Nonequilibrium Quantum Phase Transitions in a Hybrid Atom-Optomechanical System

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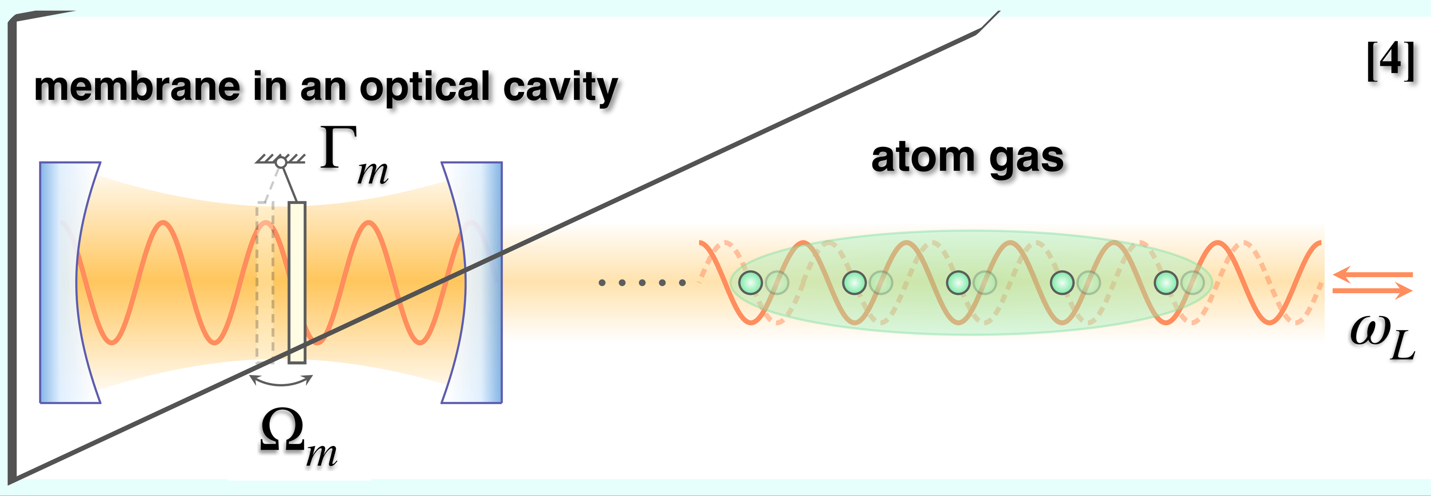
Motional Atom-Membrane Coupling

- **laser** field **induces** effective **long-range coupling** between **atoms** and **membrane**
- **displaced membrane** leads to linear displacement **force** on **atoms**
- **back-action by** displaced center-of-mass (CoM) position of **atoms** which **redistribute photons** and alter **radiation pressure** on **membrane**

$$i\partial_t \psi = \left[V\sin^2(z) - \omega_R \partial_z^2 + Ng|\psi|^2 - 2\sqrt{N\lambda}\mathrm{Re}(\alpha)\sin(2z)\right]\psi$$
$$i\partial_t \alpha = (\Omega_m - i\Gamma_m)\,\alpha - \sqrt{N\lambda}\int dz\,\sin(2z)|\psi|^2$$

- **Gaussian ansatz** for condensate wave function with 4 variables: $\zeta(t), \sigma(t), \kappa(t), \eta(t)$

$$\psi(z,t) = \left(\frac{1}{\pi\sigma(t)^2}\right)^{1/4}\exp\left(-\frac{[z-\zeta(t)]^2}{2\sigma(t)^2} + i\kappa(t)z + i\eta(t)z^2\right)$$



Cumulant Expansion with Gaussian Ansatz [1]

- **zeroth** order cumulant ⇒ time evolution of **weight**: $c_-(t), c_+(t)$
- **first** order cumulant ⇒ time evolution of CoM **position**: $\zeta(t), \kappa(t)$
- **second** order cumulant ⇒ time evolution of **width**: $\sigma(t), \eta(t)$

Nonequilibrium Quantum Phase Transition [1]

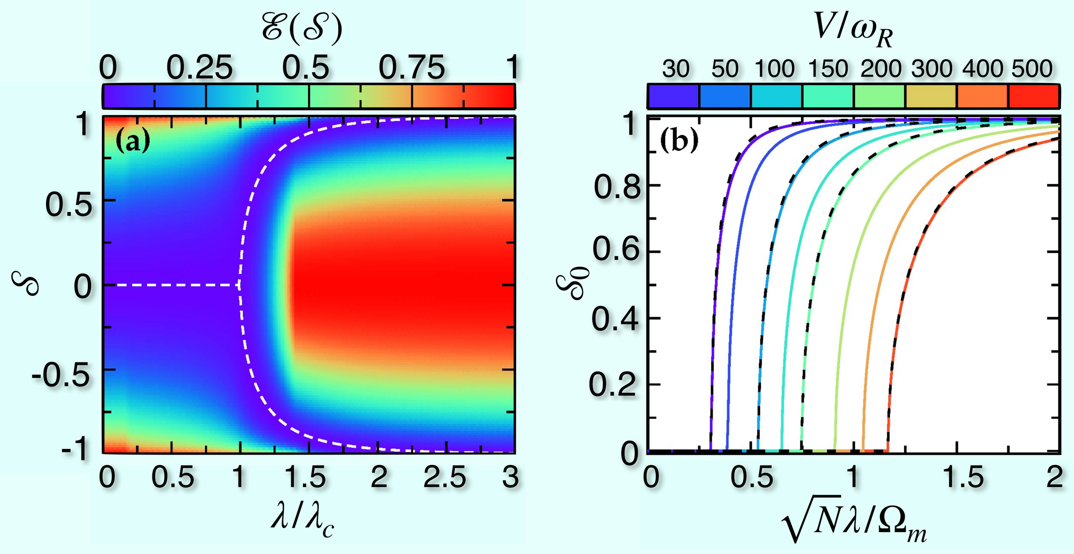
- reduced **equations of motion** for variational parameters:

$$\dot{\alpha} = -i\partial_{\alpha^*}E - \Gamma_m \alpha$$
$$(2\omega_R)^{-1}\ddot{\zeta} = -\partial_\zeta E$$
$$(4\omega_R)^{-1}\ddot{\sigma} = -\partial_\sigma E$$

- **potential energy surface** with $\mathcal{S} = \sin(2\zeta)$

$$E[\alpha,\mathcal{S},\sigma] = \Omega_m|\alpha|^2 + \frac{\omega_R}{2\sigma^2} + \frac{Ng}{\sqrt{8\pi}\sigma} - \frac{V}{2}\sqrt{1-\mathcal{S}^2}e^{-\sigma^2} - \frac{\sqrt{N\lambda}}{2}\mathrm{Re}(\alpha)\mathcal{S}e^{-\sigma^2}$$

- traces of **NQPT** via **energy surface** and **steady state**



- **critical** atom-membrane **coupling**: $a_2(\lambda_c) = 0$

$$\omega_R + \frac{Ng}{\sqrt{8\pi}}\sqrt{2\ln\frac{\lambda_c}{\lambda_V}} = 4V\left(\frac{\lambda_V}{\lambda_c}\ln\frac{\lambda_c}{\lambda_V}\right)^2$$

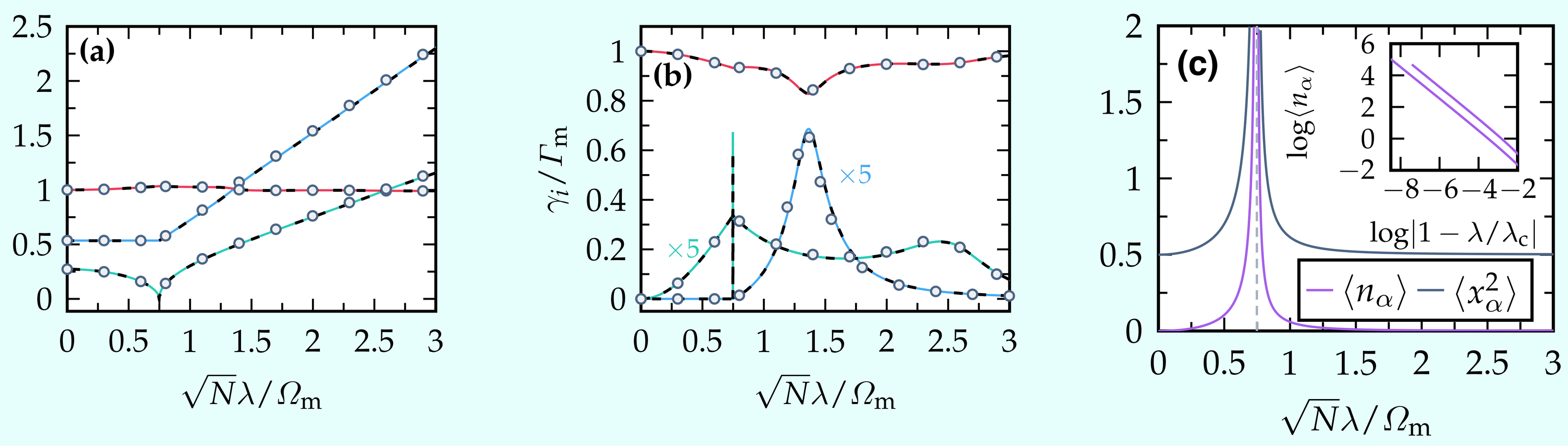
$$\sqrt{N\lambda_V} = \frac{1}{2}\sqrt{\left(\frac{\Omega_m^2+\Gamma_m^2}{\Omega_m}\right)V}$$

- **Landau expansion**: $E[\mathcal{S}] = a_0 + a_2(\lambda)\mathcal{S}^2 + a_4(\lambda)\mathcal{S}^4 + \mathcal{O}(\mathcal{S}^6) \quad \Rightarrow \quad a_4(\lambda_c) > 0$

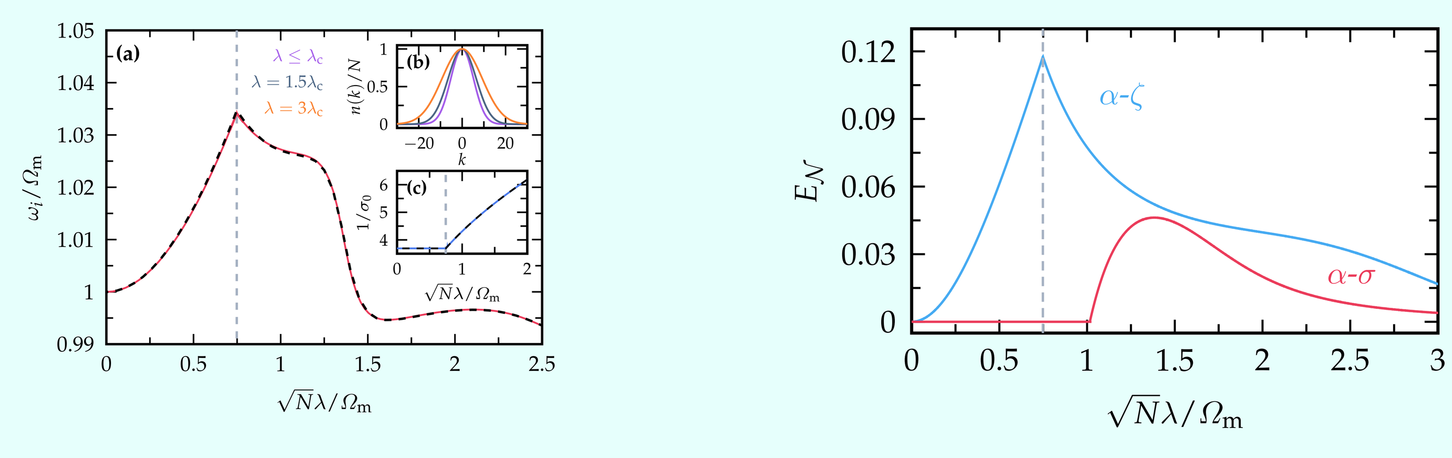
always 2nd order NQPT

Collective Phenomena & Experimental Measurement

- collective excitation **frequencies** and **decay rates**
- **membrane displacement variance** (critical exponent -1)



- **experimentally accessible** quantities: ω_α, $n(k)$, σ_0
- atom-membrane **entanglement** in **steady state** (with vacuum fluctuations)

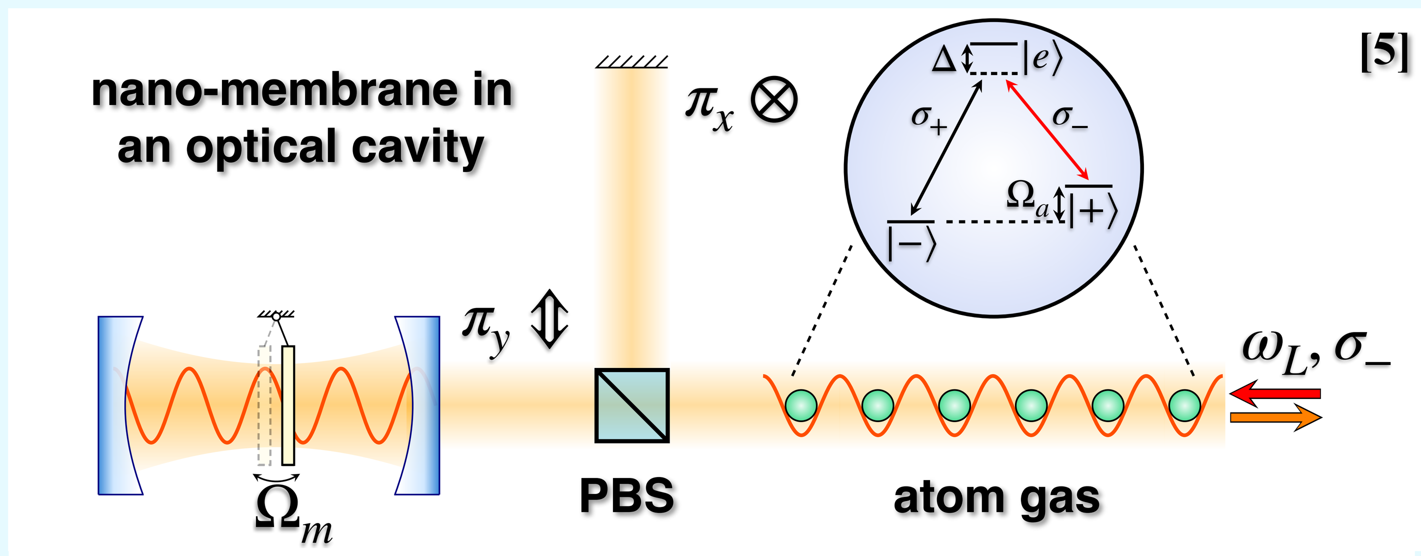


Internal State Atom-Membrane Coupling

- **polarization beam splitter** (PBS) utilized to allow for **polarization rotation**
- **displaced membrane** induces phase shift of light which translates to **polarization rotation** in PBS and may induce **two-photon transition** $|-\rangle \leftrightarrow |+\rangle$
- **back-action** induced by a transition of **atoms** under the emission of a σ_+ **photon** which alters **radiation pressure** on membrane

$$i\partial_t\psi_- = \left[-\frac{\Omega_a}{2} - \omega_R\partial_z^2 - \frac{V}{2}\cos(2z) + N\sum_{\tau=\pm} g_{\tau-}|\psi_\tau|^2\right]\psi_- - \sqrt{N\lambda}\mathrm{Re}(\alpha)\cos(2z)\psi_+$$
$$i\partial_t\psi_+ = \left[\frac{\Omega_a}{2} - \omega_R\partial_z^2 - \frac{V}{2}\cos(2z) + N\sum_{\tau=\pm} g_{\tau+}|\psi_\tau|^2\right]\psi_+ - \sqrt{N\lambda}\mathrm{Re}(\alpha)\cos(2z)\psi_- - 2\sqrt{N\lambda}\chi\mathrm{Re}(\alpha)\cos(2z)\psi_+$$
$$i\partial_t\alpha = (\Omega_m - i\Gamma_m)\,\alpha - \sqrt{N\lambda}\int dz\,\cos(2z)\mathrm{Re}(\psi_+^*\psi_-) - \sqrt{N\lambda}\chi\int dz\,\cos(2z)|\psi_+|^2$$

- **Gaussian ansatz** for condensate with 4 variables: $c_\tau(t), \sigma(t), \eta(t)$ with $\psi_\tau(z,t) = c_\tau\psi(z,t)$



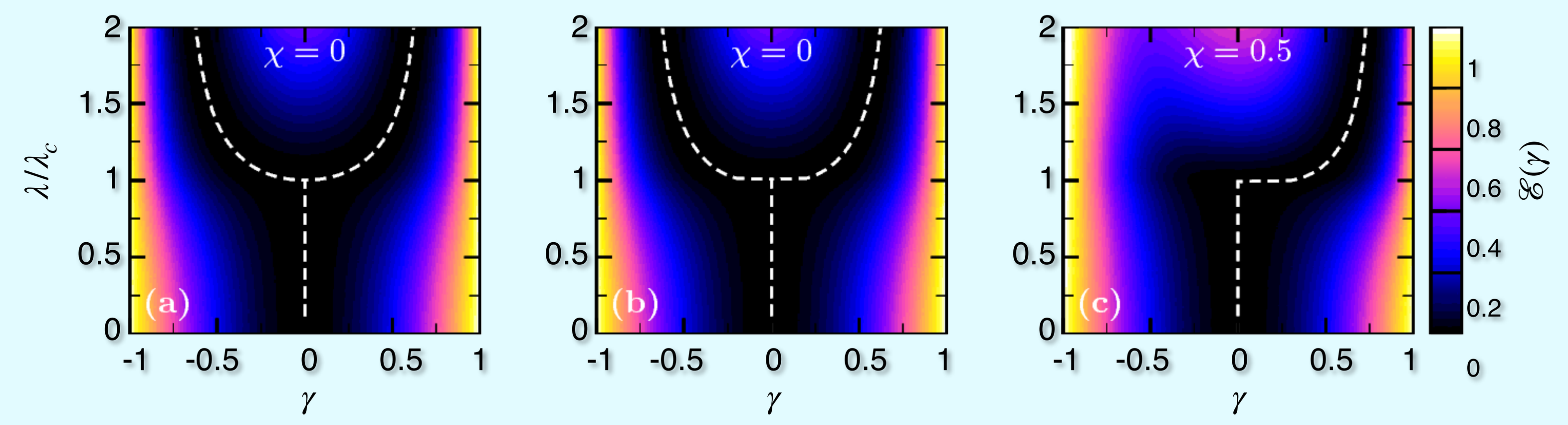
Tuning the Order of a Quantum Phase Transition [2]

- reduced **equations of motion** for variational parameters:

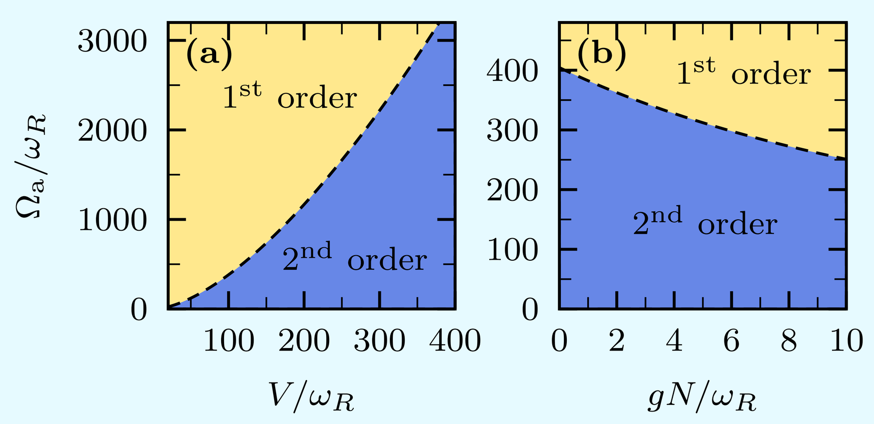
$$\dot{\alpha} = -i\partial_{\alpha^*}E - \Gamma_m\alpha$$
$$\dot{c}_\tau = -i\partial_{c_\tau^*}E$$
$$(4\omega_R)^{-1}\ddot{\sigma} = -\partial_\sigma E$$

$$E[\alpha,c_\tau,\sigma] = \Omega_m|\alpha|^2 + \frac{\Omega_a}{2}\left[|c_+|^2 - |c_-|^2\right] + \left[\frac{\omega_R}{2\sigma^2} + \frac{Ng}{\sqrt{8\pi}\sigma} - \frac{V}{2}e^{-\sigma^2}\right] - \sqrt{N\lambda}\left[\chi|c_+|^2 + 2\mathrm{Re}(c_+^*c_-)\right]\mathrm{Re}(\alpha)e^{-\sigma^2}$$

- normalized potential **energy surface** and **steady state** (dashed) with $c_+ \equiv \gamma$



- **always 1st order** NQPT if $\chi \neq 0$
- **either continuous** or **discontinuous** NQPT if $\chi = 0$



- **critical point** at which **2nd** order becomes **1st** order NQPT:

$$\Omega_c = \frac{\omega_\sigma^2}{32\omega_R\sigma_0^2}$$

1st order	2nd order
$\Omega_a \geq \Omega_c$	$\Omega_a < \Omega_c$

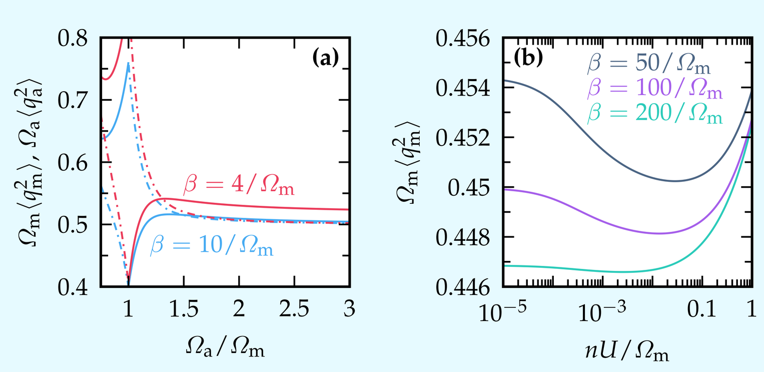
ω_σ : breathing mode frequency

Enhanced Membrane Squeezing by Atom Interactions [3]

- **lattice model**

$$H = \Omega_m b^\dagger b + \frac{\Omega_a}{2}\sum_{j\tau}\tau d_{j\tau}^\dagger d_{j\tau} - J\sum_{\langle ij\rangle\tau} d_{i\tau}^\dagger d_{j\tau} + \frac{U}{2}\sum_{j\tau\tau'} d_{j\tau}^\dagger d_{j\tau'}^\dagger d_{j\tau'} d_{j\tau} - \frac{\Lambda}{2}(a^\dagger + a)\sum_j\left(d_{j+}^\dagger d_{j-} + \mathrm{H.c.}\right)$$

- **Bogoliubov prescription** combined with **path integral formalism**
- **analytic solution** of thermal **observables**



References

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