Out-of-equilibrium dynamics of ultracold bosons in time-dependent random potentials

Milan Radonjić and Axel Pelster

Department of Physics, Technische Universität Kaiserslautern, Germany

Abstract: We investigate the impact of time-dependent random potentials on a weakly interacting Bose gas. Our ultimate aim is to explore and exploit the versatility of various scenarios of driven time-dependent random disordered potentials for studying out-of-equilibrium dynamics of ultracold bosons. Here we study smooth quench of the disordered potential from the initial disorder-free state. Depending on the quench rise time we focus on two limiting cases: adiabatic and sudden quench. In the long-time limit the former scenario reproduces the static disorder equilibrium case [1], while the latter leads to the formation of a non-equilibrium steady state.

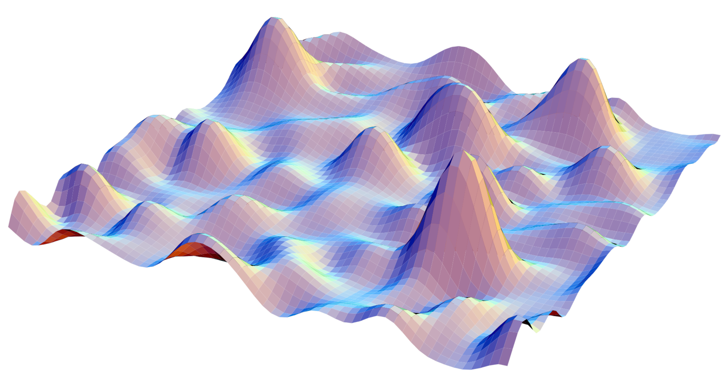
Ultracold bosons in random potentials

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + \mathcal{U}(\mathbf{x},t) - \mu_0 + g|\Psi(\mathbf{x},t)|^2\right]\Psi(\mathbf{x},t)$$

System is initially in equilibrium: $\Psi(\mathbf{x},0) = \sqrt{n}, \quad \mu_0 = gn$

Externally driven smooth quench of random potential:

$$\mathcal{U}(\mathbf{x},t) = \begin{cases} 0, & t \le 0 \\ f(t)u(\mathbf{x}), & t > 0 \end{cases}$$



Properties of driving function:

$$f(0) = 0, \qquad \lim_{t\to\infty} f(t) = 1$$

Statistical characterization of disorder:

$$\langle u(\mathbf{x})\rangle = 0, \quad \langle u(\mathbf{x})u(\mathbf{x}')\rangle = \mathcal{R}(\mathbf{x}-\mathbf{x}')$$

Off-diagonal long-range order and condensate density [2]:

$$n_0(t) = \lim_{|\mathbf{x}-\mathbf{x}'|\to\infty}\langle\Psi(\mathbf{x},t)\Psi^*(\mathbf{x}',t)\rangle = |\langle\Psi(\mathbf{x},t)\rangle|^2$$

Particle density: $n = \langle|\Psi(\mathbf{x},t)|^2\rangle$

Condensate depletion: $q(t) = n - n_0(t)$

Perturbative treatment of weak disorder

Perturbative ansatz à la [3]:

$$\Psi(\mathbf{x},t) = \Psi_0(t) + \Psi_1(\mathbf{x},t) + \Psi_2(\mathbf{x},t) + \ldots$$

$$|\Psi_\alpha(\mathbf{x},t)| \sim |u(\mathbf{x})|^\alpha, \quad \Psi_\alpha(\mathbf{x},0) = 0, \quad \alpha \ge 1$$

Zeroth order solution: $\Psi_0(t) = \sqrt{n}$

First order equations:

$$i\hbar\frac{\partial}{\partial t}\Psi_1(\mathbf{x},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + \mu_0\right)\Psi_1(\mathbf{x},t) + \mu_0\Psi_1^*(\mathbf{x},t) + \sqrt{n}f(t)u(\mathbf{x})$$

$$-i\hbar\frac{\partial}{\partial t}\Psi_1^*(\mathbf{x},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + \mu_0\right)\Psi_1^*(\mathbf{x},t) + \mu_0\Psi_1(\mathbf{x},t) + \sqrt{n}f(t)u(\mathbf{x})$$

Second order equations:

$$i\hbar\frac{\partial}{\partial t}\Psi_2(\mathbf{x},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + \mu_0\right)\Psi_2(\mathbf{x},t) + \mu_0\Psi_2^*(\mathbf{x},t) + \mathcal{F}(\mathbf{x},t)$$

$$-i\hbar\frac{\partial}{\partial t}\Psi_2^*(\mathbf{x},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + \mu_0\right)\Psi_2^*(\mathbf{x},t) + \mu_0\Psi_2(\mathbf{x},t) + \mathcal{F}^*(\mathbf{x},t)$$

$$\mathcal{F}(\mathbf{x},t) = \left[f(t)u(\mathbf{x}) + g\sqrt{n}\left(\Psi_1(\mathbf{x},t) + 2\Psi_1^*(\mathbf{x},t)\right)\right]\Psi_1(\mathbf{x},t)$$

$$\mathcal{F}^*(\mathbf{x},t) = \left[f(t)u(\mathbf{x}) + g\sqrt{n}\left(\Psi_1^*(\mathbf{x},t) + 2\Psi_1(\mathbf{x},t)\right)\right]\Psi_1^*(\mathbf{x},t)$$

Condensate depletion

Exponentially rising driving function: $f_\tau(t) = 1 - e^{-t/\tau}$

Stationary condensate depletion in 3D:

$$q_\tau \equiv \lim_{t\to\infty} q_\tau(t) = n\int_{\mathbb{R}^3}\frac{\mathrm{d}^3k}{(2\pi)^3}\hat{\mathcal{R}}(\mathbf{k})\left[\frac{\hbar^2\omega_\mathbf{k}^2}{\hbar^4\Omega_\mathbf{k}^4} + \frac{\hbar^2\omega_\mathbf{k}^2 + \hbar^2\Omega_\mathbf{k}^2}{2\hbar^4\Omega_\mathbf{k}^4(1+\Omega_\mathbf{k}^2\tau^2)}\right]$$

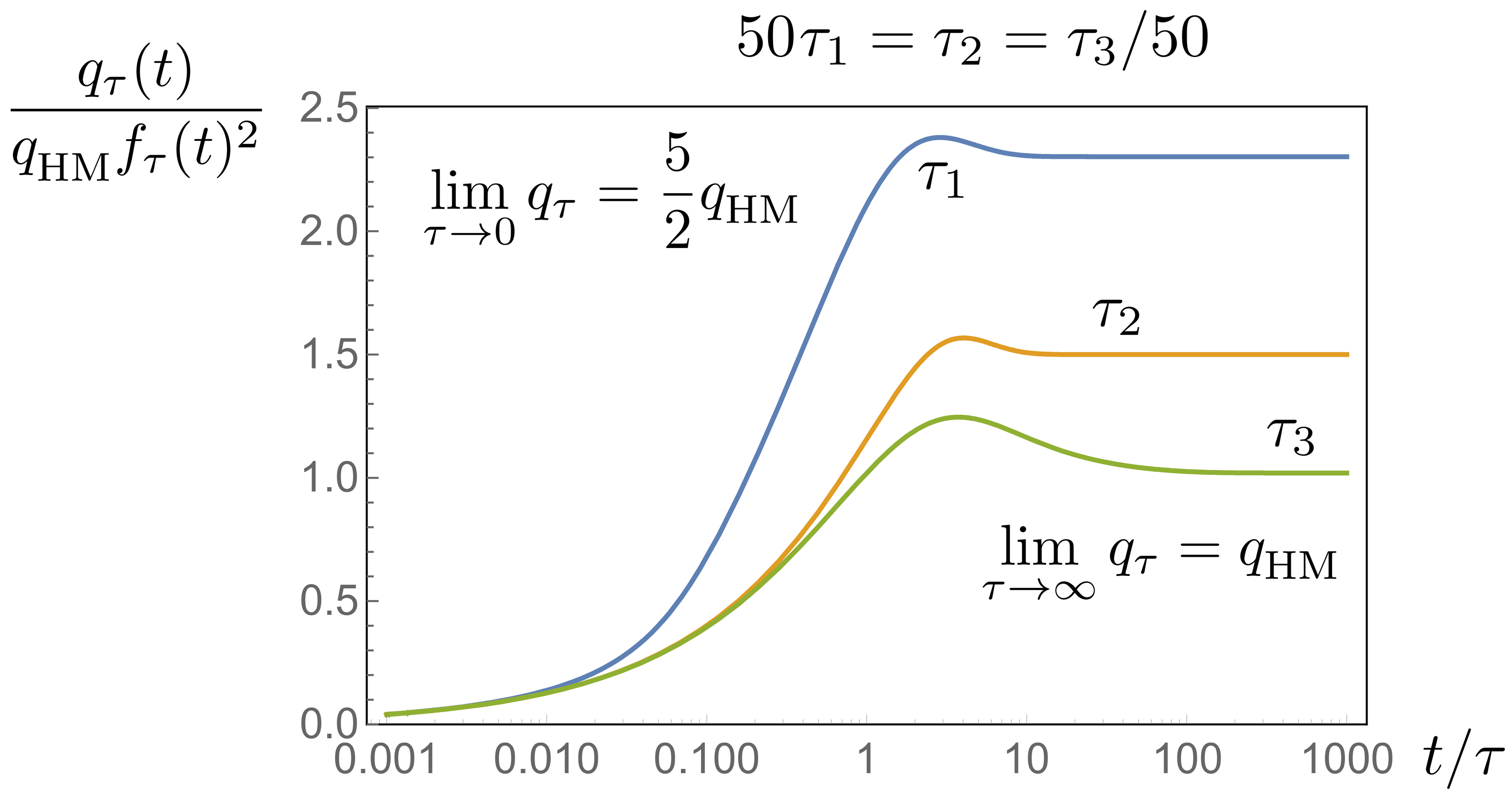
$$\hbar\omega_\mathbf{k} = \frac{\hbar^2\mathbf{k}^2}{2m}, \qquad \hbar\Omega_\mathbf{k} = \sqrt{\hbar\omega_\mathbf{k}(\hbar\omega_\mathbf{k} + 2\mu_0)}$$

$$\hat{\mathcal{R}}(\mathbf{k}) = \int_{\mathbb{R}^3}\mathrm{d}^3\mathbf{x}\, e^{-i\mathbf{k}\mathbf{x}}\mathcal{R}(\mathbf{x})$$

Delta-correlated disorder: $\mathcal{R}(\mathbf{x}) = R\delta(\mathbf{x}), \quad \hat{\mathcal{R}}(\mathbf{k}) = R$

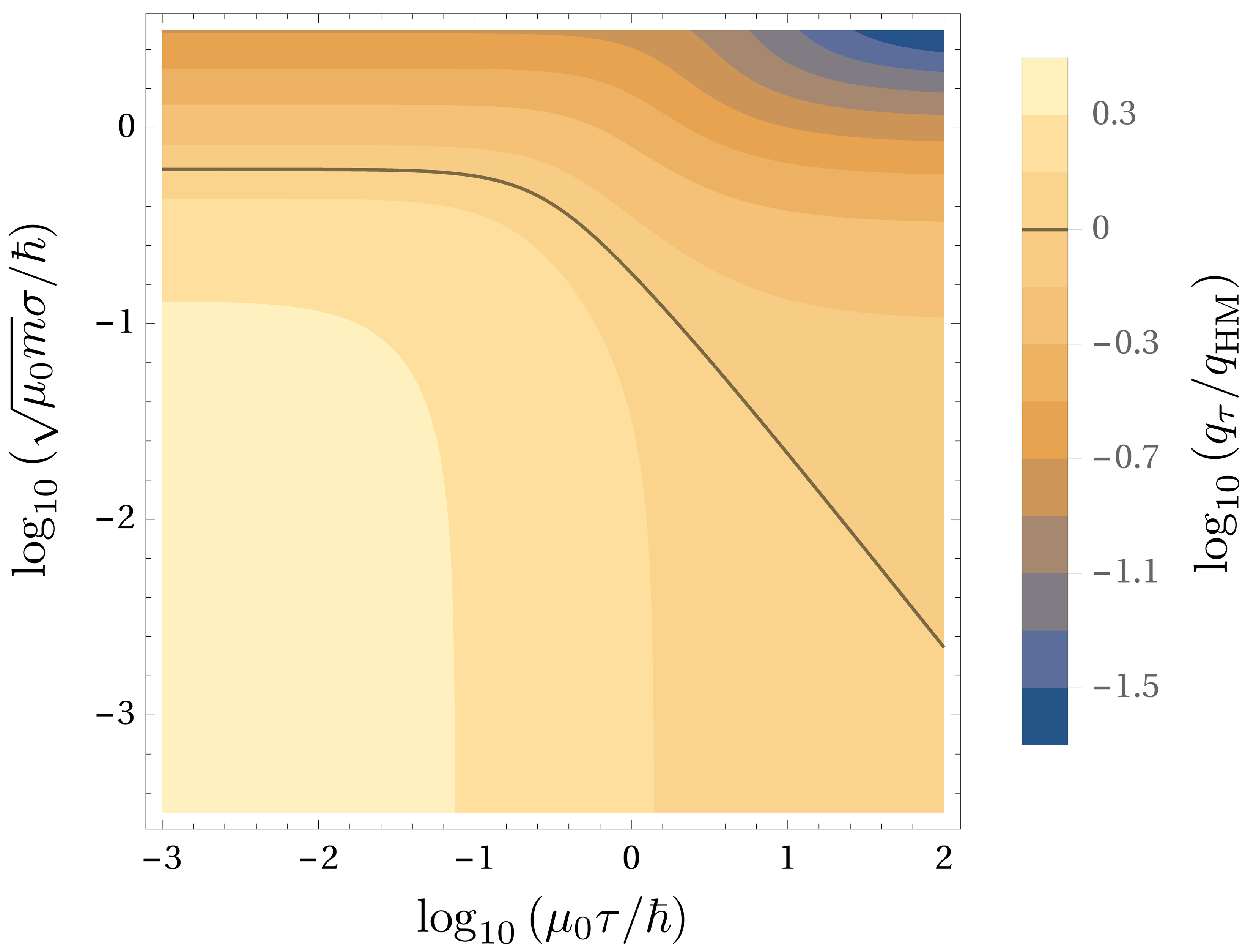
Equilibrium of Huang & Meng [1]: $q_{\mathrm{HM}} = R\sqrt{\frac{\pi n}{2g}}\left(\frac{m}{2\pi\hbar^2}\right)^{\frac{3}{2}}$

Disorder rise time influences steady state:



Gaussian-correlated disorder:

$$\mathcal{R}(\mathbf{x}) = R\frac{e^{-\mathbf{x}^2/\sigma^2}}{\pi^{3/2}\sigma^3}, \qquad \hat{\mathcal{R}}(\mathbf{k}) = Re^{-\mathbf{k}^2\sigma^2/4}$$



References:

[1] K. Huang and H.-F. Meng, *Hard-sphere Bose gas in random external potentials*, Phys. Rev. Lett. **69**, 644 (1992)

[2] R. Graham and A. Pelster, *Order via nonlinearity in randomly confined Bose gases*, Int. J. Bif. Chaos **19**, 2745 (2009)

[3] B. Nikolić, A. Balaž, and A. Pelster, *Dipolar Bose-Einstein Condensates in Weak Anisotropic Disorder*, Phys. Rev. A **88**, 013624 (2013)

Email: radonjic@physik.uni-kl.de

SFB/TR 185

OSCAR – Open System Control of Atomic and Photonic Matter

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN