Rotating BEC in Anharmonic Trap

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1. Introduction

- **Anharmonic Trap:**

$$V(\mathbf{x},\Omega)=\frac{M}{2}(\omega_\perp^2-\Omega^2)r^2+\frac{M}{2}\omega_z^2z^2+\frac{K}{4}r^4$$

- **Experimental Data [1,2]:**

$\omega_\perp = 2\pi\times 64.8$ Hz, $\omega_z = 2\pi\times 11$ Hz,

$k=\dfrac{(\hbar/M\omega_z)^2}{\hbar\omega_z}K = 0.4$, $N = 3\cdot 10^5$, $a_s = 5$ nm

$\Omega = 0...1.04\times\omega_\perp$, Mass $= {}^{87}$Rb

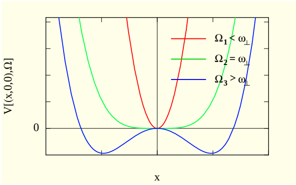

FIG. 1: *Schematic plot of the trapping potential.*

Thermodynamic

- **Grand-Canonical Ensemble:**

$$\mathcal{Z}=\sum_\nu \exp\left[-\beta(E_\nu-\mu N_\nu)\right], \qquad \textbf{Semiclassic:} \quad \sum_\nu \to \int\frac{d^3x\,d^3p}{(2\pi\hbar)^3}$$

- **Grand-Canonical Free Energy:** $\mathcal{F}=N_0(\mu_c-\mu)-\dfrac{\zeta_4(e^{\beta\mu},\Omega)}{\beta^4\hbar^3\omega_z(\omega_\perp^2-\Omega^2)}$

$$\zeta_\nu(e^{\beta\mu},\Omega)=\sum_{j=1}^\infty\frac{e^{j\beta\mu}}{j^\nu}\sqrt{j\pi\frac{M^2\beta}{4k}}(\omega_\perp^2-\Omega^2)\exp\left[j\frac{M^2\beta}{4k}(\omega_\perp^2-\Omega^2)^2\right]\operatorname{erfc}\left[\sqrt{j\frac{M^2\beta}{4k}}(\omega_\perp^2-\Omega^2)\right]$$

Dynamic

- **Gross-Pitaevskii Equation:** $i\partial_t\Psi(\mathbf{x},t)=\left[-\dfrac{\hbar^2}{2M}\Delta+V(\mathbf{x},\Omega)+\dfrac{4\pi\hbar^2a_s}{M}|\Psi(\mathbf{x},t)|^2\right]\Psi(\mathbf{x},t)$

- **Hydrodynamics [3]:** $\Psi(\mathbf{x},t)=|\Psi(\mathbf{x},t)|e^{iS(\mathbf{x},t)}$, **Thomas-Fermi Density:** $S=-\dfrac{\mu t}{\hbar}+s(\mathbf{x})$

- **Dynamic Properties:** Eigenmodes and Expansion out of the Trap with Variational Method (Ritz) [4-6]

$$\Psi(\mathbf{x},t)=A(t)\exp\left\{-\left[\frac{x^2}{2W_x^2(t)}+\frac{y^2}{2W_y^2(t)}+\frac{z^2}{2W_z^2(t)}-iS_x(t)x^2-iS_y(t)y^2-iS_z(t)z^2\right]\right\}$$

2. Thermodynamic Properties

- **Number of Particles [7]:**

$$N=N_0+\frac{\zeta_3(e^{\beta\mu},\Omega)}{\beta^3\hbar^3\omega_z(\omega_\perp^2-\Omega^2)}$$

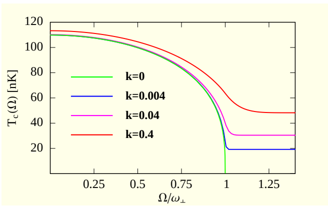

FIG. 2: *Critical temperature T_c versus rotation frequency Ω for various anharmonicities k.*

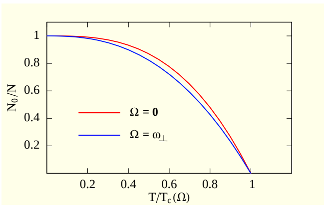

FIG. 3: *Condensate fraction N_0/N versus temperature T for two rotation frequencies Ω.*

- **Heat Capacity [7]:**

$$\mathcal{F}=U-TS-\mu N \longrightarrow C_V=\left.\frac{\partial U}{\partial T}\right|_{N,V}$$

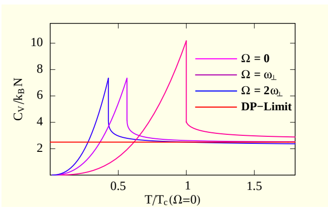

FIG. 4: *Heat capacity C_V versus temperature T. For high temperatures $T\to\infty$, the heat capacity approaches the Dulong-Petit limit (DP) of 2.5 k_BN.*

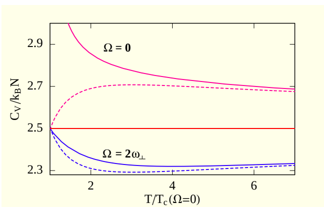

FIG. 5: *Heat capacity C_V versus temperature T. Horizontal line is the Dulong-Petit limit $T\to\infty$, the solid lines correspond to the heat capacity, and the dashed lines are approximations.*

3. Dynamic Properties

- **Thomas-Fermi Density [3,8]:**

$$N=\int d^3x\frac{M}{4\pi\hbar^2a_s}[\mu-V(\mathbf{x},\Omega)]\Theta[\mu-V(\mathbf{x},\Omega)]$$

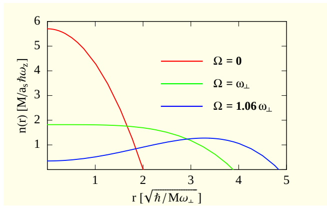

FIG. 6: *Thomas-Fermi density $n(r)$ in the xy-plane for various rotation frequencies Ω.*

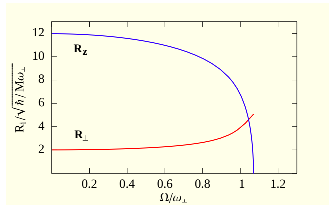

FIG. 7: *Thomas-Fermi radii versus rotation frequency Ω.*

- **Eigenmodes and Expansion [6,8]:**

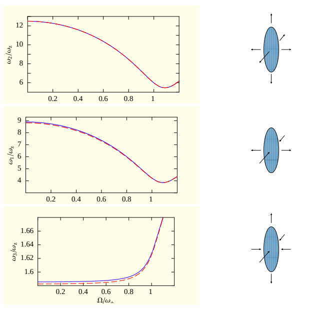

FIG. 8: *Three eigenmodes of the condensate versus rotation frequency Ω. Blue lines correspond to the variational calculation, the dashed red lines to the Thomas-Fermi approximation.*

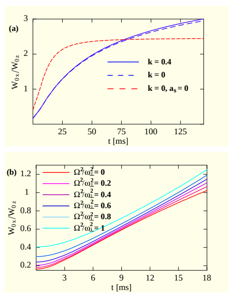

FIG. 9: *Aspect ratio of the condensate widths versus time. Part (a) compares harmonic and anharmonic trap for $\Omega=0$, part (b) shows the effect of rotation.*

4. References

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