Dimensional Crossover in Trapped Photon Gases

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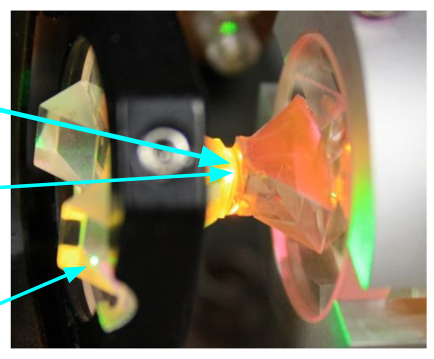
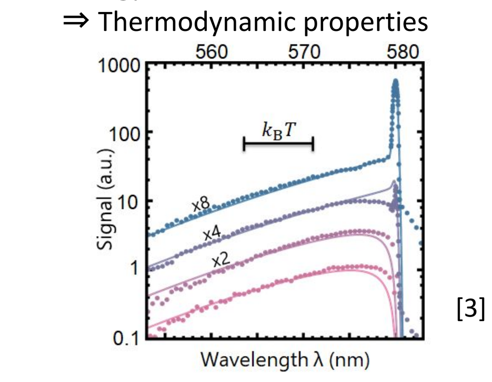
Introduction

Bose-Einstein condensation of photons

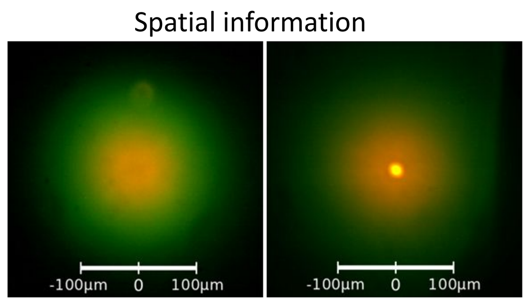
- High finesse cavity: Provides energy cutoff
- Dye solution: Heat and particle reservoir
- Pump radiation: Provides chemical potential



Energy distribution
⇒ Thermodynamic properties



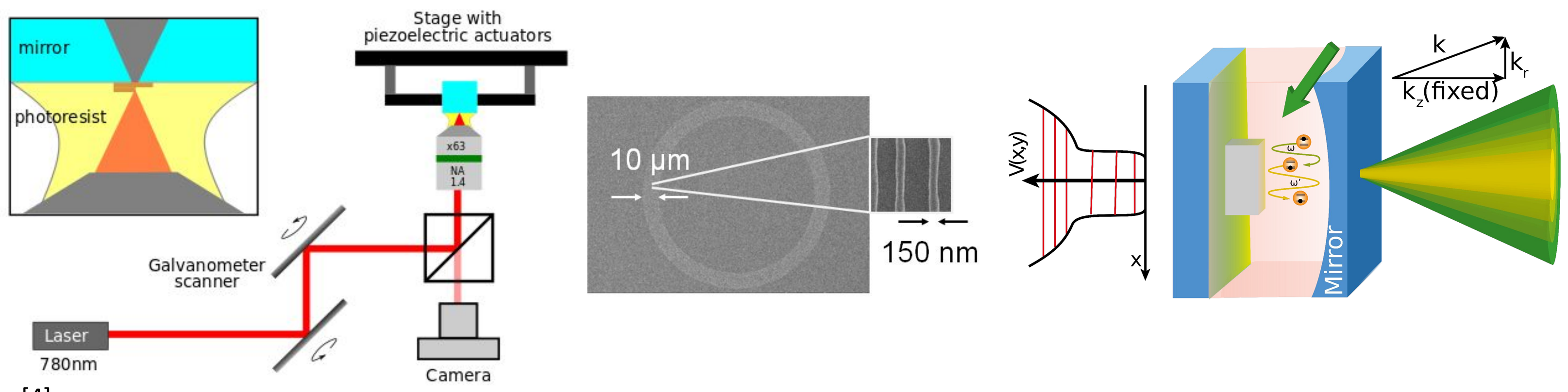
Spatial information



[1] J. Klaers et al., Nature **468**, 545 (2010)
[2] J. Klaers et al., Nat. Phys. **6**, 512 (2010)
[3] T. Damm et al., Nature Commun. **7**, 11340 (2016)

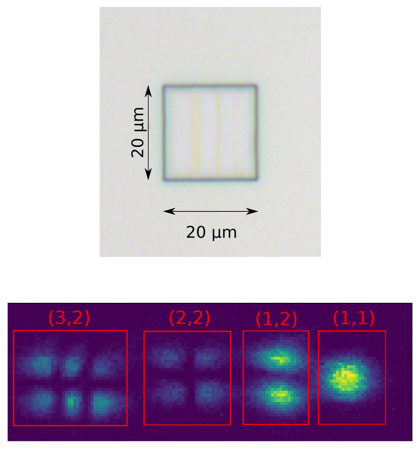
Direct Laser Writing for Realising 1D Potentials for Photon Gases

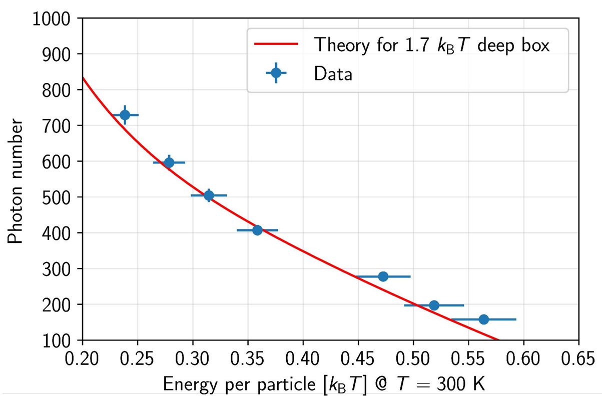
Microstructure mirror surface using DLW for producing dimple traps



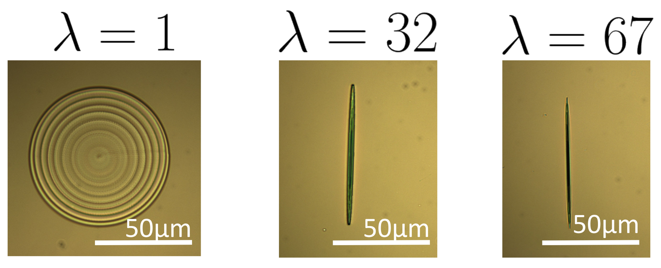
Feasibility test of direct laser writing of structures in a dye microcavity

- 3D printed box potential
- Structural and chemical stability in a dye micro cavity
- Thermalized photon gas



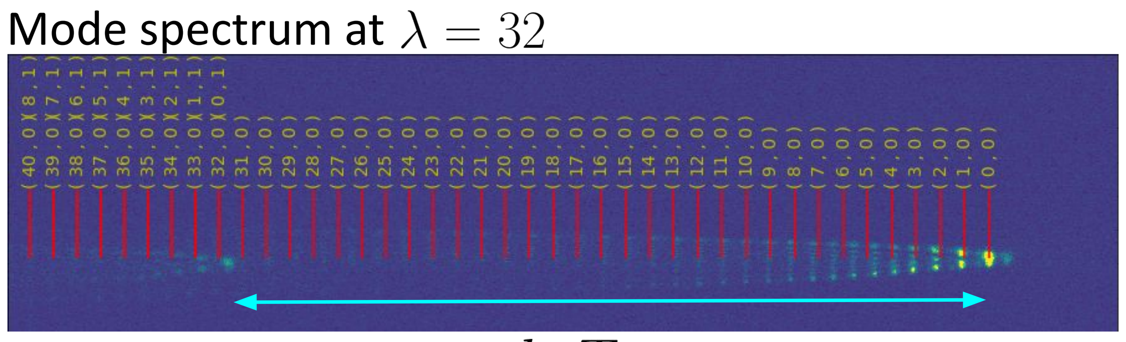


Parabolic structures for dimensional crossover study



Good agreement of mode spectrum with designed aspect ratio of printed harmonic trap

Mode spectrum at $\lambda = 32$

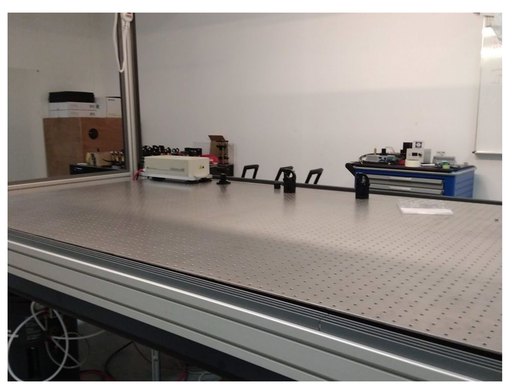


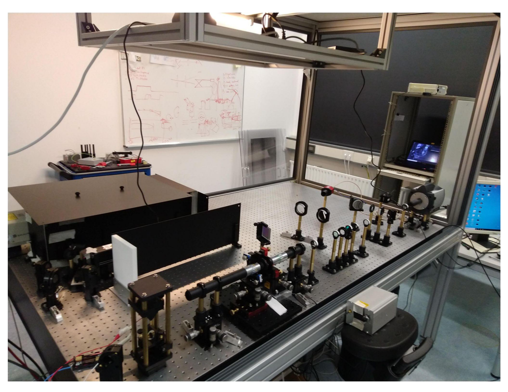
$\omega_x = 2\pi \times 0.21$ THz

$\omega_y = 32 \times \omega_x$

[4] Nanoscribe GmbH Photonic Professional (GT): User Manual
[5] Kirankumar K U, Investigation of 3D-printed potentials for photon gases (Master Thesis)

New Photon BEC Setup @ TU Kaiserslautern

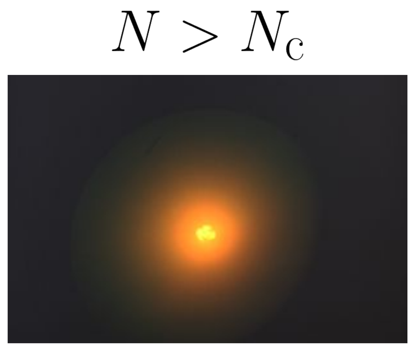




QR code of timelapse

$N < N_c$

$N > N_c$



Outlook

Experiment

- Fabricate and study photon gas in highly anisotropic parabolic structures
- Study thermodynamic properties of dimensional crossover 2D ⇔ 1D

Ideal Bose Gas: Dimensional Crossover (NJP 24, 023013 (2022))

Aim: Theoretical prediction of effective system dimension at dimensional crossover in harmonic trapping potential

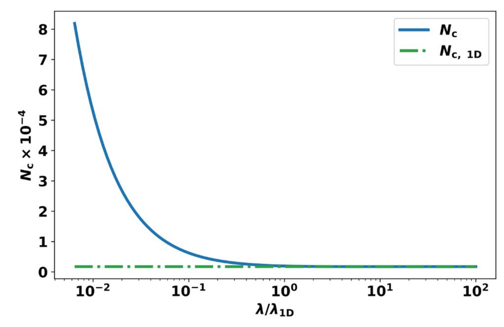
Energy levels: $E_{jn}(\lambda) = \hbar\Omega\left(j + \lambda n + \frac{1+\lambda}{2}\right)$ **, trap-aspect ratio:** $\lambda = \frac{\Omega_y}{\Omega}$

Grand-Canonical Partition Function: $\Pi = \Pi_{1D} + \Delta\Pi$

$\lambda \to \infty$

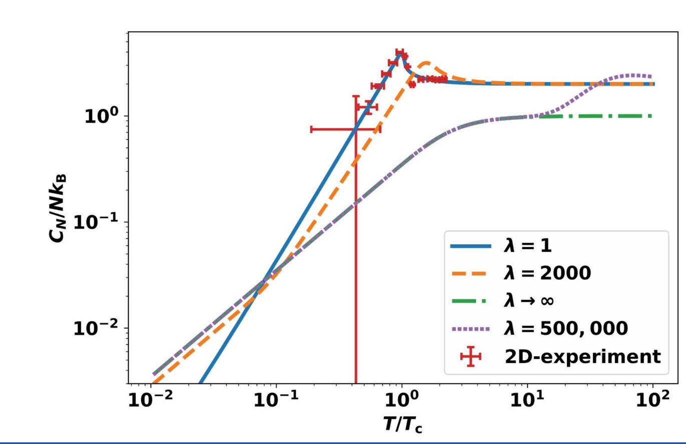
Effective 1D Limit: Π_{1D}

Example: Critical Particle Number at Crossover

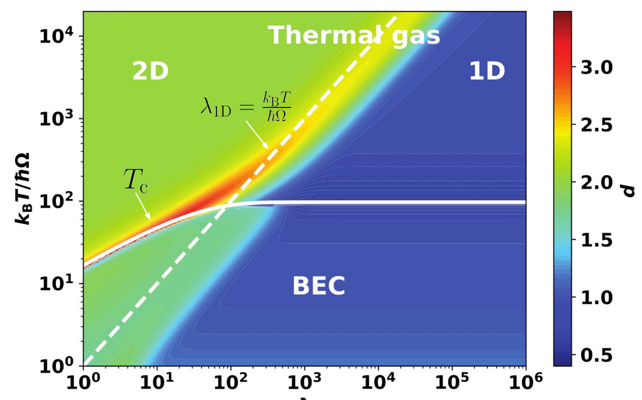


Thermodynamic Quantities:

Specific Heat

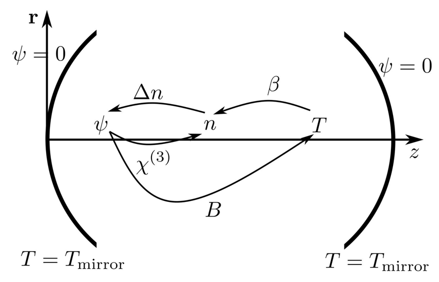


Effective Dimension: $d = \frac{1}{Nk_B}\begin{cases} C_N, & \text{thermal,} \\ \frac{\partial \ln C_N}{\partial \ln \lambda}, & \text{BEC} \end{cases}$



Photon BEC with Interaction (NJP 24, 023032 (2022))

Aim: Behaviour of ground state of photon BEC at dimensional crossover including interaction effects



Photon-Energy Functional:

$$E[\psi,\psi^*] = \int d^2x \left[\frac{\hbar^2}{2m}|\nabla\psi|^2 + \frac{m\Omega^2}{2}(x^2 + \lambda^4 y^2)|\psi|^2 + \frac{g_K}{2}|\psi|^4 + \frac{g_T}{2}\int d^2x'\, \mathcal{G}(\mathbf{x}-\mathbf{x}')|\psi(\mathbf{x})|^2|\psi(\mathbf{x}')|^2\right]$$

Kerr effect | Thermo-optic effect

Minimisation with Gaussian Ansatz:

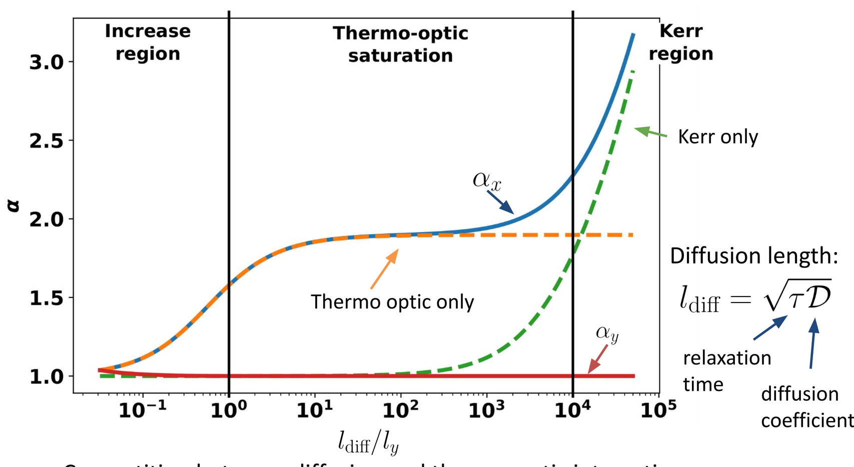
$$\psi = \sqrt{\frac{\lambda N}{\alpha_x \alpha_y \pi l_x^2}} \exp\left[-\frac{1}{2l_x^2}\left(\frac{x^2}{\alpha_x^2} + \lambda^2\frac{y^2}{\alpha_y^2}\right)\right]$$

Oscillator length: $l_x = \sqrt{\frac{\hbar}{m\Omega}}$

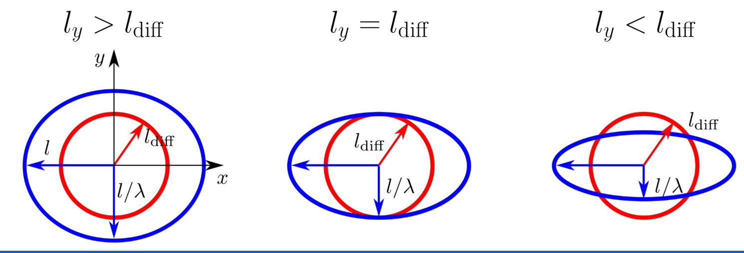
Variational parameters: α_x, α_y

Results:

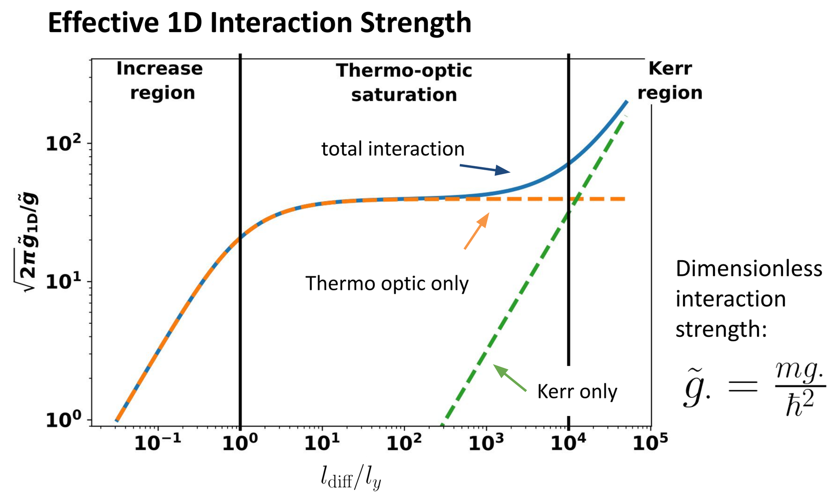
Widths



Competition between diffusion and thermo-optic interaction:



Effective 1D Interaction Strength



Dimensionless interaction strength: $\tilde{g}_\cdot = \frac{m g_\cdot}{\hbar^2}$

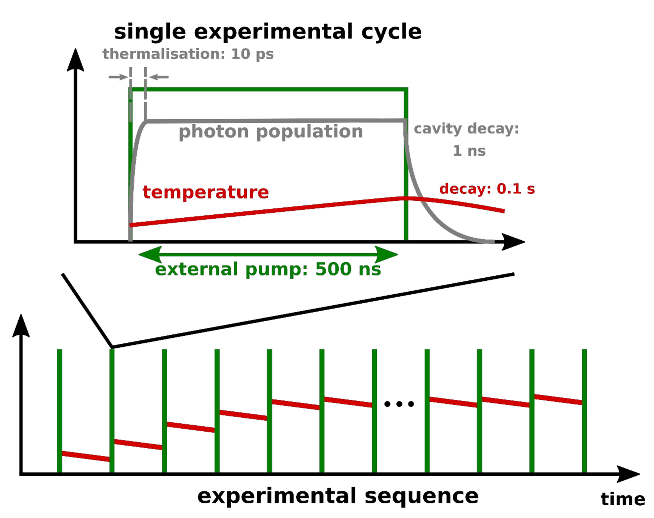
$$\tilde{g}_{1D}(\lambda) = \frac{1}{\sqrt{2\pi}}\left[\tilde{g}_K\lambda + \frac{\tilde{g}_T l_x}{l_{\text{diff}}}\sqrt{\frac{\pi}{2}}e^{l_y^2/2l_{\text{diff}}^2}\operatorname{erfc}\left(\frac{l_y}{\sqrt{2}l_{\text{diff}}}\right)\right]$$

Large trap anisotropy:

$$\lim_{\lambda\to\infty}\tilde{g}_{1D}(\lambda) = \frac{1}{\sqrt{2\pi}}\left(\tilde{g}_K\lambda + \frac{\tilde{g}_T l_x}{l_{\text{diff}}}\right)$$

Exact Diagonalisation of Thermo-Optics (arXiv:2203.16955)

Aim: Behaviour of photon gas in a single pump pulse
⇒ Hartree-Fock analogue



Adiabatic Hamiltonian:

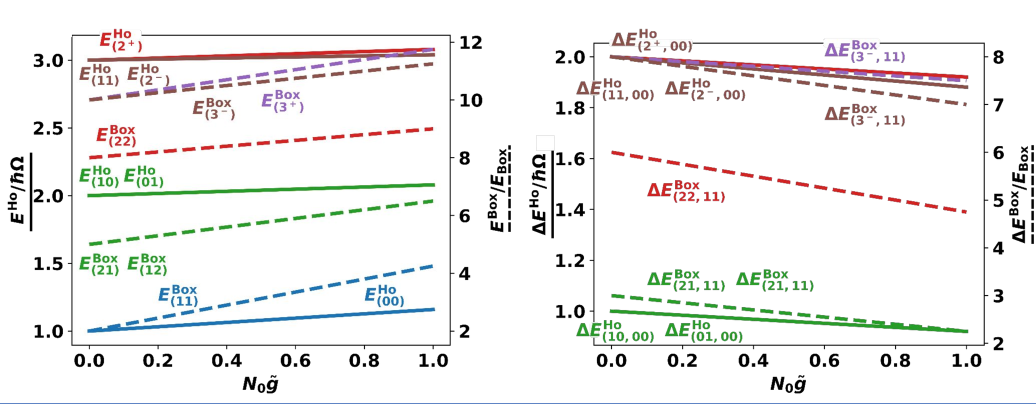
$$\hat{H}(t) = \int d^2x\, \hat{\Psi}^\dagger(\mathbf{x},t)\left\{H^{(0)}(\mathbf{x}) + g(t)n(\mathbf{x},0)\right\}\hat{\Psi}(\mathbf{x},t)$$

Thermo-optic interaction | Initial photon density

(increasing linearly in time)

Benefits: - Thermal cloud included
- Spectroscopic measurement of interaction strength

Perturbative calculation of energy differences:



Condensate width @ dimensional crossover: