B6 Hydrodynamic description of Vortices in Photon Bose-Einstein Condensates

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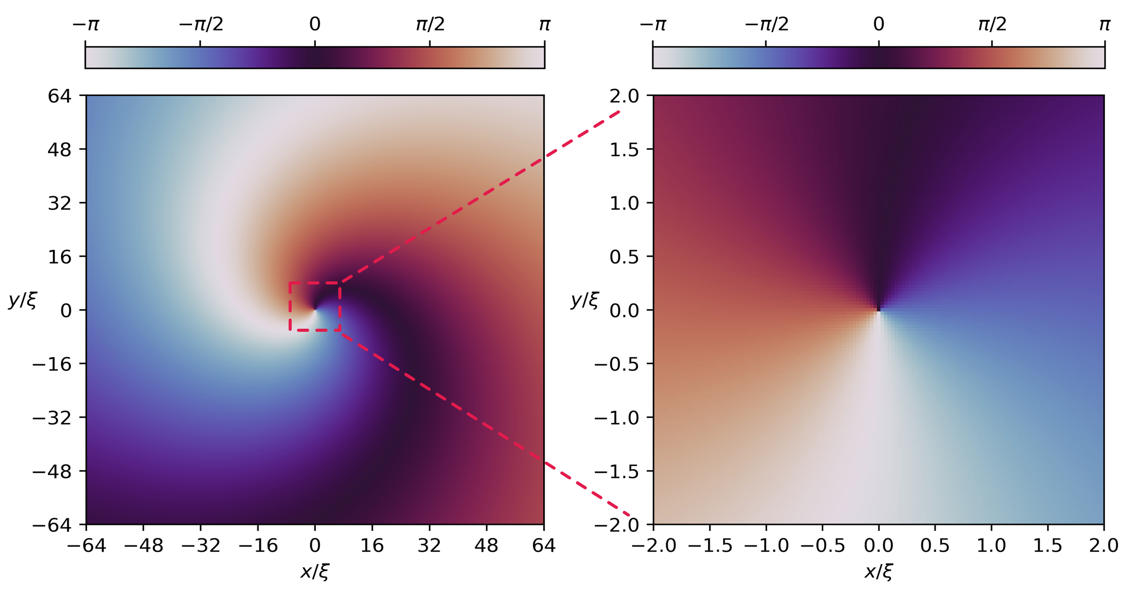
Introduction

Vortices in Bose-Einstein Condensation of Photons

- Spiral shaped velocity field: Vortex act like particle cannon
- Different behaviour depending on observation area

Phase of wavefunction



Hydrodynamic description

- Decompose wave function

Madelung transformation

$$\psi(\mathbf{x},t) = \sqrt{n(\mathbf{x},t)}\,e^{i\Phi(\mathbf{x},t)} \qquad \mathbf{v}(\mathbf{x},t) = \frac{\hbar}{m}\nabla\Phi(\mathbf{x},t)$$

density dependent amplitude; phase

current density; open system inhomogeneity

$$\frac{\partial n(\mathbf{x},t)}{\partial t} + \nabla\cdot\left[n(\mathbf{x},t)\,\mathbf{v}(\mathbf{x},t)\right] = n(\mathbf{x},t)\left[\gamma - \Gamma n(\mathbf{x},t)\right]$$

open system continuity equation

transport derivative; external potential; interaction; quantum pressure; Lorentz force like term

$$\frac{\partial \mathbf{v}(\mathbf{x},t)}{\partial t} + \left[\mathbf{v}(\mathbf{x},t)\cdot\nabla\right]\mathbf{v}(\mathbf{x},t) = -\nabla\left[\frac{U(\mathbf{x})}{m} + \frac{g n(\mathbf{x},t)}{m} - \frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{n(\mathbf{x},t)}}{\sqrt{n(\mathbf{x},t)}}\right] + \boldsymbol{\omega}(\mathbf{x},t)\times\mathbf{v}(\mathbf{x},t)$$

quantum Bernoulli equation

$\boldsymbol{\omega}(\mathbf{x},t) = \nabla\times\mathbf{v}(\mathbf{x},t)$ vorticity

$$\frac{\partial \boldsymbol{\omega}(\mathbf{x},t)}{\partial t} + \left[\mathbf{v}(\mathbf{x},t)\cdot\nabla\right]\boldsymbol{\omega}(\mathbf{x},t) = \left[\boldsymbol{\omega}(\mathbf{x},t)\cdot\nabla\right]\mathbf{v}(\mathbf{x},t) - \boldsymbol{\omega}(\mathbf{x},t)\left[\nabla\cdot\mathbf{v}(\mathbf{x},t)\right]$$

Helmholtz vorticity equation

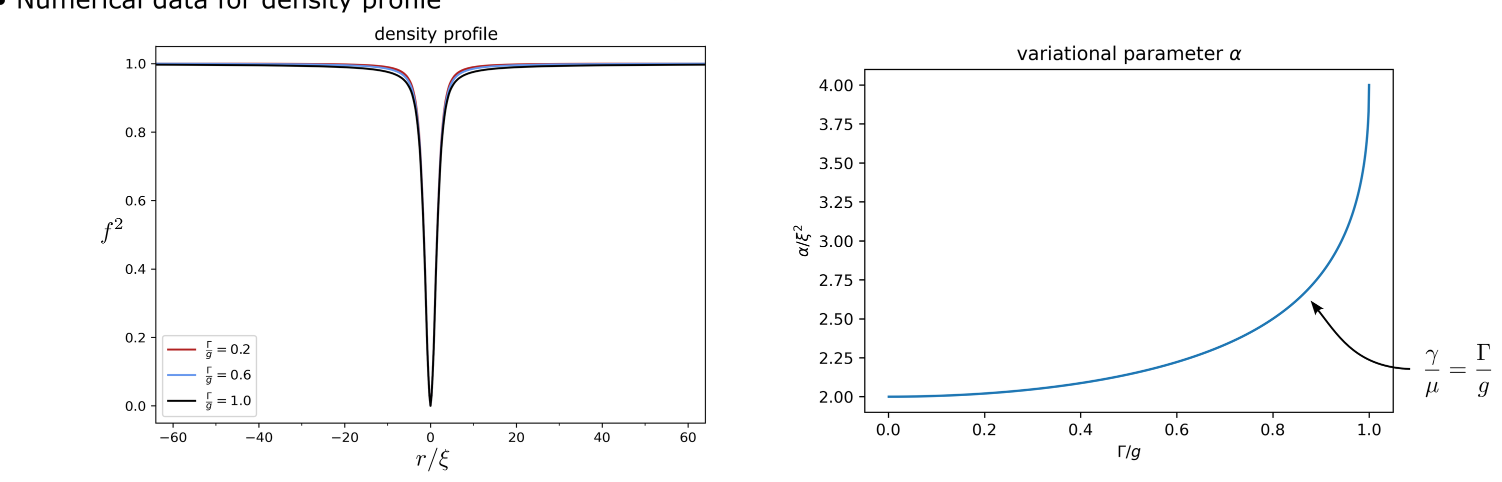
Single vortex solution: Density profile

- Variational solution for density profile

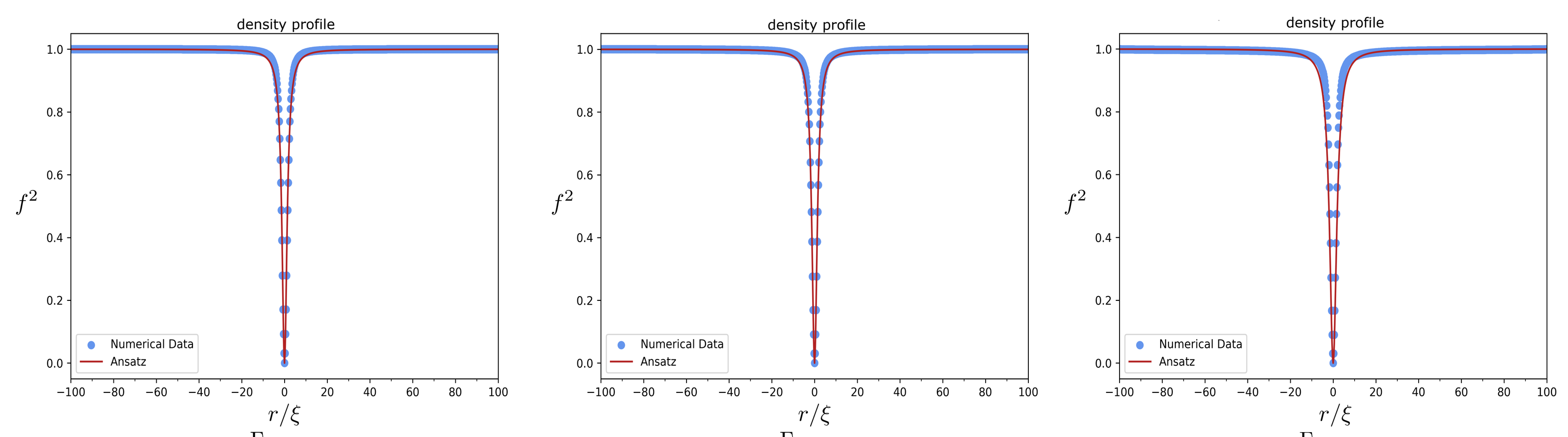
$$f(r) = \sqrt{\frac{r^2}{r^2+\alpha}} \qquad \alpha = \frac{\hbar^2}{2m\mu}\,4\frac{\mu}{\gamma}\frac{g}{\Gamma}\left[1 \pm \sqrt{1-\left(\frac{\Gamma}{g}\right)^2}\right] \longrightarrow 0 \le \left|\frac{\Gamma}{g}\right| \le 1 \longrightarrow 0 \le \left|\frac{\gamma}{\mu}\right| \le 1$$

healing length ξ^2; only minus physical

- Numerical data for density profile



- Numerical data for density profile compared with analytic solution



$\frac{\Gamma}{g} = 0.2$ ⟶ $\frac{\Gamma}{g} = 0.6$ ⟶ $\frac{\Gamma}{g} = 1.0$

Outlook

- Vortex motion with initial velocity
- Off-centered vortex motion
- Dynamical stability of variational approach
- Two vortex motion
- BKT, Kibble-Zurek, KPZ physics in open dissipative systems

Model

- Open dissipative system due to particle gains and losses
  - Additional terms in Gross-Pitaevski equation needed [1]

kinetic term; external potential; particle interaction; density dependent loss term; particle gains

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \left\{-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{x}) + g\left|\psi(\mathbf{x},t)\right|^2 + \frac{i}{2}\left[\gamma - \Gamma\left|\psi(\mathbf{x},t)\right|^2\right]\right\}\psi(\mathbf{x},t)$$

Gross-Pitaevski equation; pumping term; particle losses

Single vortex system without trapping potential

- Consider steady state $\psi(r,\varphi,t) = \psi(r,\varphi)\,e^{-\frac{i}{\hbar}\mu t} = \sqrt{n_\infty n(r)}\,e^{i\Phi(r,\varphi)}e^{-\frac{i}{\hbar}\mu t}$

$\mathbf{v}(\mathbf{x},t) = \mathbf{v}(\mathbf{x})$, $n(\mathbf{x},t) = n(\mathbf{x})$

- Helmholtz vector decomposition theorem in 2D [6]
- Large distance behaviour

$$\mu = g n_\infty \qquad n_\infty = \frac{\gamma}{\Gamma} \qquad \mathbf{v}(r,\varphi) = \frac{\hbar}{m}\left[\nabla\varphi_S(\varphi) + \nabla\varphi_R(r)\right]$$

polar coordinates; rotational part; non-rotational part

- Equations of motion

$$0 = \mu f(r) + \frac{\hbar^2}{2m}\left\{\nabla^2 f(r) - \frac{f(r)}{r^2} - f(r)\left[\nabla\varphi_R(r)\right]^2\right\} - g n_\infty f^3(r)$$

⟶ density equation

Real part of cGPE

$f(r) = \sqrt{n(r)}$

one parameter left due to large distance behaviour

$$0 = \frac{\hbar^2}{2m}\left\{f(r)\nabla^2\varphi_R(r) + 2\nabla f(r)\cdot\nabla\varphi_R(r)\right\} - \frac{1}{2}\left[\gamma - \Gamma n_\infty f^2(r)\right]f(r)$$

⟶ radial velocity equation

imaginary part of cGPE

- Variational density ansatz $f(r) = \sqrt{\frac{r^2}{r^2+\alpha}}$

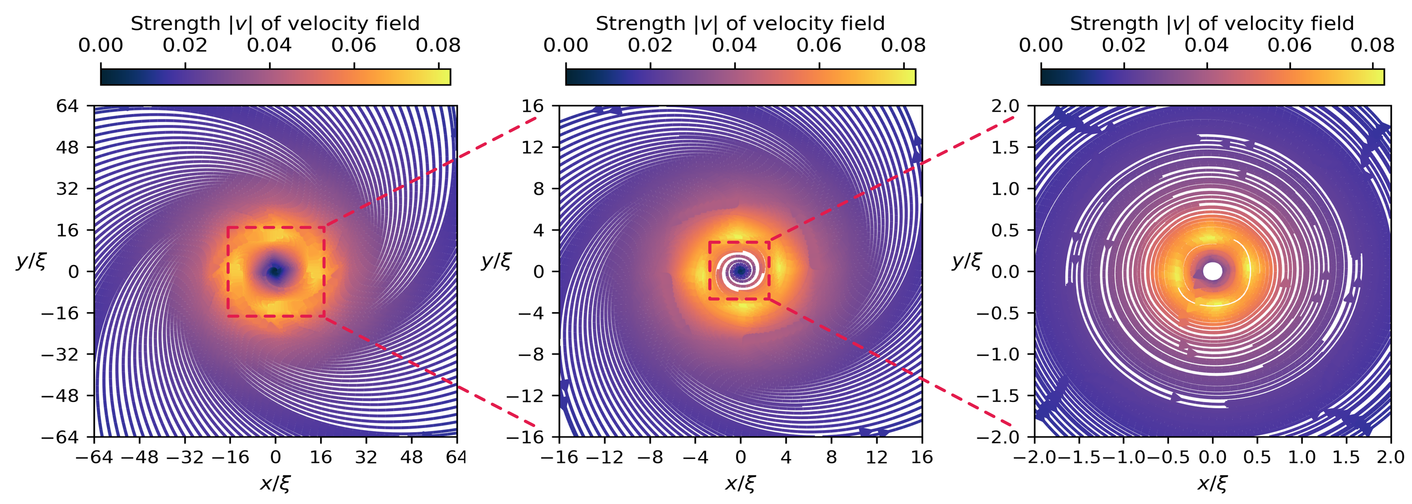
⟶ variational parameter α and variational function $\varphi_R(r)$

Single vortex solution: Velocity field

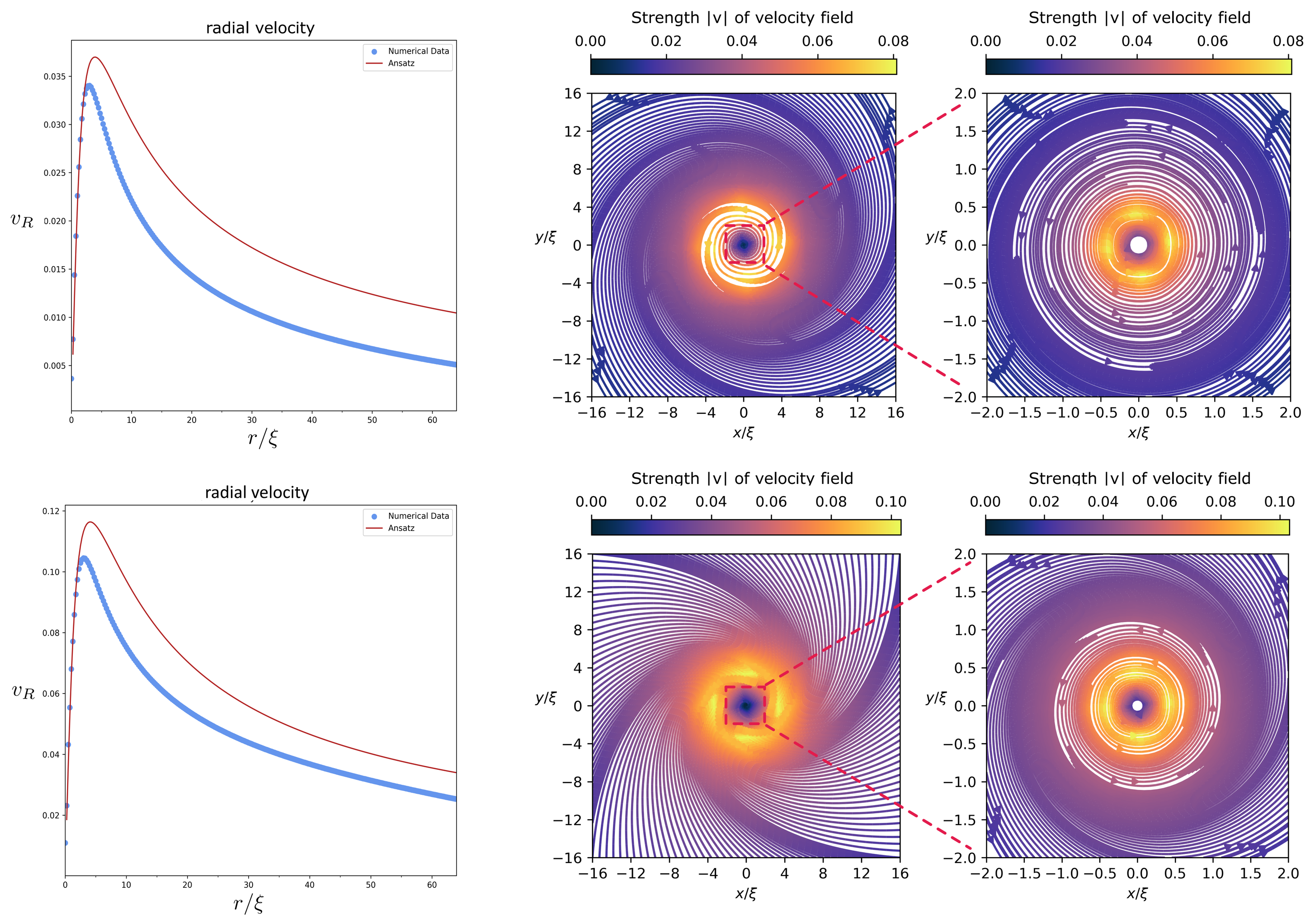
- Variational solution for radial velocity

$$v_R = \frac{d\varphi_R(r)}{dr} = \frac{2m\mu}{\hbar^2}\frac{\alpha\gamma}{4\mu}\left[\ln\left(\frac{r^2+\alpha}{\alpha}\right)\frac{r^2+\alpha}{r^3} - \frac{1}{r}\right]$$

- Velocity field for: $\frac{\gamma}{\mu} = \frac{\Gamma}{g} = 0.2$



- Numerical velocity field



$\frac{\Gamma}{g} = 0.2$ ⟶ $\frac{\Gamma}{g} = 0.6$

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