Bose Condensate in Strong Disorder Potential With Arbitrary Correlation Length

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Gross-Pitaevskii Eqn. in disordered potential ($T = 0$)

$$\left(-\frac{\nabla^2}{2m} + U(\mathbf{r}) + g|\psi(\mathbf{r})|^2 - \mu\right)\psi(\mathbf{r}) = 0$$

Fourier decomposition: $\psi(\mathbf{r}) = \frac{1}{V}\sum_{\mathbf{k}} e^{i\mathbf{k}.\mathbf{r}}\alpha_{\mathbf{k}} \qquad U(\mathbf{r}) = \frac{1}{V}\sum_{\mathbf{k}} e^{i\mathbf{k}.\mathbf{r}}U_{\mathbf{k}}$

$$\Rightarrow \left(\frac{\mathbf{k}^2}{2m} - \mu\right)\alpha_{\mathbf{k}} + \sum_{\mathbf{q}} U_{\mathbf{q}}\alpha_{\mathbf{k}-\mathbf{q}} + \frac{g}{V}\sum_{\mathbf{k}',\mathbf{q}} \alpha^*_{\mathbf{k}'}\alpha_{\mathbf{k}+\mathbf{q}}\alpha_{\mathbf{k}'-\mathbf{q}} = 0$$

- Gaussian distribution: $\langle U_{\mathbf{q}}\rangle = 0$ and $\langle U_{\mathbf{q}}U_{\mathbf{q}'}\rangle = \delta_{\mathbf{q},-\mathbf{q}'}R(\mathbf{q})/V$
- Lorentzian correlation function: $R(\mathbf{q}) = R/(1+\xi^2\mathbf{q}^2)$
- Disorder correlation length (see Clement et al., PRL **95**, 170409 (2005)):

$$\xi = 1.22\lambda l/D \sim 5.2\mu m \gg \xi_h \sim 0.11\mu m$$

Moving condensate

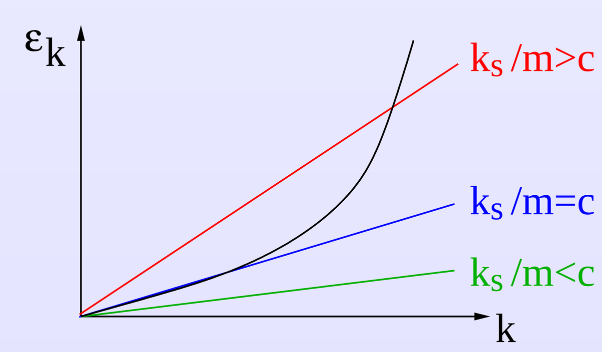
Homogeneous ($\mathbf{k} = \mathbf{k_s}$) and inhomogeneous ($\mathbf{k} \neq \mathbf{k_s}$) parts:
Perturbation theory:

$$\alpha_{\mathbf{k}} = \sqrt{n_0}\delta_{\mathbf{k},\mathbf{k_s}} + \delta\alpha_{\mathbf{k}} \qquad \Rightarrow \delta\alpha_{\mathbf{k}+\mathbf{k_s}} = \frac{\left(\frac{\mathbf{k}^2}{2m} - \frac{\mathbf{k}.\mathbf{k_s}}{m}\right)\sqrt{n_0}U_{\mathbf{k}}}{\epsilon^2_{\mathbf{k}} - (\frac{\mathbf{k}.\mathbf{k_s}}{m})^2}$$

Bogoliubov energy: $\epsilon_{\mathbf{k}} = \sqrt{\frac{\mathbf{k}^2}{2m}(\frac{\mathbf{k}^2}{2m} + 2gn_0)} \overset{\mathbf{k}\to 0}{\to} \mathbf{c}|\mathbf{k}|$

Stability condition: $\epsilon_k > |\mathbf{k}.\mathbf{k_s}/m|$ otherwise singularity

$\Rightarrow$ Landau criterion must be satisfied



Debye-like screening of the potential

Density response to the external potential ($\mathbf{k_s} = \mathbf{0}$): $\delta n(\mathbf{r}) = \sum_{\mathbf{q}} \frac{e^{i\mathbf{q}.\mathbf{r}}}{\sqrt{V}}\delta\rho_{\mathbf{q}}$

Condensate $\to$ Filter for wavelength longer than the healing length

$$U_{\mathbf{q},eff} = U_{\mathbf{q}} + g\delta\rho_{\mathbf{q}} = \frac{U_{\mathbf{q}}}{\mathcal{K}(\mathbf{q},\omega=0)}$$

Screening of the potential at long wavelength:

$$\mathcal{K}(\mathbf{q},\omega=0) = 1 + 2/(\mathbf{q}^2\xi_h^2) \overset{\mathbf{q}\to 0}{\to} \infty$$

Healing length $\xi_h = 1/\sqrt{2mgn} \equiv$ Debye length for the condensate

First-order results

- Solution ($\mathbf{k_s} = 0$): $n_0 = n - \sum_{\mathbf{k}\neq 0} n_{\mathbf{k}}$

$$n_{\mathbf{k}\neq\mathbf{0}} = \langle|\alpha_{\mathbf{k}}|^2\rangle = \frac{R(\mathbf{k})n_0}{|\mathbf{k}^2/(2m)\mathcal{K}(\mathbf{k},\omega=0)|^2} = \frac{R(\mathbf{k})n_0}{(\mathbf{k}^2/(2m) + 2gn_0)^2}$$

  Bose glass phase: fully inhomogeneous condensate
- Superfluid component: $n_s = |\mathbf{P}|/|\mathbf{k_s}| \neq n_0$ where $\mathbf{P} = \sum_{\mathbf{k}} \mathbf{k}n_{\mathbf{k}}$
- Lowest order in R and $\xi = 0$ (Huang and Meng, PRL **69**, 644 (1992)):

$$n - n_0 = VR\frac{m^2}{8\pi^{3/2}}(\frac{n_0}{a})^{1/2}$$
$$n - n_s = \frac{4}{3}(n - n_0)$$

Random Phase Approximation

Higher order expansion: strong depletion possible
On average we neglect off-diagonal combination compared to diagonal one:

$$\rho_{\mathbf{k},\mathbf{q}} = \alpha^*_{\mathbf{k}}\alpha_{\mathbf{k}+\mathbf{q}} \ll n_{\mathbf{k}} \equiv |\alpha_{\mathbf{k}}|^2$$

Non perturbative approach $\Rightarrow$ Dielectric function:

$$\mathcal{K}(\mathbf{q},\omega) = \frac{-\frac{2gn_{\mathbf{k_s}}}{V}\frac{\mathbf{q}^2}{m}}{(\omega + i0_+ - \frac{\mathbf{k_s}.\mathbf{q}}{m})^2 - (\frac{\mathbf{q}^2}{2m})^2 + \frac{gn_{\mathbf{k_s}}}{V}\frac{\mathbf{q}^2}{m}} + \mathcal{K}_n(\mathbf{q},\omega)$$

contains contribution of the inhomogeneous part:

$$\mathcal{K}_n(\mathbf{q},\omega) = 1 - \frac{2g}{V}\sum_{\mathbf{k}\neq\mathbf{k_s}} \frac{n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{q}}}{\omega + i0_+ - \frac{\mathbf{k}.\mathbf{q}}{m} - \frac{\mathbf{q}^2}{2m}}$$

$\Rightarrow$ self-consistent set of equations for $n_{\mathbf{k}}$ by using the new $\mathcal{K}(\mathbf{q},\omega=0)$

Results: 1st order transition

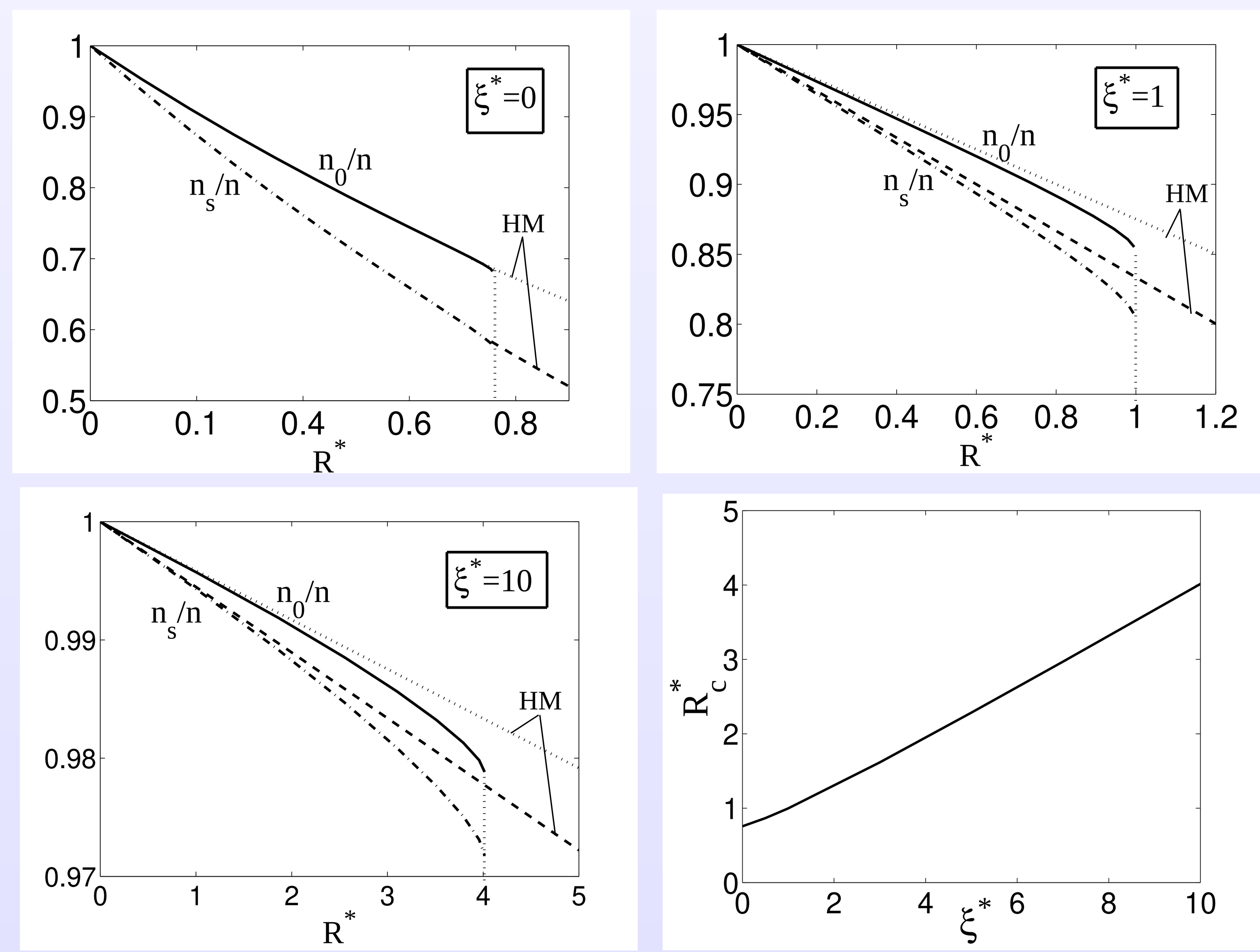


Fig. 1-2-3: 1st order Bose Glass Trans. at $R^* = R/\sqrt{4\pi^2 gn/m^3} = 0.75$ for $\xi^* = \xi/\xi_h = 0$, at $R^* = 0.98$ for $\xi^* = 1$ and at $R^* = 4$ for $\xi^* = 10$. Fig.4: Critical value R^*_c vs. ξ^*.

- Relation $n - n_s = \frac{4}{3}(n - n_0)$ still valid
- At the transition: $\frac{dn_0}{dR} \to \infty \Rightarrow \frac{dn_s}{dR} \to \infty$

Conclusions

1. RPA applied on G-P Eqn. allows to recover results obtained from Bog. approach by Huang and Meng
2. First order transition from superfluid to Bose glass

See cond-mat/0607546

Supports: AvH, SFB-TR12