Phase diagram of "dirty" bosons in a lattice with discrete and continuous disorder distribution

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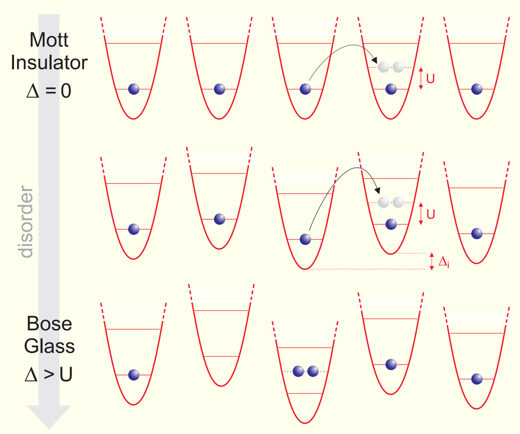
Hamiltonian

M.P.A.Fisher, P.B.Weichman, G.Grinstein, and D.S.Fisher, PRB **40**, 546 (1989)



$$\hat{H}_{BH} = -J\sum_{<\mathbf{i},\mathbf{j}>}\hat{a}^\dagger_{\mathbf{i}}\hat{a}_{\mathbf{j}} + \frac{U}{2}\sum_{\mathbf{i}}\hat{a}^\dagger_{\mathbf{i}}\hat{a}^\dagger_{\mathbf{i}}\hat{a}_{\mathbf{i}}\hat{a}_{\mathbf{i}} - \sum_{\mathbf{i}}(\mu+\epsilon_{\mathbf{i}})\,\hat{a}^\dagger_{\mathbf{i}}\hat{a}_{\mathbf{i}}$$

Definitions of phases ($T=0$)

- Superfluid (SF): non-zero superfluid density
- Mott insulator (MI): zero compressibility, gap for particle-hole excitations
- Bose glass (BG): non-zero compressibility, no gap, divergent superfluid susceptibility

Mean-field approximation

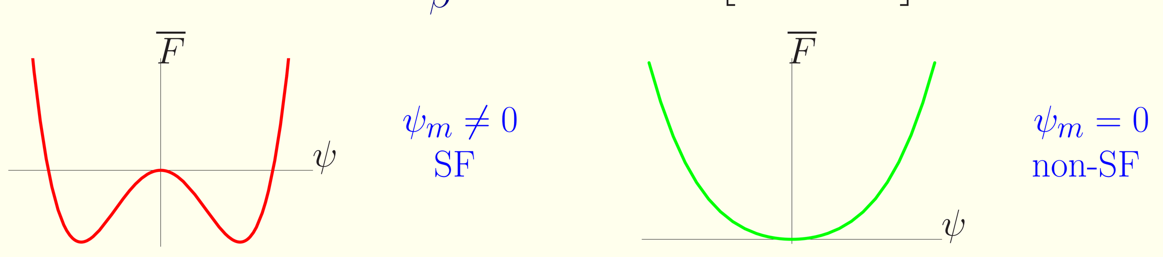
M.P.A.Fisher et al. (1989), Sheshadri et al. (1993), D. van Oosten et al. (2001)
S. Sachdev, *Quantum phase transitions* (2001)

$$\psi = \overline{\langle\hat{a}_{\mathbf{i}}\rangle} \qquad \langle\ldots\rangle = \frac{\mathrm{Tr}\left[\ldots\exp(-\beta\hat{H})\right]}{\mathrm{Tr}\left[\exp(-\beta\hat{H})\right]} \qquad \overline{\ldots} = \Pi_{\mathbf{i}}\int_{-\infty}^{+\infty}\ldots p(\epsilon_{\mathbf{i}})d\epsilon_{\mathbf{i}}$$

$$\hat{a}^\dagger_{\mathbf{i}}\hat{a}_{\mathbf{j}} \approx \psi^*\hat{a}_{\mathbf{j}} + \hat{a}^\dagger_{\mathbf{i}}\psi - |\psi|^2$$

$$\hat{H} = -2dJ\left(\psi\hat{a}^\dagger + \psi^*\hat{a} - |\psi|^2\right) + \frac{U}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\,\hat{a} - (\mu+\epsilon)\,\hat{a}^\dagger\hat{a}$$

$$\overline{F} = -\frac{1}{\beta}\overline{\ln Z} \qquad Z = \mathrm{Tr}\left[\exp(-\beta\hat{H})\right]$$



$$\overline{N} = -\frac{\partial\overline{F}}{\partial\mu} \qquad \overline{\kappa} = -\frac{\partial\overline{N}}{\partial\mu}$$

$$\overline{G(t)} = -i\left[\Theta(t)\overline{G_>(t)} + \Theta(-t)\overline{G_<(t)}\right]$$

$$G_>(t) = \langle\hat{a}(t)\hat{a}^\dagger(0)\rangle \qquad G_<(t) = \langle\hat{a}^\dagger(0)\hat{a}(t)\rangle$$

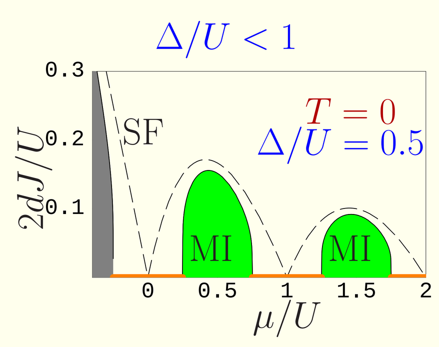
$$\overline{\rho(\omega,\mu)} = -\frac{1}{\pi}\,\mathrm{Im}\,\overline{G(\omega)} \qquad \overline{\chi} = \int_0^\infty \overline{G_>(t)}dt$$

Uniform disorder distribution

$$p(\epsilon) = \begin{cases}\frac{1}{\Delta} & \text{if } \epsilon\in\left[-\frac{\Delta}{2},\frac{\Delta}{2}\right]\\ 0 & \text{otherwise}\end{cases}$$

K.V.Krutitsky, A.Pelster and R.Graham, New J. Phys. **8**, 187 (2006)

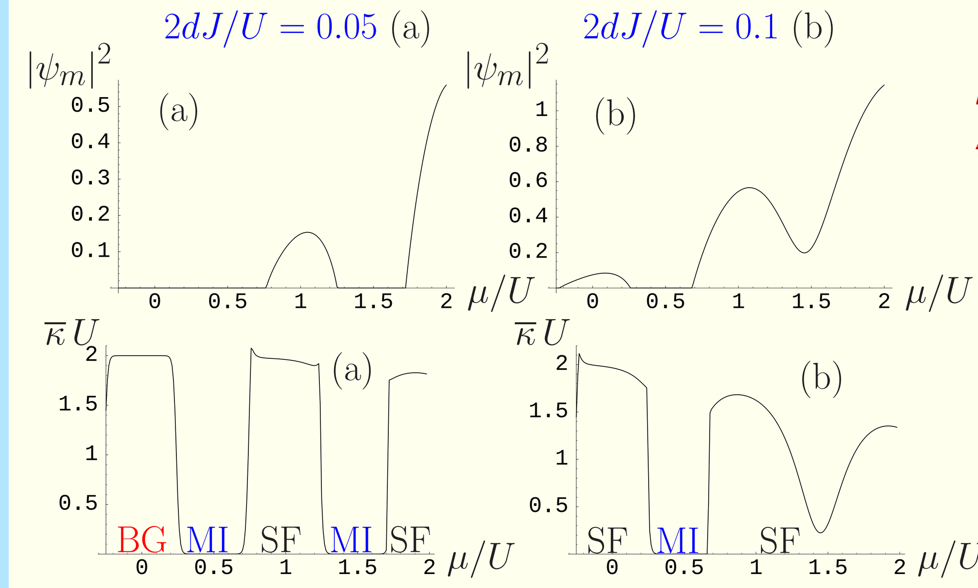
$T=0$



$$\overline{\kappa(\mu)} = \frac{n_+ - n_-}{\Delta} \qquad n_\pm > \frac{\mu\pm\Delta/2}{U}$$

$$\overline{\rho(0,\mu)} = \begin{cases}\frac{n}{\Delta} & \frac{\mu}{U}\in\left[n-1, n-1+\frac{\Delta}{2U}\right]\\ 0 & \frac{\mu}{U}\in\left[n-1+\frac{\Delta}{2U}, n-\frac{\Delta}{2U}\right]\\ \frac{n+1}{\Delta} & \frac{\mu}{U}\in\left[n-\frac{\Delta}{2U}, n\right]\end{cases}$$

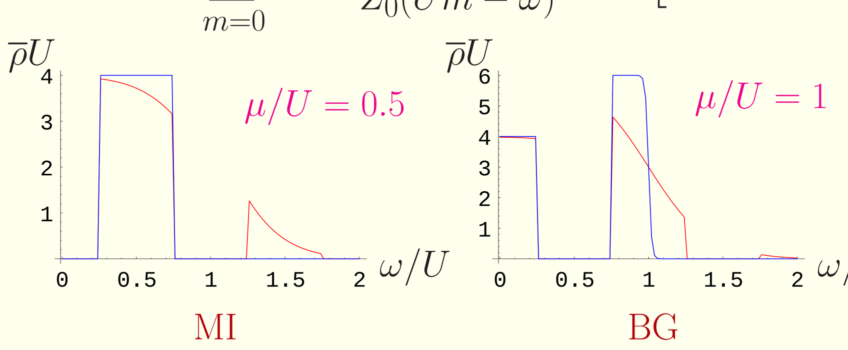
Non-zero temperature



$$\beta U \gg 1: \quad \overline{\kappa(\mu)} \approx \frac{n_+ - n_-}{\Delta} + \alpha\,e^{-\beta\delta}$$

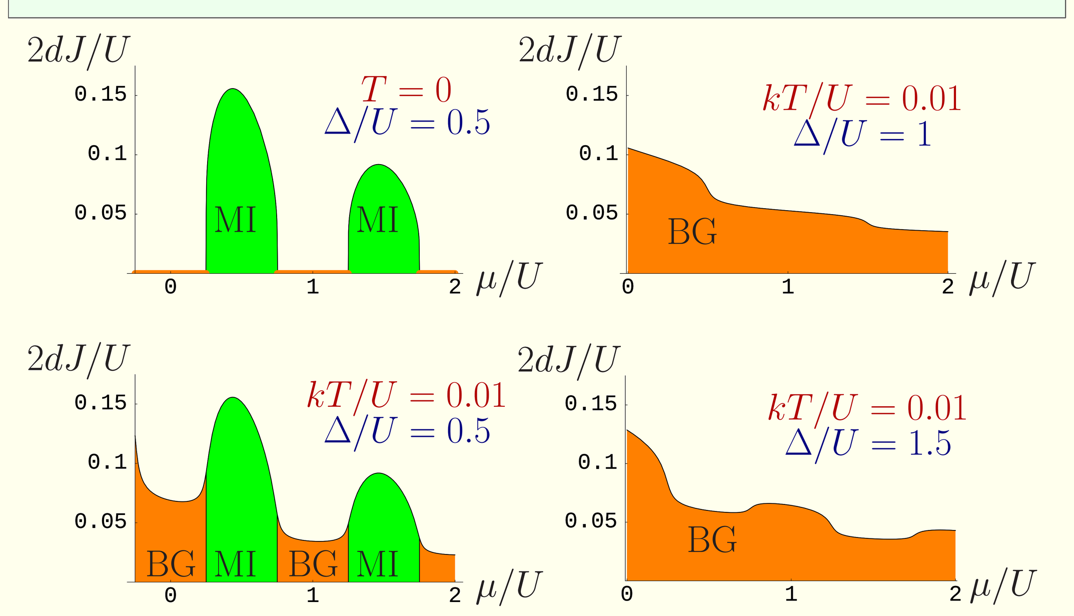
Density of states

$$\overline{\rho(\omega,\mu)} = \sum_{m=0}^{\infty}\frac{(m+1)\,p(Um-\mu-\omega)}{Z_0(Um-\omega)}\left[e^{-\beta E_{m+1}(Um-\omega)} + e^{-\beta E_m(Um-\omega)}\right]$$



$$\overline{\rho(0,\mu)} = \begin{cases}\frac{2n}{\Delta Z_0(U(n-1))}\,e^{-\beta E_{n-1}(U(n-1))} & \frac{\mu}{U}\in\left[n-1, n-1+\frac{\Delta}{2U}\right]\\ 0 & \frac{\mu}{U}\in\left[n-1+\frac{\Delta}{2U}, n-\frac{\Delta}{2U}\right]\\ \frac{2(n+1)}{\Delta Z_0(Un)}\,e^{-\beta E_n(Un)} & \frac{\mu}{U}\in\left[n-\frac{\Delta}{2U}, n\right]\end{cases}$$

Phase diagrams



Conclusions

Bosons in a periodic lattice with on-site disorder at non-zero temperature are considered within a mean-field theory.

Continuous disorder distribution:

- The criteria used for the definition of the superfluid, Mott insulator and Bose glass are analysed.
- The phases are distinguished by the superfluid density and the density of states of the low-energy exitations.
- The phase diagram of the system is calculated.

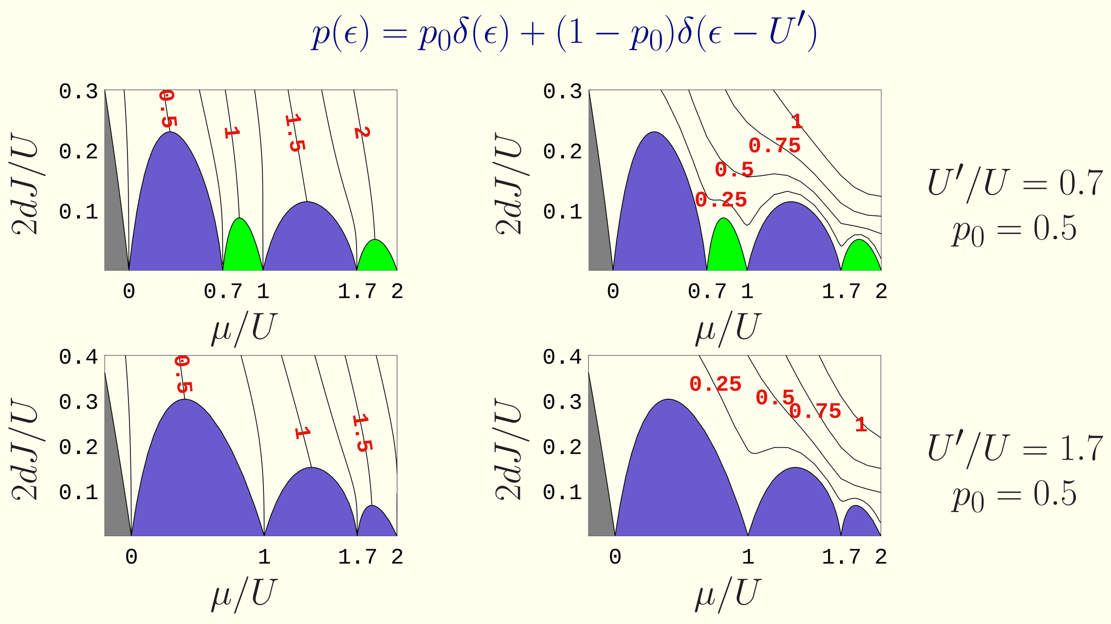
Discrete disorder distribution:

- Insulating phases with noninteger occupation numbers and nonvanishing particle number fluctuations.

Discrete disorder distribution

Bosons in the presence of random "heavy" impurities

$$p(\epsilon) = p_0\delta(\epsilon) + (1-p_0)\delta(\epsilon-U')$$



$$U(n-1) < \mu < Un$$

$$\frac{2dJ}{U} = \frac{\left(\frac{\mu}{U}-n+1\right)\left(n-\frac{\mu}{U}\right)\left(\frac{\mu_1}{U}-n_1+1\right)\left(n_1-\frac{\mu_1}{U}\right)}{p_0\left(\frac{\mu}{U}+1\right)\left(\frac{\mu_1}{U}-n_1+1\right)\left(n_1-\frac{\mu_1}{U}\right)+p_1\left(\frac{\mu_1}{U}+1\right)\left(\frac{\mu}{U}-n+1\right)\left(n-\frac{\mu}{U}\right)}$$

$$\mu_1 = \mu - U' \qquad U(n_1-1) < \mu_1 < Un_1$$

$$\overline{N(\mu)} = p_0 n + (1-p_0)n_1 \qquad D_n(\mu) = p_0(1-p_0)\,(n-n_1)^2$$

$0 < U'/U < 1$

$$\overline{N(\mu)} = \begin{cases} n-(1-p_0) & \text{if } \frac{\mu}{U}\in\left[n-1, n-1+U'/U\right]\\ n & \text{if } \frac{\mu}{U}\in\left[n-1+U'/U, n\right]\end{cases}$$

$$D_n(\mu) = \begin{cases} p_0(1-p_0) & \text{if } \frac{\mu}{U}\in\left[n-1, n-1+U'/U\right]\\ 0 & \text{if } \frac{\mu}{U}\in\left[n-1+U'/U, n\right]\end{cases}$$

Non-zero temperature

$kT/U = 0$
0.01
0.02