Tuning the Quantum Phase Transition of Bosons in Optical Lattices via Periodic Modulation of s-Wave Scattering Length

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Abstract

We consider interacting bosons in a 2D square and a 3D cubic optical lattice with a periodic modulation of the s-wave scattering length. At first we map the underlying periodically driven Bose-Hubbard model for large enough driving frequencies to an effective time-independent Hamiltonian with a conditional hopping. Combining different analytical approaches with quantum Monte Carlo simulations then reveals that the superfluid-Mott insulator quantum phase transition still exists despite the periodic driving and that the location of the quantum phase boundary turns out to depend quite sensitively on the driving amplitude. A more detailed quantitative analysis shows even that the effect of driving can be described within the usual Bose-Hubbard model provided that the hopping is rescaled appropriately with the driving amplitude. This finding indicates that the Bose-Hubbard model with a periodically driven s-wave scattering length and the usual Bose-Hubbard model belong to the same universality class from the point of view of critical phenomena [1].

Floquet Theory

- Bose-Hubbard Hamiltonian with time-periodic driving:

$$\hat{H}(t) = \hat{H}_{\rm BH} + A\cos(\omega t)\sum_i g(\hat{n}_i)\,, \quad \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$$

$$\hat{H}_{\rm BH} = \sum_i f(\hat{n}_i) - t\sum_{\langle ij\rangle} \hat{a}_i^\dagger \hat{a}_j\,, \quad f(\hat{n}_i) = \frac{U}{2}\left(\hat{n}_i^2 - \hat{n}_i\right) - \mu\hat{n}_i$$

condition: $U, t \ll \hbar\omega \ll \Delta$
time average over driving period [2]

- Effective time-independent Bose-Hubbard Hamiltonian:

$$\hat{H}_{\rm eff} = \sum_i f(\hat{n}_i) - t\sum_{\langle ij\rangle} \hat{a}_i^\dagger J_0\left(G(\hat{n}_i,\hat{n}_j)\right)\hat{a}_j$$

$$G(\hat{n}_i,\hat{n}_j) = \frac{g(\hat{n}_j) - g(\hat{n}_j - 1) + g(\hat{n}_i) - g(\hat{n}_i + 1)}{\hbar\omega}$$

- Examples:

Shaken lattice

$$\hat{H}(t) = \hat{H}_{\rm BH} + A\cos(\omega t)\sum_i \hat{n}_i$$

$$\hat{H}_{\rm eff} = \sum_i f(\hat{n}_i) - tJ_0\left(\frac{A}{\hbar\omega}\right)\sum_{\langle ij\rangle}\hat{a}_i^\dagger\hat{a}_j$$

$\Longrightarrow$ **renormalized hopping** [3-5]

Modulated interaction [6]

$$\hat{H}(t) = \hat{H}_{\rm BH} + A\cos(\omega t)\sum_i \frac{1}{2}\left(\hat{n}_i^2 - \hat{n}_i\right)$$

$$\hat{H}_{\rm eff} = \sum_i f(\hat{n}_i) - t\sum_{\langle ij\rangle}\hat{a}_i^\dagger J_0\left(\frac{A}{\hbar\omega}(\hat{n}_j - \hat{n}_i)\right)\hat{a}_j$$

$\Longrightarrow$ **conditional hopping** [7,8]

Landau Theory

- Bose-Hubbard Hamiltonian with current: $\hat{H}_{\rm BH}(J^*, J) = \hat{H}_{\rm BH} + \sum_i\left(J^*\hat{a}_i + J\hat{a}_i^\dagger\right)$
- Grand-canonical free energy: $F = -\frac{1}{\beta}\ln{\rm Tr}\left[e^{-\beta\hat{H}_{\rm BF}(J^*,J)}\right]$

$$\psi = \langle\hat{a}_i\rangle = \frac{1}{N_{\rm s}}\frac{\partial F(J^*,J)}{\partial J^*}\quad;\quad \psi^* = \langle\hat{a}_i^\dagger\rangle = \frac{1}{N_{\rm s}}\frac{\partial F(J^*,J)}{\partial J}$$

- Legendre transformation: $\Gamma(\psi^*,\psi) = \psi^* J + \psi J^* - F/N_{\rm s}$

$$\frac{\partial\Gamma}{\partial\psi^*} = J\quad;\quad\frac{\partial\Gamma}{\partial\psi} = J^*$$

$\Longrightarrow$ Limit of vanishing current: order parameter ψ^*, ψ from extremizing effective potential Γ [9]

- Landau expansion: $\Gamma = a_0 + a_2|\psi|^2 + a_4|\psi|^4 + \cdots$
  $\Longrightarrow$ Landau coefficients determined in tunneling expansion

First Hopping Order (3D)

- Gutzwiller mean-field theory (dots):

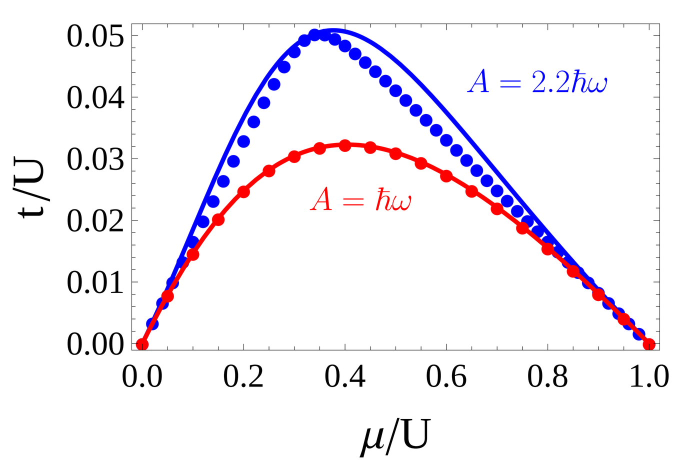
$$1 + zt\left[\frac{n}{f(n)-f(n-1)} + \frac{n+1}{f(n)-f(n+1)}\right] + \frac{n(n+1)z^2t^2\left[1 - J_0^2\left(\frac{A}{\hbar\omega}\right)\right]}{[f(n)-f(n-1)][f(n)-f(n+1)]} = 0$$

- Landau theory (solid lines):

$$1 + zt\left[\frac{n}{f(n)-f(n-1)} + \frac{n+1}{f(n)-f(n+1)}\right] + \frac{n(n+1)2zt\left[J_0\left(\frac{A}{\hbar\omega}\right) - 1\right]}{[f(n)-f(n-1)][f(n)-f(n+1)]} = 0$$

- Restriction of first hopping order:

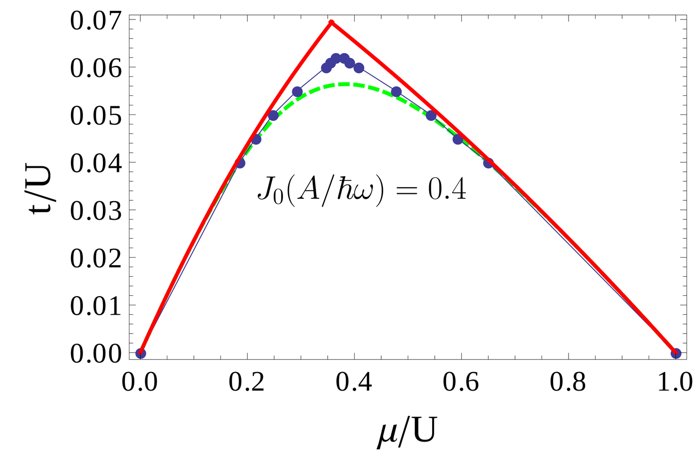
$$0 < \frac{A}{\hbar\omega} < x_1 \approx 1.52\,, \qquad J_0(x_1) = \frac{1}{2}$$



Second Hopping Order (2D)

- **Strong-coupling method [10]**
- **Quantum Monte Carlo [11]**
- **Second-order Landau theory [12]**

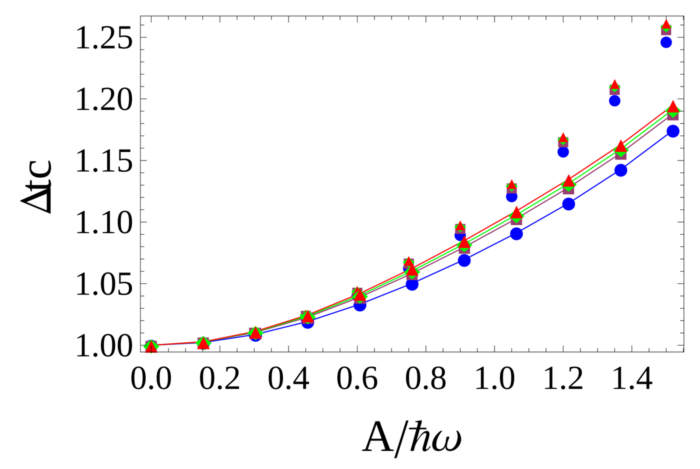
$\Longrightarrow$ error less than 6 % for
$0 < \frac{A}{\hbar\omega} < x_2 \approx 2.4\,, \quad J_0(x_2) = 0$



Observation

Critical hopping:

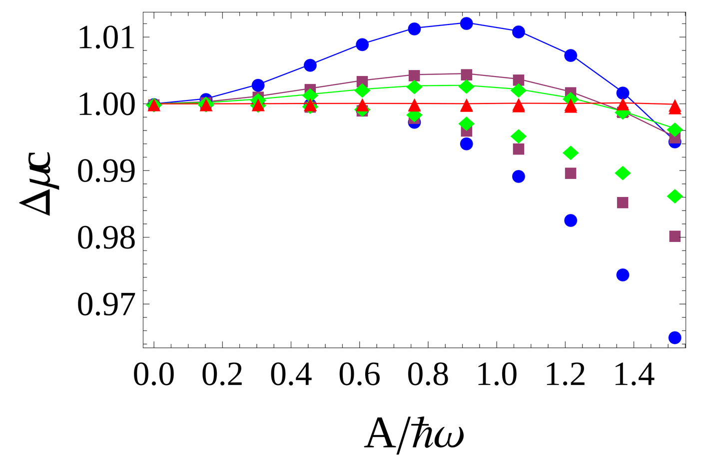
$$\Delta t_c = \frac{t_c(A)}{t_c(A=0)}$$



$\Longrightarrow$ **uniform renormalization**

Critical chemical potential:

$$\Delta\mu_c = \frac{\mu_c(A) - (n-1)U}{\mu_c(A=0) - (n-1)U}$$



$\Longrightarrow$ **unaffected by driving**

● $n=1$, ■ $n=2$, ◆ $n=3$, ▲ $n=100$ (3D: dots without lines, 2D: dots with lines)

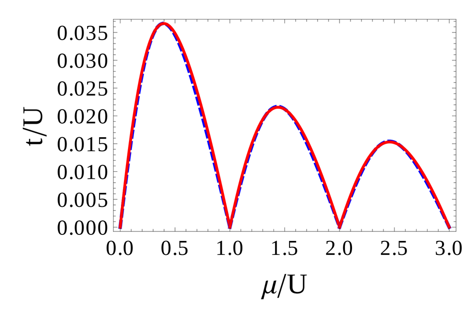
Effective Bose-Hubbard Model

$$\hat{H}_{\rm eff} = \sum_i\left[\frac{U}{2}\hat{n}_i(\hat{n}_i - 1) - \mu\hat{n}_i\right] - t\sum_{\langle ij\rangle}\hat{a}_i^\dagger J_0\left(\frac{A}{\hbar\omega}(\hat{n}_j - \hat{n}_i)\right)\hat{a}_j$$

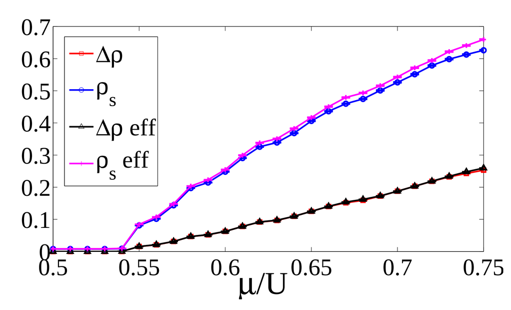
Approximation due to uniform renormalization of critical hopping with driving amplitude

$$\hat{H}(x) = \sum_i\left[\frac{U}{2}\hat{n}_i(\hat{n}_i - 1) - \mu\hat{n}_i\right] - t\lambda(x)\sum_{<ij>}\hat{a}_i^\dagger\hat{a}_j$$

$$\lambda(x) = 1 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots\,,\quad x = \frac{A}{\hbar\omega}$$



3D: $A = \hbar\omega$



QMC, 2D: $t = 0.05U, A = 0.4\hbar\omega$

References

1. T. Wang, X.-F. Zhang, F.E.A. dos Santos, S. Eggert, and A. Pelster, Phys. Rev. A **90**, 013633 (2014).
2. E. Arimondo, D. Ciampini, A. Eckardt, M. Holthaus, and O. Morsch, Adv. At. Mol. Opt. Phys. **61**, 515 (2012).
3. A. Eckardt, C. Weiss, and M. Holthaus, Phys. Rev. Lett. **95**, 260404 (2005).
4. H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo, Phys. Rev. Lett. **99**, 220403 (2007).
5. A. Zenesini, H. Lignier, D. Ciampini, O. Morsch, and E. Arimondo, Phys. Rev. Lett. **102**, 100403 (2009).
6. S.E. Pollack, D. Dries, R.G. Hulet, K.M.F. Magalhaes, E.A.L. Henn, E.R.F. Ramos, M.A. Caracanhas, and V. S. Bagnato, Phys. Rev. A **81**, 053627 (2010).
7. A. Rapp, X. Deng, and L. Santos, Phys. Rev. Lett. **109**, 203005 (2012).
8. J. Hubbard, Proc. Roy. Soc. A **276**, 238 (1963).
9. F.E.A. dos Santos and Pelster, Phys. Rev. A **79**, 013614 (2009).
10. J.K. Freericks and H. Monien, Phys. Rev. B **53**, 2691 (1996).
11. A.W. Sandvik, Phys. Rev. B **59**, R14157 (1999).
12. T. Wang, X.-F. Zhang, S. Eggert, and A. Pelster, Phys. Rev. A **87**, 063615 (2013).

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