Bose-Einstein Condensates with Strong Disorder: Gaussian Approximation for Correlation Functions

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Abstract

Ultracold bosonic atoms in potentials with quenched disorder represent a notoriously difficult problem due to the competition of localization and superfluidity. Whereas some initial promising results are known for weak disorder within a Bogoliubov theory of dirty bosons [1,2], the case of strong disorder is still quite elusive [3]. Here we work out a non-perturbative approach towards the dirty boson problem at zero temperature which is based on a Gaussian approximation for correlation functions of the disorder potential and the condensate wave function solving the Gross-Pitaevskii equation. For contact interaction we find that the case of delta-correlated disorder can be treated analytically, whereas the case of a Lorentzian disorder correlation necessitates a numerical solution of a set of self-consistency equations. For weak disorder we reproduce the condensate depletion of Huang and Meng and for strong disorder we yield a quantum phase transition to a Bose-glass phase.

Model

- **Disorder**
  - Disorder Ensemble Average

$$\overline{\bullet} = \int \mathcal{D}V \bullet P[V], \qquad \int \mathcal{D}V\, P[U] = 1$$

  - Assumptions

$$\overline{V(\mathbf{x_1})} = 0, \qquad \overline{V(\mathbf{x_1})V(\mathbf{x_2})} = R(\mathbf{x_1} - \mathbf{x_2})$$

- **Gross-Pitaevskii equation:**

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) - \mu + g\psi^2(\mathbf{x})\right]\psi(\mathbf{x}) = 0$$

$\Longrightarrow$ stochastic nonlinear partial differential equation [3]

Gaussian factorization of correlation functions

- Gaussian assumption: cumulants of 3rd and higher order are zero
- Second cumulant in homogeneous systems is

$$G_{\psi V}(\mathbf{x}_1 - \mathbf{x}_2) = \overline{\psi(\mathbf{x}_1)V(\mathbf{x}_2)} - \overline{\psi(\mathbf{x}_1)}\ \overline{V(\mathbf{x}_2)}$$

- Example of expansion (with $\overline{V(\mathbf{x})} = 0$)

$$\overline{\psi(\mathbf{x}_1)\psi(\mathbf{x}_2)V(\mathbf{x}_3)} = \overline{\psi(\mathbf{x}_1)}G_{\psi V}(\mathbf{x}_2 - \mathbf{x}_3) + \overline{\psi(\mathbf{x}_2)}G_{\psi V}(\mathbf{x}_1 - \mathbf{x}_3)$$

Self-consistency equations

- **Derivation:**

$$\overline{\mathrm{GP}}: \quad G_{\psi V}(\mathbf{0}) - \mu\psi + gn\psi + 2gG_{\psi\psi}(\mathbf{0})\psi = 0$$

$$\overline{\mathrm{GP}\ V(\mathbf{x})}: \quad \left(\frac{\hbar^2\mathbf{k}^2}{2m} - \mu + 3ng\right)G_{\psi V}(\mathbf{k}) = -\psi R(\mathbf{k})$$

$$\overline{\mathrm{GP}\ \psi(\mathbf{x})}: \quad \left(\frac{\hbar^2\mathbf{k}^2}{2m} - \mu + 3ng\right)G_{\psi\psi}(\mathbf{k}) = -\psi G_{V\psi}(\mathbf{k})$$

- **Closed system:**

$$\psi^2 = \frac{n}{1 + I_{\psi\psi}}, \qquad I_{\psi\psi} = \int \frac{d^3k}{(2\pi)^3}\frac{R(\mathbf{k})}{\left(\frac{\hbar^2\mathbf{k}^2}{2m} - \mu + 3ng\right)^2}$$

$$I_{\psi V} + \mu = gn\frac{1 + 3I_{\psi\psi}}{1 + I_{\psi\psi}}, \qquad I_{\psi V} = \int \frac{d^3k}{(2\pi)^3}\frac{R(\mathbf{k})}{\frac{\hbar^2\mathbf{k}^2}{2m} - \mu + 3ng}$$

$\Longrightarrow$ Lorentz correlation $R(\mathbf{k}) = \frac{R}{1+\sigma^2\mathbf{k}^2}$ yields algebraically solvable equations

- **Bose glass:** $\psi^b = 0, \quad I^b_{\psi\psi} = \infty, \quad \mu^b = 3ng$
  following from renormalization procedure

$$0 = I^b_{\psi V} = \int \frac{d^3k}{(2\pi)^3}\frac{R(\mathbf{k})}{\frac{\hbar^2\mathbf{k}^2}{2m}} \sim \frac{1}{\sigma}; \qquad \sigma: \text{correlation length}$$

Perturbative results

- **Self-consistency equations:** homogeneous case and $R(\mathbf{k}) = R$

$$n = n_0 + \sqrt{\pi}R\left(\frac{m}{2\pi\hbar^2}\right)^{3/2}\frac{n_0}{\sqrt{3gn - \mu}}$$

$$3gn - \mu = 2gn_0 - 2\sqrt{\pi}R\left(\frac{m}{2\pi\hbar^2}\right)^{3/2}\sqrt{3gn - \mu}$$

- **Condensate depletion in leading order:**

$$n_0 + n - \sqrt{\frac{\pi n}{2g}}R\left(\frac{m}{2\pi\hbar^2}\right)^{3/2} + \dots$$

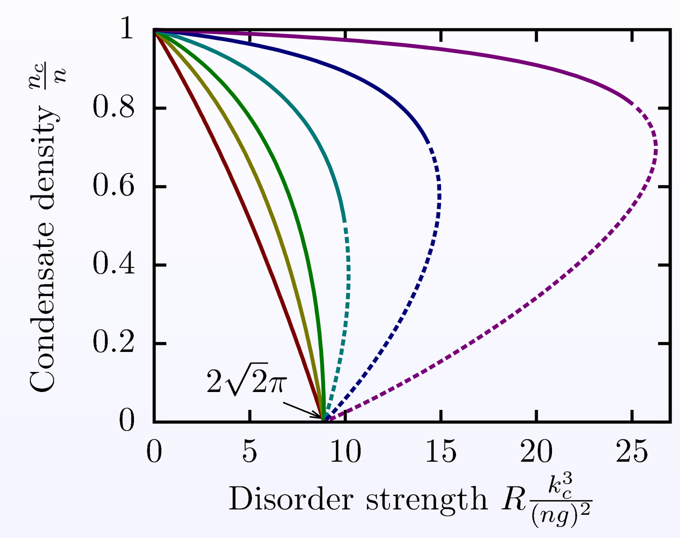
- **Equation of state in leading order:**

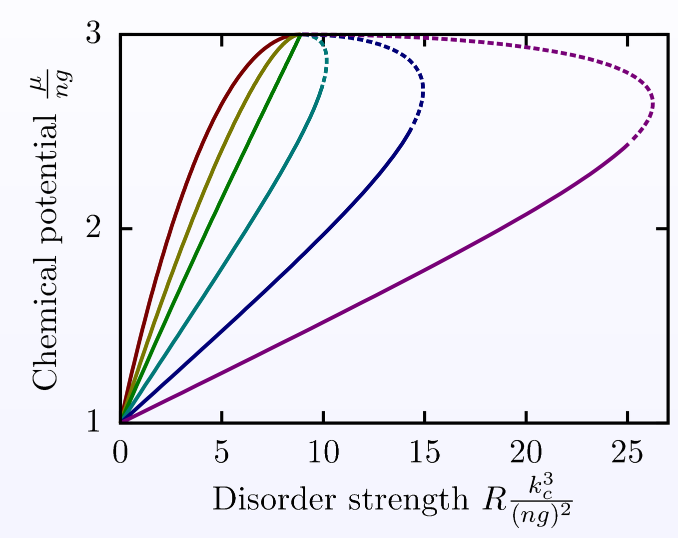
$$\mu = gn + 3\sqrt{2\pi gn}R\left(\frac{m}{2\pi\hbar^2}\right)^{3/2} + \dots$$

$\Longrightarrow$ Results of Bogoliubov theory of dirty bosons [1,2]

Non-perturbative results

- **Condensate density and chemical potential:**

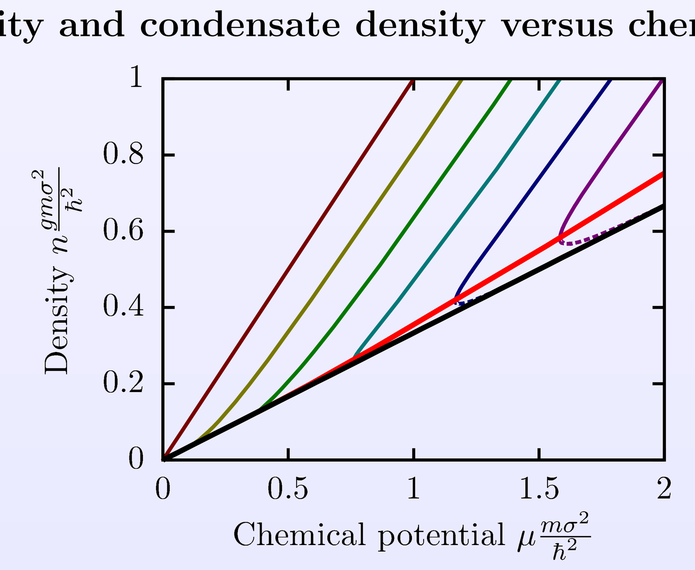


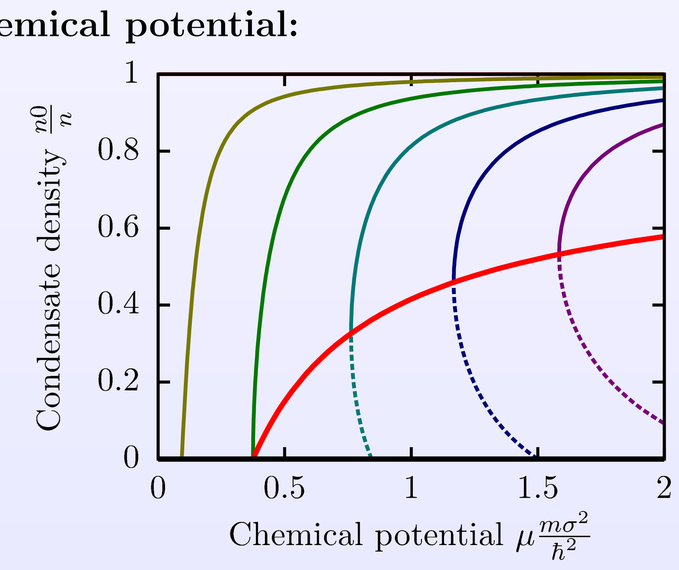


correlation lengths: $\sigma\frac{\sqrt{2mng}}{\hbar} = 0, \frac{1}{4}, \frac{1}{2}, 1, 2,$ and 4

$\Longrightarrow$ border of first/second order transition: $R_{1-2}\sigma_{1-2} = \frac{\pi\hbar^4}{m^2}$

- **Density and condensate density versus chemical potential:**

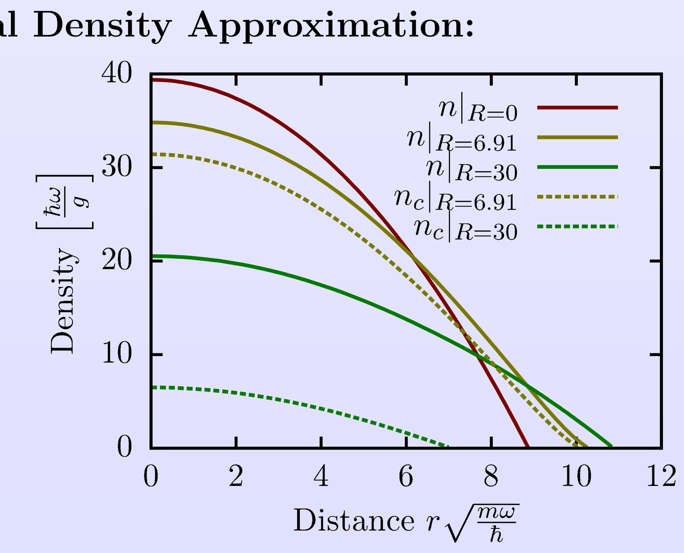


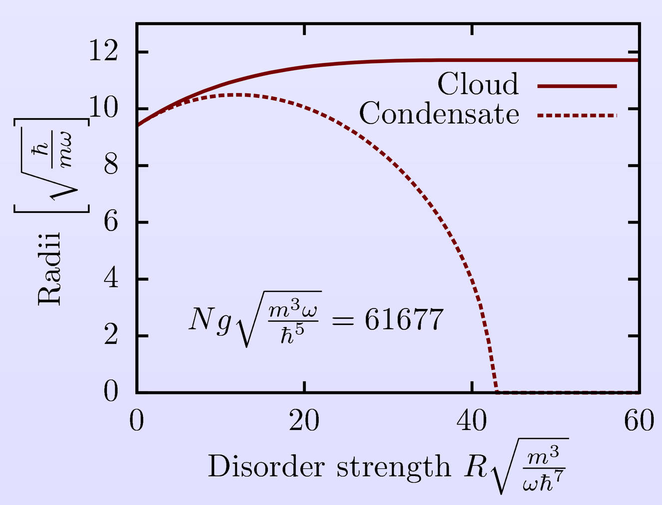


correlation lengths: $R\frac{m^2\sigma}{\hbar^4} = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi,$ and $\frac{5\pi}{2}$

black and red lines show Bose glass phase and first-order phase transition, respectively.

- **Local Density Approximation:**





Conclusions and outlook

- Density dependence of condensate and cloud radii confirmed by non-perturbative replica method [4]
- Numerical solution of Gross-Pitaevskii equation with disorder potential necessary [5]
- Trap calculation for finite correlation length
- Time-dependent generalization
- Generalization of preliminary results for dipolar Bose-Einstein condensates to strong disorder [6,7]

References

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