Spinor Fermi Gases

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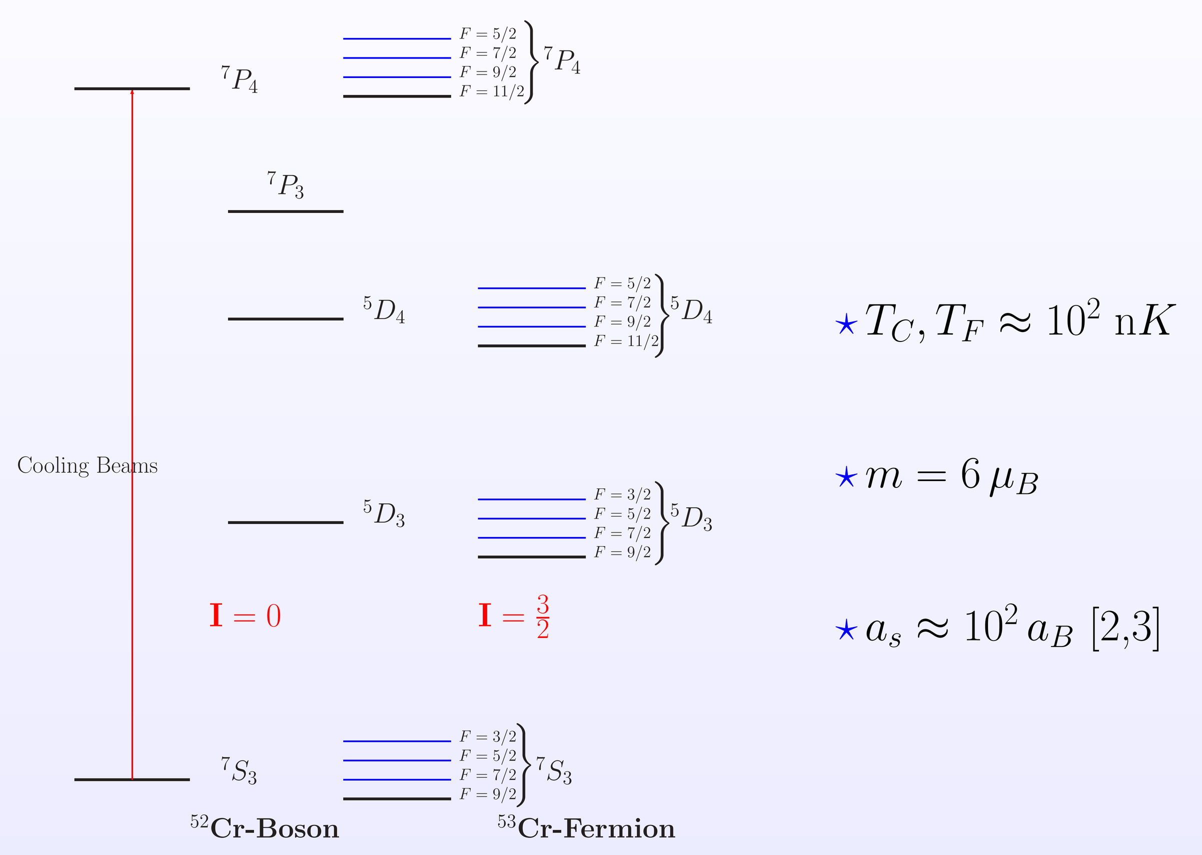
Chromium 53: A Spinor Fermi Gas

• ^{52}Cr × ^{53}Cr: Lowest Energy Levels [1]



⋆ $T_C, T_F \approx 10^2$ nK

⋆ $m = 6\,\mu_B$

⋆ $a_s \approx 10^2\,a_B$ [2,3]

• Ideal Spinor Fermi Gas

⋆ Partition function

$$\mathcal{Z}^{(0)} = \oint \mathcal{D}\Psi^\dagger \oint \mathcal{D}\Psi \, \exp\left(-\mathcal{A}^{(0)}[\Psi^\dagger, \Psi]/\hbar\right)$$

⋆ Ensemble average

$$\langle \bullet \rangle = \frac{\oint \mathcal{D}\Psi^\dagger \oint \mathcal{D}\Psi \, \bullet \exp\left(-\mathcal{A}^{(0)}[\Psi^\dagger, \Psi]/\hbar\right)}{\oint \mathcal{D}\Psi^\dagger \oint \mathcal{D}\Psi \, \exp\left(-\mathcal{A}^{(0)}[\Psi^\dagger, \Psi]/\hbar\right)}$$

⋆ Free action

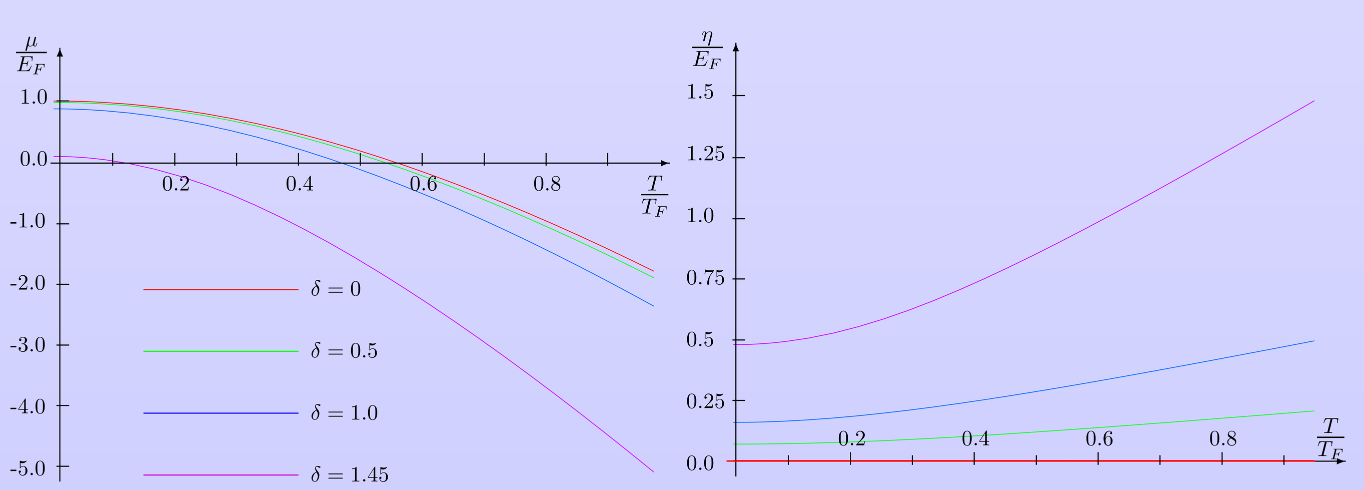
$$\mathcal{A}^{(0)}[\Psi^\dagger, \Psi] = \int_0^{\hbar\beta} d\tau \int d^3x \, \psi_i^*(\mathbf{x},\tau)\mathcal{L}_{ij}\psi_j(\mathbf{x},\tau)$$

$$\mathcal{L}_{ij} = \left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2}{2M}\Delta + U(\mathbf{x}) - \mu\right]\delta_{ij} - \eta F^z_{ij}$$

Lagrange parameters

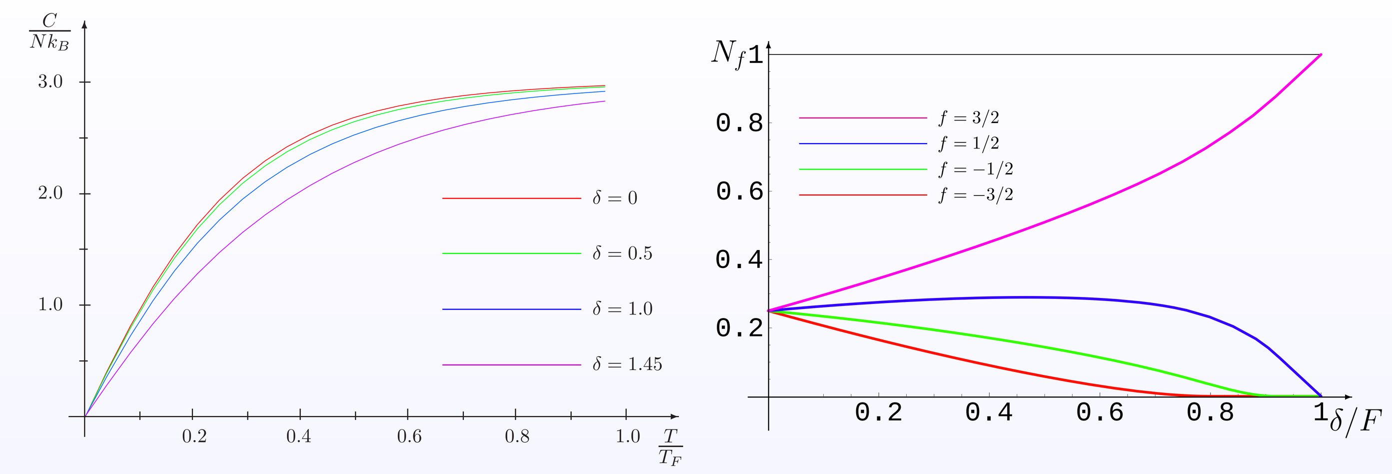
Harmonic trap: $U(\mathbf{x}) = \sum_{i=1}^{3} \frac{1}{2} M\omega_i^2 x_i^2$

• Chemical and Magneto-Chemical Potential ($F = 3/2$)



$$\delta = \langle \Psi^\dagger F^z \Psi \rangle / \langle \Psi^\dagger \Psi \rangle = \text{Magnetization per particle}$$

• Heat Capacity and Occupation Numbers ($F = 3/2$)



Threshold Magnetization $N_f(\delta_{th}^{(f)}) = 0$

Example: $\delta_{th}^{(-F)} = \frac{(1+F)(1+3F)(-1+6F)}{15F(1+2F)}$ $\qquad \delta_{th}^{(-3/2)} = 11/9 \to 81\%\, 3/2$

Weakly Interacting Fermi Gas

• Perturbation Theory at T = 0

$$\mathcal{A}^{(\text{int})}[\Psi, \Psi^*] = \frac{1}{2}\int_0^{\hbar\beta} d\tau \int d^3x \int d^3x' \, V^{(\text{int})}_{ij i'j'}(\mathbf{x},\mathbf{x}')\psi_i^*(\mathbf{x},\tau)\psi_j(\mathbf{x},\tau)\psi_{i'}^*(\mathbf{x}',\tau)\psi_{j'}(\mathbf{x}',\tau)$$

$$V^{(\text{int})}_{iji'j'}(\mathbf{x},\mathbf{x}') = V^{(\text{contact})}_{iji'j'}(\mathbf{x},\mathbf{x}') + V^{(\text{dd})}_{iji'j'}(\mathbf{x},\mathbf{x}')$$

1) Contact Interaction

⋆ General form [4,5]

$$V^{(\text{contact})}(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x}-\mathbf{x}') \sum_{f=|F_1-F_2|}^{F_1+F_2} g_f \mathcal{P}_f, \qquad g_f = \frac{4\pi\hbar^2 a_f}{M}$$

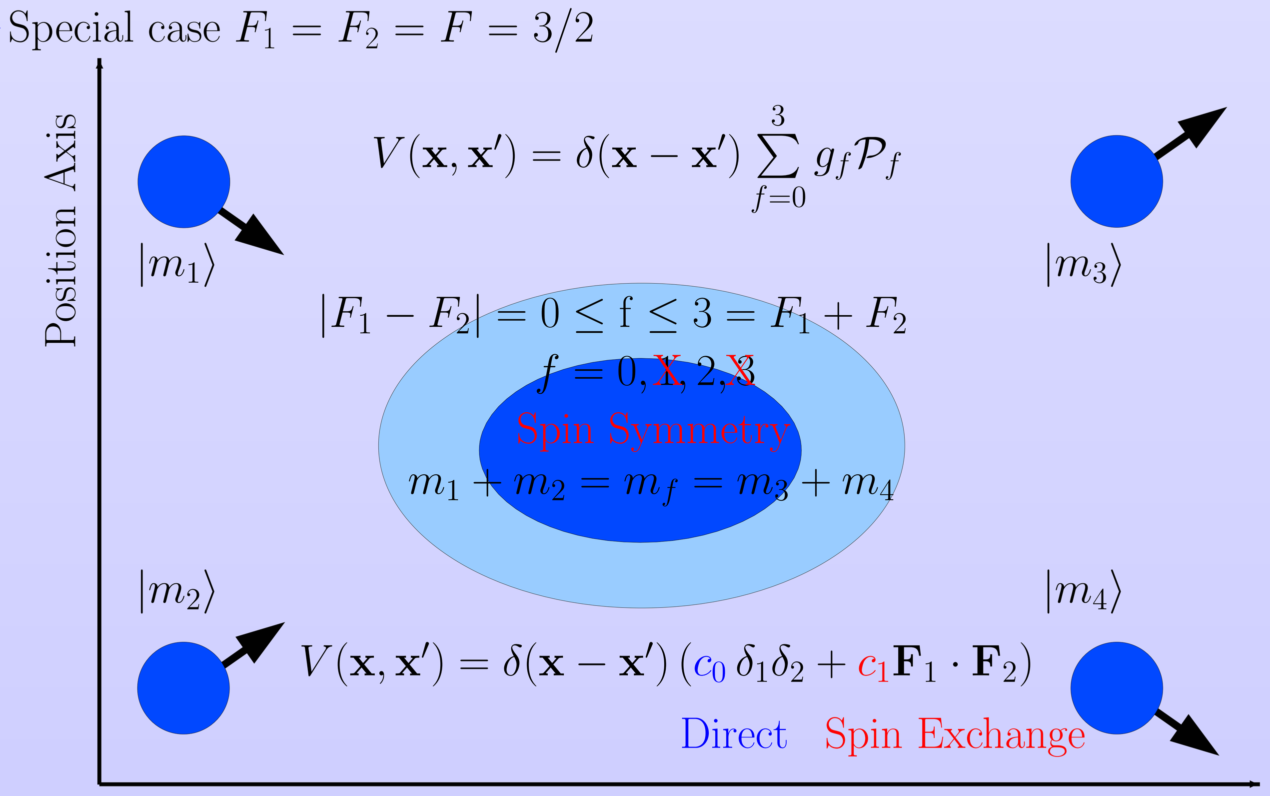
a_f: scattering length in $|\mathbf{F}_1 + \mathbf{F}_2| = f$-channel

$\mathcal{P}_f$ projects $|F_1, F_2; m_1, m_2\rangle$ into $|F_1, F_2; f, m\rangle$:

$$\sum_{f=|F_1-F_2|}^{F_1+F_2} \lambda_f^n \mathcal{P}_f = (\mathbf{F}_1 \cdot \mathbf{F}_2)^n, \;\; \lambda_f = \tfrac{1}{2}\left[f(f+1) - F_1(F_1+1) - F_2(F_2+1)\right]$$

$$V^{(\text{contact})}_{iji'j'}(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x}-\mathbf{x}') \sum c_n \left(\mathbf{F}_{ij} \cdot \mathbf{F}_{i'j'}\right)^n$$

⋆ Special case $F_1 = F_2 = F = 3/2$



$$c_0 = \frac{(-g_0 + 5g_2)}{4} \qquad c_1 = \frac{(-g_0 + g_2)}{3}$$

• First-Order Correction to Ground-State Energy

⋆ Homogeneous case $\left(F = 1/2, E_F = \left(3\pi^2 n\right)^{2/3}\hbar^2/2M\right)$ [6]

$$\frac{E}{NE_F} = -\frac{1}{5}\left[(1+2\delta)^{5/6} + (1-2\delta)^{5/6}\right] + \frac{1}{2}\left[(1+2\delta)^{4/3} + (1-2\delta)^{4/3}\right] + \frac{2ak_F}{3\pi}(1+2\delta)^{1/3}(1-2\delta)^{1/3}$$

⋆ Harmonic case $\left(F = 3/2, E_F = \hbar\overline{\omega}(3N/2)^{1/3}\right)$

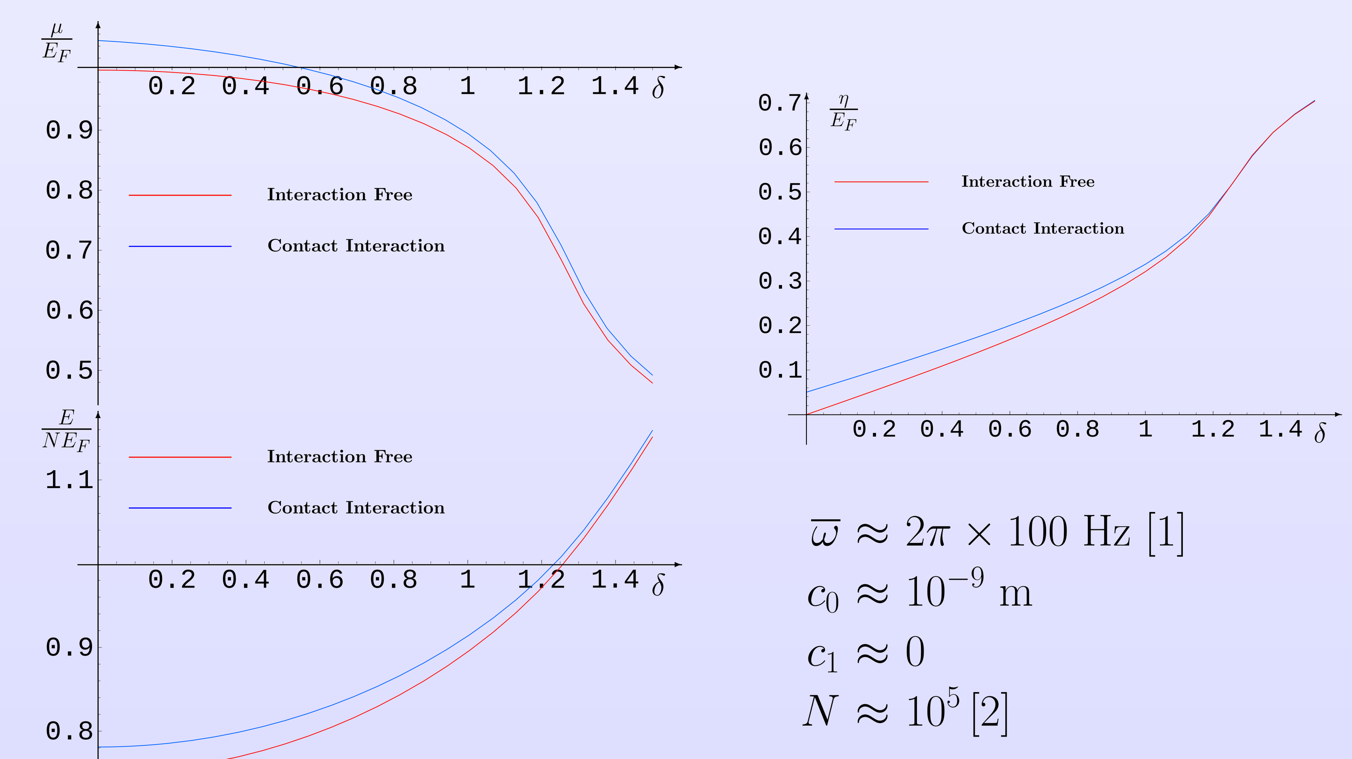
$$\frac{E}{NE_F} = \frac{3}{4} + \frac{7N^{1/6}}{\overline{a}_{\text{osc}} 2^{1/3}\sqrt{\pi}}\Bigg\{(5a_2 - a_0)\sum_{f,f'\neq f} A(y_<, y_>)\left[1 + f(f'+1)\right] + \frac{(a_2 - a_0)}{6}\sum_{f=-3/2}^{3/2}\left[\left(\frac{3}{2}-f\right)\left(\frac{5}{2}+f\right)A(y_f, y_{f+1}) + \left(\frac{3}{2}+f\right)\left(\frac{5}{2}-f\right)A(y_{f-1}, y_f)\right]\Bigg\}$$

$$\overline{a}_{\text{osc}} = \sqrt{\hbar/M\overline{\omega}}, \qquad \overline{\omega} = (\omega_x\omega_y\omega_z)^{1/3}$$

$$A(y_<, y_>) = {}_2F_1\left(-\tfrac{3}{2}, \tfrac{3}{2}; 4; \tfrac{y_<}{y_>}\right)\frac{y_<^3}{\Gamma(4)}\frac{y_>^{3/2}}{\Gamma(5/2)}\Theta(y_<)$$

$y_<$ ($y_>$) is the smallest (largest) of y_f and $y_{f'}$; $y = \mu + f\eta$

• Chemical, Magneto-Chemical Potential and Ground-state Energy ($F = 3/2$)



$\overline{\omega} \approx 2\pi \times 100$ Hz [1]

$c_0 \approx 10^{-9}$ m

$c_1 \approx 0$

$N \approx 10^5$ [2]

2) Dipole-Dipole Interaction

$$V^{(\text{dd})}_{iji'j'}(\mathbf{x},\mathbf{x}') = \frac{\mu_0\, m^2}{4\pi}\left\{\frac{\mathbf{F}_{ij}\cdot\mathbf{F}_{i'j'}}{|\mathbf{x}-\mathbf{x}'|^3} - 3\frac{\left[\mathbf{F}_{ij}\cdot(\mathbf{x}-\mathbf{x}')\right]\left[\mathbf{F}_{i'j'}\cdot(\mathbf{x}-\mathbf{x}')\right]}{|\mathbf{x}-\mathbf{x}'|^5}\right\}$$

• Well-understood in polarized ^{52}Cr [7–9] and bosonic spinors [10]

• Already introduced in polarized Fermi gases [11]

Perspectives

• Dipolar Mixtures: ^{52}Cr and ^{53}Cr can be simultaneously trapped [1]

• Extension of the BCS-BEC crossover theory [12–14]

1) Spin degree of freedom

2) Dipolar interaction

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