Dissipative Two-Mode Tavis-Cummings Model with Time-Delayed Feedback Control

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Abstract

We investigate the dynamics of a two-mode laser system based on a quantum mechanical Tavis-Cummings model [1]. Using mean-field equations, we determine the phase diagram and apply on top three different time delayed feedback schemes of Pyragas form to influence the system behavior.

2-Mode-TC

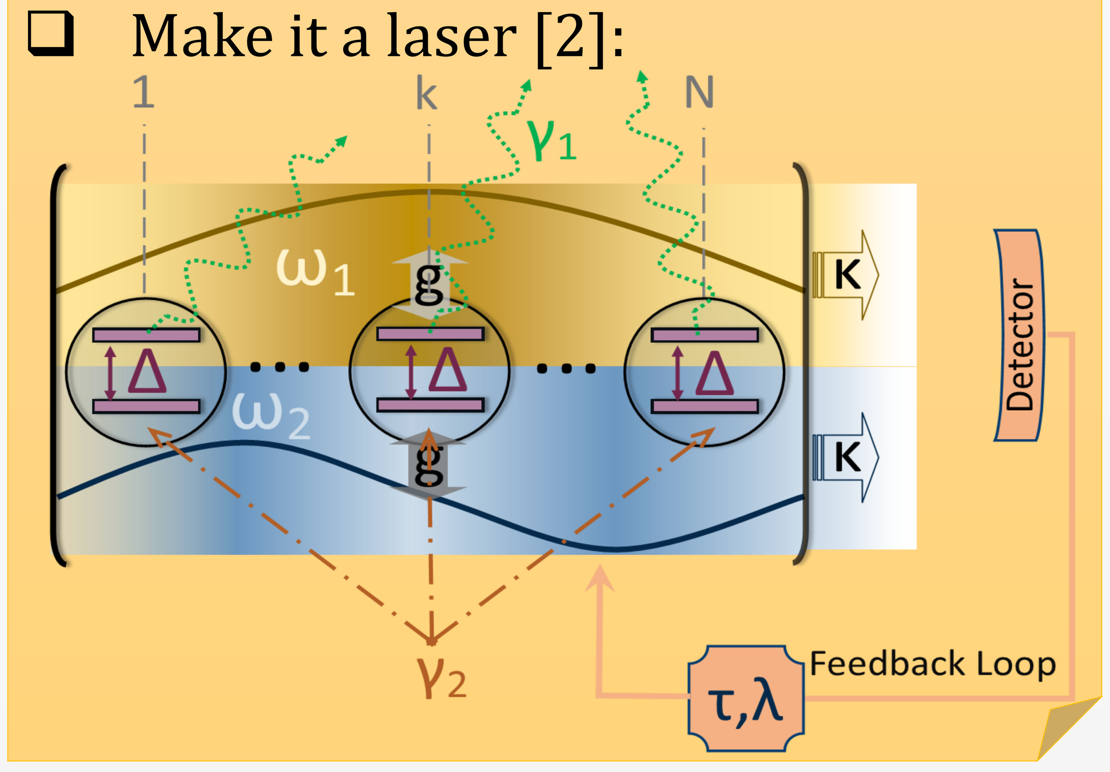
- Hamiltonian of a closed system [1]

$$\hat{H} = \sum_{m=1}^{2} \omega_m \hat{a}_m^\dagger \hat{a}_m + \Delta \hat{J}_z + \frac{g}{\sqrt{N}} \sum_{m=1}^{2} (\hat{a}_m \hat{J}^+ + \hat{a}_m^\dagger \hat{J}^-)$$

with

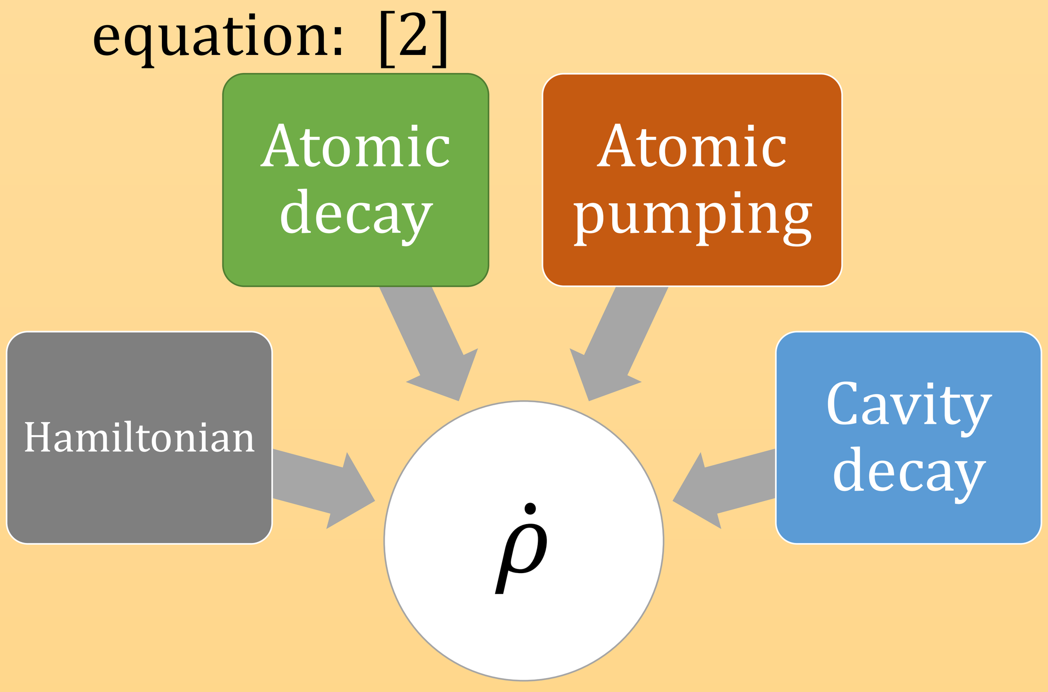
- $\hat{J}_z, \hat{J}^\pm$ – collective spin operators
- $\hat{a}_{1,2}^{(\dagger)}$ – ladder operators of the optical mode
- g – atom-field-coupling
- ω_1, ω_2 – frequencies of the 2 modes
- Δ – atomic frequency
- N – number of atoms

- Make it a laser [2]:



2-Mode-TC Laser

- Description based on master equation: [2]



- Resulting rescaled mean-field eqs.

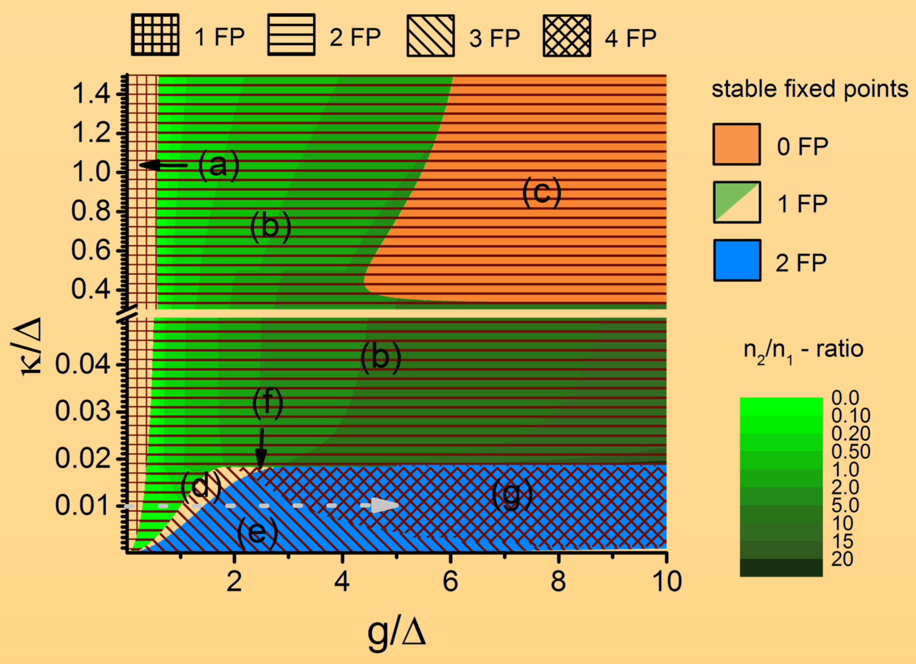
$$\begin{aligned}
\dot{a}_1 &= (-i\,\omega_1 - \kappa)a_1 - i\,g\,J^- \\
\dot{a}_2 &= (-i\,\omega_2 - \kappa)a_2 - i\,g\,J^- \\
\dot{J}^+ &= (i\Delta - \Gamma_\downarrow)J^+ - i\,2g(a_1^* + a_2^*)J_z \\
\dot{J}_z &= -i\,g(a_1 + a_2)J^+ + i\,g(a_1^* + a_2^*)J^- \\
&\quad + \Gamma_\uparrow(z_0 - J_z)
\end{aligned}$$

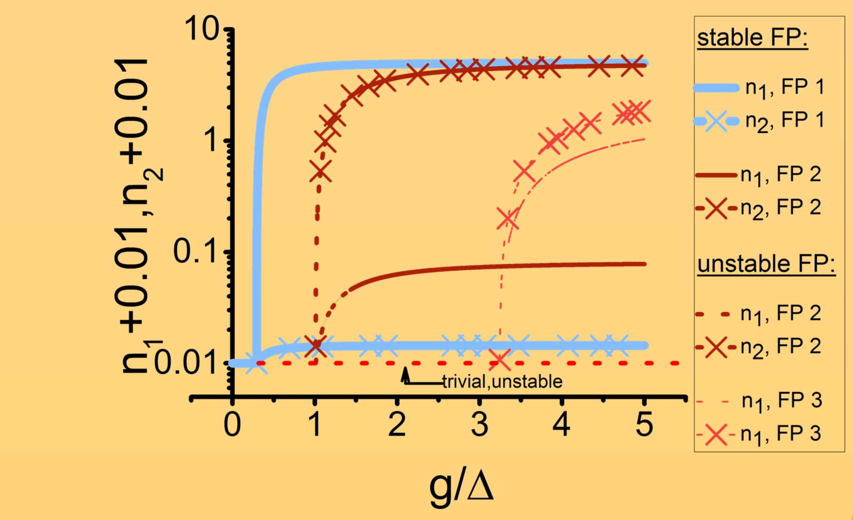
Fixed Point Analysis

- Calculate fixed points by switching to a rotated frame with self-consist. determined frequency ω
  - Shift frequencies: $\omega_m \to \omega_m - \omega$; $\Delta \to \Delta - \omega$
  - $\partial_t(a_1, a_2, J^+, J_z) = (0,0,0,0) \Rightarrow J_z^0 = \Re(J_z^0) + \Im(J_z^0) \Rightarrow \omega \Rightarrow (a_1, a_2, J^+, J_z)^0$
  - Up to 4 possible fixed points
- Stability analysis
  - Up to 2 stable non-trivial fixed points
  - Areas with one dominating mode
  - Complete absence of stability is possible
- Apply Pyragas feedback to modify behavior of the system [3,4]

Phase and Bifurcation Diagram

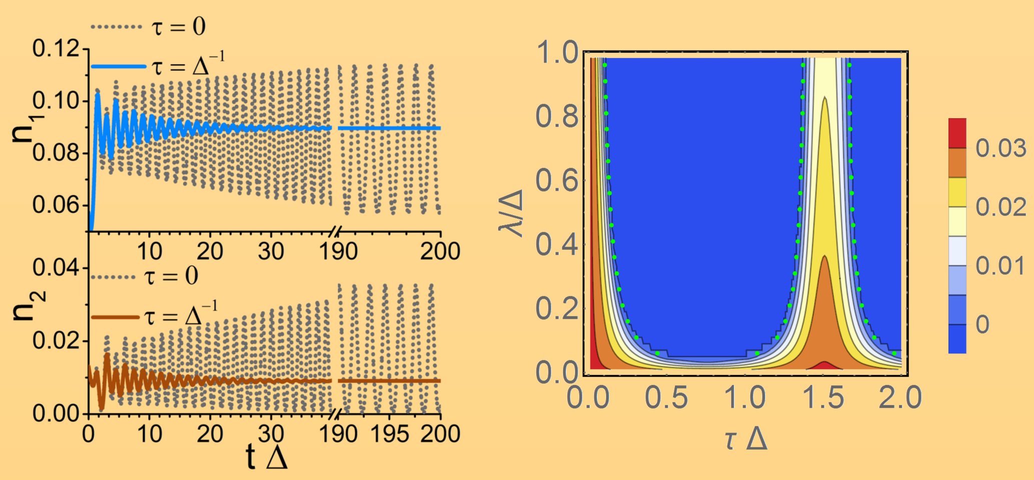
Area	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Fixed Points	1	2	2	3	3	4	4
Stable	1	1	0	1	2	1	2





Pyragas Feedback: Stabilization

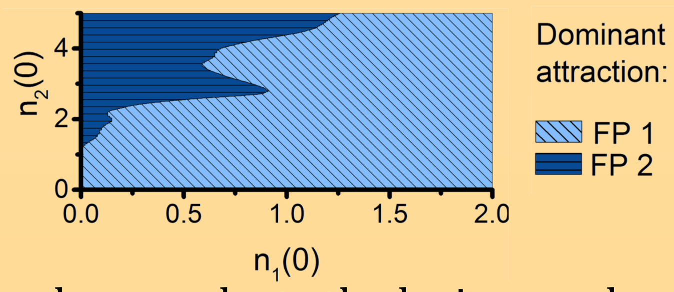
- Feedback: $\dot{J}_z = \cdots + \lambda(J_z[t-\tau] - J_z[t])$
- Its effect: stabilizes the unstable mode
- Works in: area (c)
- Realization: pumping



- Without feedback the fixed point (black dotted lines) is unstable, thus the mode occupation oscillates forever ($\tau = 0$).
- With feedback, the fixed point gets stable ($\tau = \frac{1}{\Delta}$).
- The control diagram in the (τ, λ) - plane. In the blue area the fixed point is stable.

Pyragas Feedback: Selecting the Lasing Mode

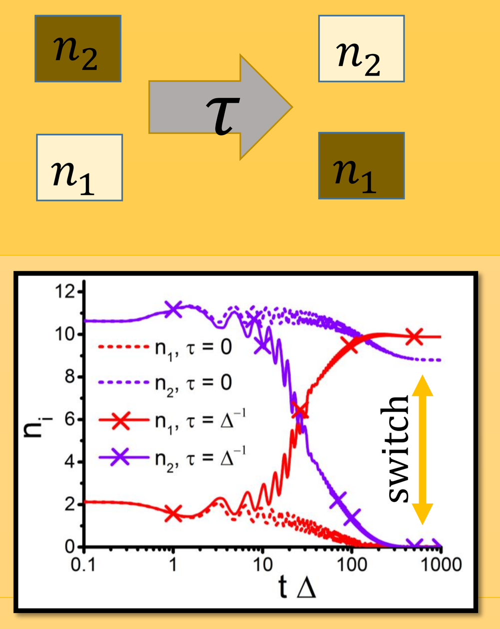
- Without feedback initial condition selects the lasing mode:

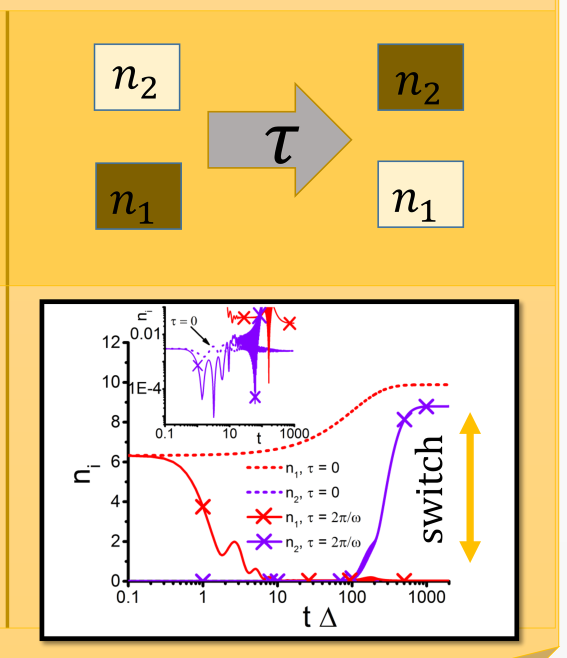


- The feedback scheme selects the lasing mode irrespective of initial condition (here $\omega_1 < \omega_2$).

Scheme 1	Scheme 2
Feedback: $\omega_1 \to (\omega_1 + \lambda(n_2(t-\tau) - n_2))$	Feedback: $\dot{a}_1 = \cdots + \lambda(a_1(t-\tau) - a_1)$
Its action: ω_1- mode acquires large population [5]	Its action: ω_2- mode acquires large population
Realization: moving mirror	Realization: extra mirror

Feedback switch, works in area (e)





Conclusions

- Complex phase diagram with multiple fixed points is found even without feedback.
- Feedback opens the possibility to select the radiating mode, or to stabilize the system.
- Feedback works well if there is only one unstable or two stable points. Mixture of stable/unstable points often leads to oscillating behavior or partial destabilization.
- Extension of the model with a thermalization mechanism would yield a minimal model to study the transitions between a condensate- and a laser-like state, which originate from a macroscopic occupation of the lower and higher cavity mode, respectively [6].

References

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