Nonlinear Magnetic-Susceptibility of Ideal Quantum Gases

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Ideal Spinor-Quantum Gases

• **Hamiltonian**

$$\hat{H} = \sum_{\sigma=-F}^{F} \sum_{\mathbf{k}} \hat{a}^{+}_{\mathbf{k}\sigma} \left(\frac{\hbar^2 \mathbf{k}^2}{2m} - \mu - \alpha\sigma + U(\mathbf{x}) \right) \hat{a}_{\mathbf{k}\sigma}$$

Harmonic trapping potential: $U(\mathbf{x}) = \frac{m}{2} \sum_{i=1}^{3} \omega_i^2 x_i^2$

Magnetochemical potential: $\alpha = \mu_B g_F B_z$

• **System of Equations**[1, 2]

⋆ Particle-number identity

$$1 = \sum_{\sigma=-F}^{F} C_{\mathbf{0}\sigma} + \frac{T^r}{\eta c} \sum_{\sigma=-F}^{F} \zeta_r\left(\eta \mathrm{e}^{\beta(\mu+\alpha\sigma)}\right)$$

$$C_{\mathbf{0}\sigma} := \tfrac{N_{\mathbf{0}\sigma}}{N}; \quad r := \begin{cases} 3/2 & \text{homogenous} \\ 3 & \text{trapped} \end{cases}; \quad \eta := \begin{cases} 1 & \text{bosons} \\ 0 & \text{MB-gas} \\ -1 & \text{fermions} \end{cases}$$

⋆ Magnetization identity

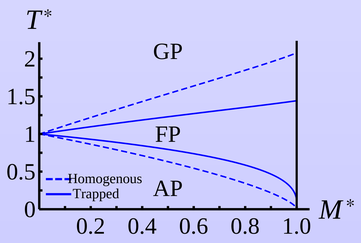
$$M^* = \sum_{\sigma=-F}^{F} \sigma C_{\mathbf{0}\sigma} + \frac{T^r}{\eta c} \sum_{\sigma=-F}^{F} \sigma \zeta_r\left(\eta \mathrm{e}^{\beta(\mu+\alpha\sigma)}\right)$$

⋆ Stationary Gross-Pitaevskii-equations

$$\left(\varepsilon_0 - \mu - \alpha\sigma\right) C_{\mathbf{0}\sigma} = 0; \quad \sigma = -F, .., F$$

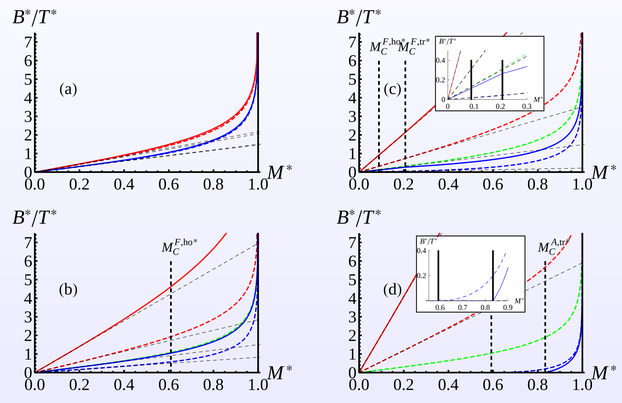
$$\left.\begin{aligned} C_{\mathbf{0}\sigma} &= 0; & \forall\ \sigma \\ C_{\mathbf{0}-F} = .. = C_{\mathbf{0}F-1} &= 0; & \mu = \varepsilon_0 - \alpha F \\ \alpha &= 0; & \mu = \varepsilon_0 \end{aligned}\right\} \rightarrow 3 \text{ Solutions} \rightarrow 3 \text{ Phases}$$

• **Magnetization-Temperature Phase Diagram for $F = 1$-Bosons**[2, 3, 4, 5]



Nonlinear Magnetic-Susceptiblity

• **Nonlinear Magnetic-Susceptibility**[1, 2]



	Homogenous	Trapped	Spin
Bosons	———	- - -	F=1
Fermions	———	- - -	F=1/2
MB−gas	———	- - -	F=1
Lin.approx.	- - - - - - - - - -		

$T^* = 10$ (a), 1.65 (b), 1.1 (c), 0.55 (d)

• **Linear Approximations**[1, 6]

$$\alpha = \theta(T - T_{\mathrm{C}}^0)\, \frac{3k_B T M^*}{F(F+1)}\, \frac{\zeta_r\left(\eta \mathrm{e}^{\beta(\varepsilon_0+\mu)}\right)}{\zeta_{r-1}\left(\eta \mathrm{e}^{\beta(\varepsilon_0+\mu)}\right)} \xrightarrow{T\to\infty} \frac{3k_B T M^*}{F(F+1)}$$

• **MB-Gas**

$$M^{*,\mathrm{MB}}(\alpha, \beta) = F B_F(F\alpha\beta)$$

Brillouin-functions $B_k(x) = \frac{2k+1}{2k} \coth\left(\frac{2k+1}{2k}x\right) - \frac{1}{2k} \coth\left(\frac{1}{2k}x\right)$

• **Discussion**

⋆ $T \to \infty$: Effects of quantum statistics vanish → Classical Curie-behaviour

⋆ $T < T_{\mathrm{C}}^0$: Meissner like effect for non-charged bosons

⋆ Homogenous bosons: 3rd order phase transitions
Trapped bosons: 2nd order phase transitions

⋆ $T \to T_{\mathrm{C}}^0$: No linear susceptibility for bosons

⋆ $T = T_C^0$: Crossing of susceptibilities of homogenous and trapped bosons

⋆ MB-susceptibility formally invariant up to trapping

$T = 0$-Fermions

• **System of Equation**[1, 2]

$$1 = \frac{1}{c} \sum_{\sigma=-F+i}^{F} (\mu + \alpha\sigma)^r$$

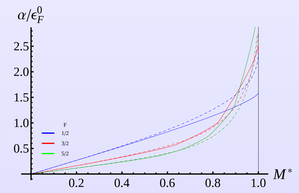
$$M^* = \frac{1}{c} \sum_{\sigma=-F+i}^{F} \sigma(\mu + \alpha\sigma)^r$$

• **Critical Magnetizations**[1, 2]

$$M_{\mathrm{cr}}^{*(i)} = \frac{\mathrm{H}_{2F-i+1}^{(-1-r)}}{\mathrm{H}_{2F-i+1}^{(-r)}} - F; \qquad M_{\mathrm{cr}}^{*(i)} < M^* < M_{\mathrm{cr}}^{*(i+1)}$$

$F > \frac{3}{2}$: Successive exclusion of Zeeman-states at $M_{\mathrm{cr}}^{*(i)}$

• **Nonlinear Magnetic-Susceptibility of $T = 0$-Fermions**



⋆ $F = \frac{1}{2}$: Nonlinear extension of Pauli-susceptibility[1, 2]

$$\mu(T=0) = \tfrac{\varepsilon_F^0}{2}\left[(1+2M^*)^r + (1-2M^*)^r\right]$$

$$\alpha(T=0) = \varepsilon_F^0\left[(1+2M^*)^r - (1-2M^*)^r\right]$$

⋆ $F \geq \frac{3}{2}$: Discontinuities in the 3rd (4th) derivative of the Free-Energy

Outlook: See our recent paper for further discussion[1]

⋆ Border of $T = 0$ cloud is fully polarized.

⋆ Qualitative influence of magnetization on the $T = 0$-structure-factor.

⋆ Critical temperature for fermions?

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References

[1] B. Liebchen, J. Bosse, A. Pelster Ideal Spinor Qunatum Gases, 2010 In Preparation

[2] B. Liebchen Statik und Dynamik von Spinor-Quantengasen *Diplomarbeit. Freie Universität Berlin*, 2010 http://users.physik.fu-berlin.de//~pelster/Theses/liebchen.pdf

[3] T. Isoshima, T. Ohmi, K. Machida Double phase transitions in magnetized spinor Bose-Einstein condensation *Journal-Physical Society of Japan*, 69:3864, 2000

[4] W. Zhang, S. Yi, L. You Bose-Einstein condensation of trapped interacting spin-1 atoms *Physical Review A*, 70:43611, 2004.

[5] S.-P. Parvis Thermodynamic Properties of $F = 1$ Spinor-Bose-Einstein Condensates *Diplomarbeit. Freie Universitaet Berlin*, 2006 http://users.physik.fu-berlin.de/~pelster/Theses/soltan.pdf

[6] J. Bosse, K.N. Pathak, G.S. Singh Dynamics of Uniform Quantum Gases II: Magnetic Susceptibility *Physica A: Statistical Mechanics and its Applications*, 389:1173, 2010.