Bose-Einstein Condensates in Moving Random Potentials

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Abstract

The hydrodynamic properties of a dilute stationary Bose-Einstein condensate are determined from solving the coupled continuity and Euler equations. Here we develop a perturbative solution approach by assuming that the disorder potential is weak and moving slower than the sound velocity. In this way we find explicit expressions for the disorder ensemble averages of both the condensate and the superfluid density for a Lorentzian correlated disorder. In the special case of a static delta correlated disorder in 3d the results reduce to the ones derived originally by Huang and Meng [1-4]. Furthermore, we specialize our results to Bose-Einstein condensates with quasi 1d ring geometry which have been experimentally realized in different laboratories worldwide. In particular, we discuss how the ring length affects the respective hydrodynamics properties.

Mean Field

- External random potential $U(\mathbf{r})$
  - irregularities in wire traps, laser spackles
  - properties $\langle U(\mathbf{r})\rangle = 0$, $\langle U(\mathbf{r})U(\mathbf{r}')\rangle = R(\mathbf{r}-\mathbf{r}')$
- Gross-Pitaevskii equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}} + U\left(\mathbf{r}-\frac{\hbar}{m}\mathbf{k}_U t\right) + \int d^3\mathbf{r}' V(\mathbf{r}-\mathbf{r}')\Psi^*(\mathbf{r}',t)\Psi(\mathbf{r}',t)\right]\Psi(\mathbf{r},t) = i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}$$

- Hydrodynamic equations - solution of the form $\Psi(\mathbf{r},t) = a(\mathbf{r})e^{i\phi(\mathbf{r})}\exp\left(i\mathbf{k}_S\mathbf{r} - \frac{i}{\hbar}\left(\mu + \frac{\hbar^2 k_S^2}{2m}\right)t\right)$ with substitution $\mathbf{r}\to\mathbf{r}+\frac{\hbar}{m}\mathbf{k}_U t$, and relative velocity $\mathbf{K} = \mathbf{k}_S - \mathbf{k}_U$

$$\left[-\frac{\hbar^2}{2m}(\nabla^2 - (\nabla\phi(\mathbf{r}))^2) + \frac{\hbar^2}{m}\mathbf{K}\cdot\nabla\phi(\mathbf{r}) + U(\mathbf{r}) - \mu + \int d^3\mathbf{r}' V(\mathbf{r}-\mathbf{r}')a^2(\mathbf{r}')\right]a(\mathbf{r}) = 0$$
$$\nabla\left[a^2(\mathbf{r})(\nabla\phi(\mathbf{r}) + \mathbf{K})\right] = 0$$

- Spatial average = Disorder average, $\overline{A} = \langle A\rangle$ [1]
- Condensate density as order parameter $n_c = \lim_{\mathbf{r}-\mathbf{r}'\to\infty}\langle\Psi^*(\mathbf{r}')\Psi(\mathbf{r})\rangle = \left\langle ae^{i\phi}\right\rangle\left\langle ae^{-i\phi}\right\rangle$
- Perturbation expansion: $a = a_0 + a_1 + a_2 + \ldots$, $\nabla\phi = \nabla\phi_0 + \nabla\phi_1 + \nabla\phi_2 + \ldots$
  - 0^{th} order: $a_0(\mathbf{r}) = a_0$ and $\phi_0(\mathbf{r}) = 0$, $\mu = a_0^2 V(\mathbf{k}=0)$ [1,2]
  - 1^{st} order: the Fourier transforms of solutions

$$a_1(\mathbf{k}) = \frac{-a_0 U(\mathbf{k})}{\frac{\hbar^2\mathbf{k}^2}{2m} - 2\frac{\hbar^2}{m}\frac{(\mathbf{Kk})^2}{\mathbf{k}^2} + 2a_0^2 V(\mathbf{k})}, \quad (\nabla\phi_1)(\mathbf{k}) = 2\frac{(\mathbf{Kk})\mathbf{k}}{\mathbf{k}^2}\frac{U(\mathbf{k})}{\frac{\hbar^2\mathbf{k}^2}{2m} - 2\frac{\hbar^2}{m}\frac{(\mathbf{Kk})^2}{\mathbf{k}^2} + 2a_0^2 V(\mathbf{k})}$$

Two-fluid model

- Separate momentum and kinetic energy densities, $\mathbf{k}'_S = \mathbf{k}_S - \Delta\mathbf{k}_S$

$$\frac{m}{\hbar}\langle\mathbf{j}(\mathbf{r})\rangle = n_S\mathbf{k}'_S + n_N\mathbf{k}_U = (n_S+n_N)\mathbf{k}_S - (n_S\Delta\mathbf{k}_S + n_N\mathbf{K})$$
$$= \frac{1}{2i}\left\langle\Psi^\dagger\nabla\Psi - \Psi\nabla\Psi^\dagger\right\rangle = n\mathbf{k}_S + \Delta\mathbf{j}$$
$$\frac{2m}{\hbar^2}\langle\epsilon_{\text{kin}}(\mathbf{r})\rangle = n_S{\mathbf{k}'_S}^2 + n_N\mathbf{k}_U^2 = (n_S+n_N)\mathbf{k}_S^2 - 2(n_S\Delta\mathbf{k}_S + n_N\mathbf{K})\mathbf{k}_S + n_S{\Delta\mathbf{k}_S}^2 + n_N\mathbf{K}^2$$
$$= \left\langle\nabla\Psi^\dagger\nabla\Psi\right\rangle - \epsilon_0 = n\mathbf{k}_S^2 + 2\Delta\mathbf{j}\mathbf{k}_S + \epsilon_s$$

- Equating and solving gives

$$n_S = n\frac{1}{1+\delta}, n_N = n\frac{\delta}{1+\delta}, \ \Delta\mathbf{k}_S = -\frac{1+\delta}{n}\Delta\mathbf{j} - \delta\mathbf{K}, \text{ with } \delta = \frac{n\epsilon_s - \Delta\mathbf{j}^2}{(n\mathbf{K}+\Delta\mathbf{j})^2}$$

- With hydrodynamics: $n = \langle a^2\rangle$, $\Delta\mathbf{j} = \langle a^2\nabla\phi\rangle$, $\epsilon_s = \langle a^2(\nabla\phi)^2\rangle$, $\epsilon_0 = \langle(\nabla a)^2\rangle$

1D ring system

- $\int\frac{d^D\mathbf{k}}{(2\pi)^D}f(\mathbf{k}) \to \frac{1}{2\pi\rho}\sum_{\alpha=-\infty}^{\infty} f\left(\frac{\alpha}{\rho}\right)$, ring radius ρ
- Exact solution of continuity equation

$$\phi(r) = \int_0^r dr' K\left[\frac{1}{a^2(r')\overline{a^{-2}}}\left(\frac{l}{K\rho}+1\right) - 1\right], \ l = \rho\overline{\phi'(r)} \in \mathbb{Z}$$

- Two-fluid model

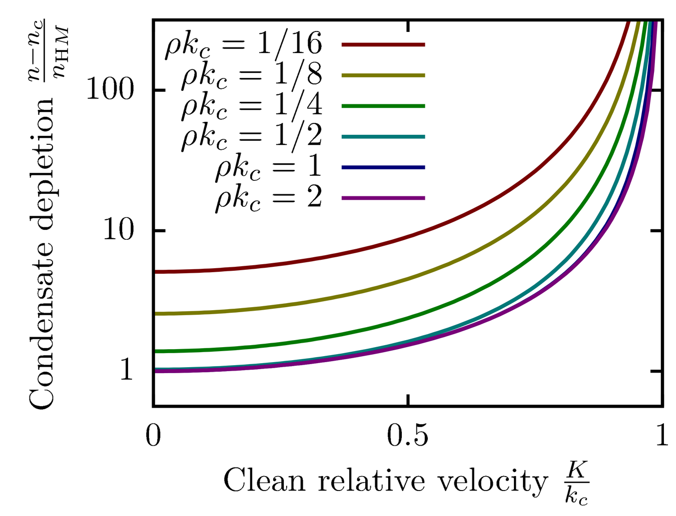
$$n = \overline{a^2}, \ \Delta j = \overline{a^2}\left(\frac{l/\rho + K}{\overline{a^2}\,\overline{a^{-2}}} - K\right), \ \epsilon_s = \overline{a^2}\left(\frac{l^2/\rho^2 - K^2}{\overline{a^2}\,\overline{a^{-2}}} + K^2\right), \ \delta = \frac{\overline{a^{-2}}}{\overline{a^2}} - 1, \ \Delta k_S = -\frac{l}{\rho}$$

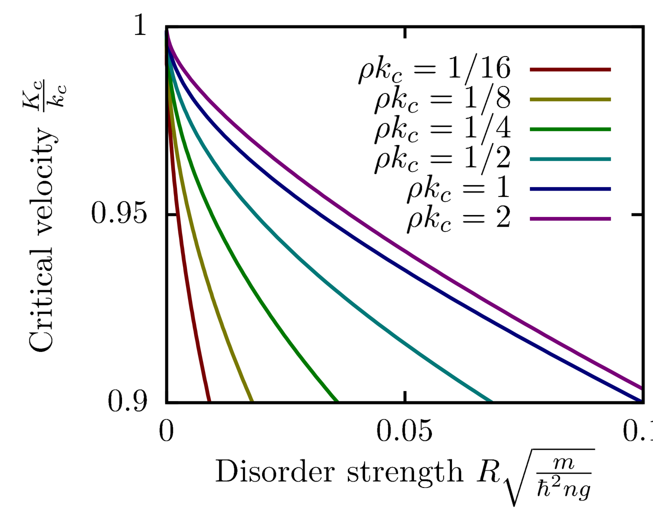
- Discretized version of the perturbation theory for δ-correlated disorder: $a_{1\alpha} = \frac{-mU_\alpha/\hbar^2}{\frac{\alpha^2}{2} + 2\left(\frac{gNm\rho}{2\pi\hbar^2} - K^2\rho^2\right)}$

$$n - n_c = \overline{a_1^2} = n\frac{R\sqrt{m}}{2\pi\hbar(gn)^{3/2}}\left(\sqrt{\frac{gnm}{\hbar^2}}\rho\right)^3 f\left[\left(1 - \frac{\hbar^2K^2}{gnm}\right)\frac{gnm}{\hbar^2}\rho^2\right]$$
$$n_N = 4\overline{a_1^2}, \ f(x) = \frac{\pi\coth(2\pi\sqrt{x})}{4x^{\frac{3}{2}}} + \frac{\pi^2}{2x\coth^2(2\pi\sqrt{x})}$$

1D Results

- Infinite ring condensate depletion $n_{\text{HM}} = n\frac{R\sqrt{m}}{8\hbar(gn)^{3/2}}$ [3]
- Increases with K, clean case wavevector $k_c = \frac{\sqrt{ngm}}{\hbar}$
- $\rho \geq 1/k_c$, small finite size effects
- Superfluid depletion 4 times larger

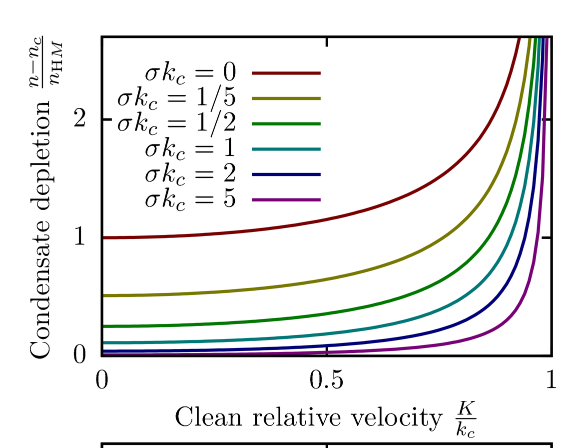


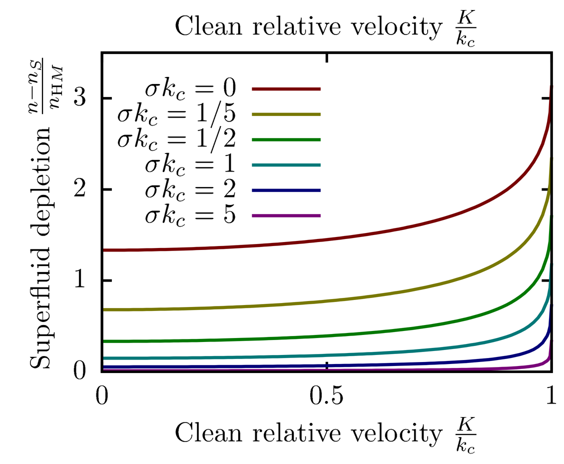


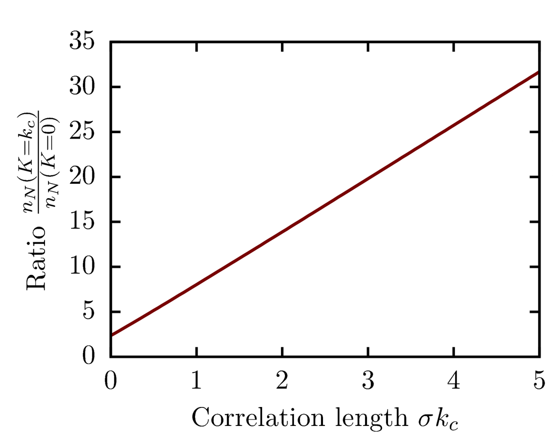
- Implicit equation $n_c(R, K_c) = 0$
- Finite ring $K_c = \frac{\sqrt{gmn}}{\hbar}\left(1 - \frac{\sqrt{R}}{4gn\sqrt{2\pi\rho}} - \frac{R}{64g^2n^2\pi\rho} + \ldots\right)$
- Infinite ring $K_c = \frac{\sqrt{gmn}}{\hbar}\left[1 - \frac{1}{8gn}\left(\frac{R\sqrt{m}}{\hbar}\right)^{\frac{2}{3}} + \ldots\right]$

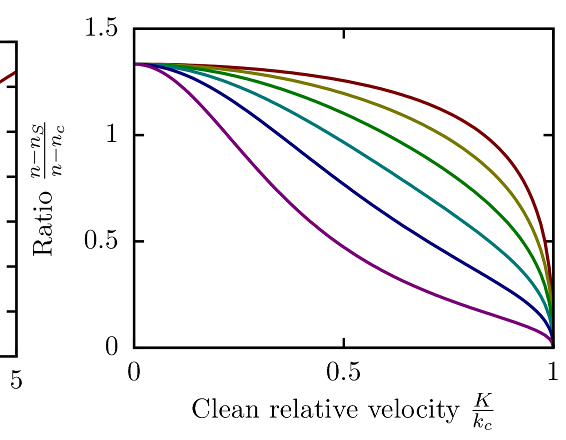
3D Results

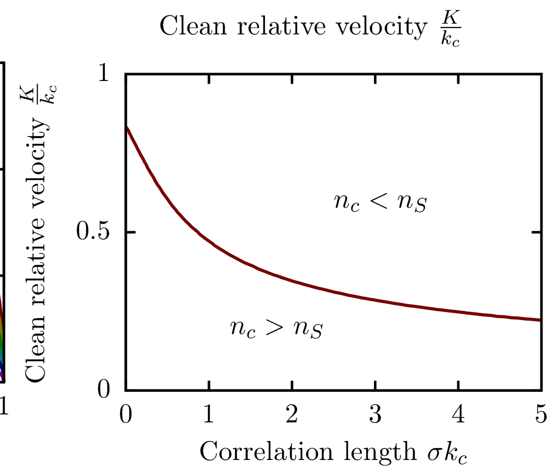
- $n_{\text{HM}} = \frac{Rk_c^3}{4\pi ng^2}$[3], $k_c = \frac{\sqrt{gnm}}{\hbar}$
- Depletion increases with K, and decreases with σ
- $K = K_c$ for small R
- Superfluid depletion 4/3 times larger [2,4] than condensate depletion for $\sigma = 0$ and $K = 0$, for large σ or K this changes [5]
- Superfluid depletion finite for $K = k_c$ it gets larger compared to $K = 0$ for increasing σ











Conclusions and outlook

- Time evolution to determine integration constants and obtain second order results
- Diplar interaction

References

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