potential trap** $V(x)=\frac{1}{2}\,m\,\Omega^2\,x^2$

Method

- **Generating random potential [5]**

$$U(x)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\left[A_n\cos(k_n x)+B_n\sin(k_n x)\right]$$

with

$$\langle A_nB_n\rangle=0 \qquad \langle A_nA_m\rangle=\langle B_nB_m\rangle=R(0)^2\delta nm \qquad p(k_n)=\frac{\varepsilon^2}{\sqrt{2\pi}\lambda}\exp\left\{-\frac{k_n^2}{2\lambda^2}\right\}\,.$$

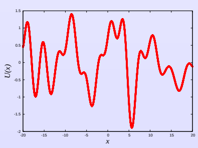

Fig. 1: Sample potential for Gaussian correlation in one dimension with $N=100$, $\varepsilon=1$, and $\lambda=1$.

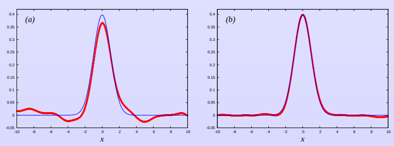

Fig. 2: Correlation $\langle U(x)U(0)\rangle$ with $N=100$ averaged over (a) 1000 and (b) 10 000 sample potentials compared to $R(x)$ (blue).

- **C program for solving time-(in)dependent Gross-Pitaevskii equation in one space dimension [1]**

Results

- **Particle density**

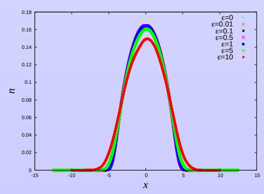

Fig. 3: Particle density n for $N=10000$ and $\lambda=1$.

- **Condensate density**

With increasing disorder strength, the global condensate density decreases

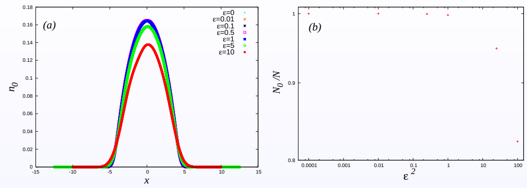

Fig. 4: Condensate density n_0 (a) and fractional number of condensed particles N_0/N (b), where $N_0=\int n_0\,dx$, $N=10000$ and $\lambda=1$.

- **Condensate depletion**

With increasing disorder strength, the depletion increases

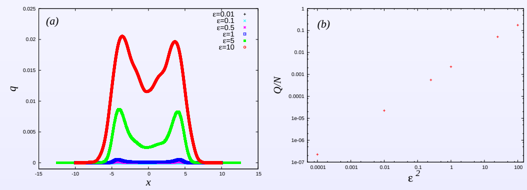

Fig. 5: Depletion q (a) and fractional number of particles Q/N (b) in the disconnected local minicondensates, where $Q=\int q\,dx$, $N=10000$ and $\lambda=1$.

- **Bose-glass phase**

Existence of a Bose-glass phase in the intermediate region

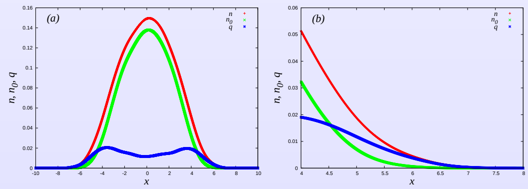

Fig. 6: Particle density n, condensate density n_0, depletion q (a) and intermediate region (b) for $N=10000$, $\lambda=1$, and $\varepsilon=10$.

- **Comparison with the Huang-Meng theory**

Depletion is proportional to the disorder strength [6–9]

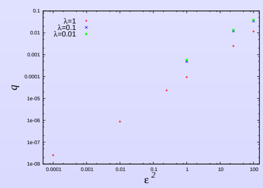

Fig. 7: Depletion q as a function of the disorder strength ε^2 in the center of the BEC ($x=0$) for $N=10000$.

Perspectives

- We plan to follow Ref. [2] and develop a theory which allows us to solve this problem analytically, then compare the results with the numerical ones.
- Extend the study to 3 dimensions.

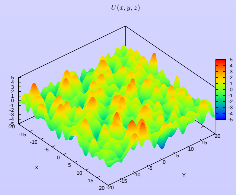

Fig. 8: Sample potential for Gaussian correlation in 3D with $N=10000$, $z=1$, $\varepsilon=1$, $\lambda_x=1$, $\lambda_y=\frac{1}{2}$, and $\lambda_z=\frac{1}{2}$.

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DAAD

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