Dipolar Bose-Einstein Condensates with Periodic Modulated Contact Interaction

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Abstract

Harmonically trapped Bose-Einstein condensates (BECs) with a sufficiently strong dipolar interaction possess both a stable and an unstable equilibrium. Following Ref. [1] we investigate how the stability of both equilibria change under parametric excitation by a periodic modulation of the s-wave scattering length [2, 3]. To this end we perform a linear stability analysis based on a Gaussian variational ansatz of the underlying Gross-Pitaevskii Lagrangian. We find that parametric excitation can stabilize a previously unstable dipolar BEC and lead to bistability in case of two equal energy stable solutions for a certain choice of driving amplitude and frequency.

Introduction

- **Gross-Pitaevskii Lagrangian**

$$L(t) = \int \left[\frac{i\hbar}{2}\left(\psi(\mathbf{r},t)\frac{\partial\psi(\mathbf{r},t)^*}{\partial t} - \psi(\mathbf{r},t)^*\frac{\partial\psi(\mathbf{r},t)}{\partial t}\right) - \frac{\hbar^2}{2m}|\nabla\psi(\mathbf{r},t)|^2 - \frac{m\omega_\rho^2}{2}(\rho^2+\lambda^2 z^2)|\psi(\mathbf{r},t)|^2\right] d\mathbf{r}$$
$$-\frac{g}{2}\int |\psi(\mathbf{r},t)|^2 \left[(1+q\cos(\Omega t))\delta(\mathbf{r}-\mathbf{r}') + \frac{3\varepsilon\,\mathrm{sign}(g)}{4\pi|\mathbf{r}-\mathbf{r}'|^3}\left(1 - 3\frac{((\mathbf{r}-\mathbf{r}')\mathbf{e}_z)^2}{|\mathbf{r}-\mathbf{r}'|}\right)\right]|\psi(\mathbf{r}',t)|^2\, d\mathbf{r}\, d\mathbf{r}'$$

- **Gaussian variational ansatz**, for Tomas-Fermi variational treatment see Ref. [4]

$$\psi^{G}(\rho,z,t) = \frac{\sqrt{N}}{\pi^{\frac{3}{4}}\tilde{u}_\rho\sqrt{\tilde{u}_z}}\exp\left[-\frac{\rho^2}{2\tilde{u}_\rho^2} + i\rho^2\phi_\rho - \frac{z^2}{2\tilde{u}_z^2} + iz^2\phi_z\right]$$

dimensionless quantities: $u_{\rho,z} = \tilde{u}_{\rho,z}/a_{\mathrm{ho}}$, $a_{\mathrm{ho}} = \sqrt{\hbar/(m\omega_\rho)}$, $t' = \Omega t/2$

- **Euler-Lagrange equations**

$$\frac{\Omega^2}{4\omega_\rho^2}\ddot{u}_\rho + u_\rho = \frac{1}{u_\rho^3} + \frac{p}{u_\rho^3 u_z}\left[1 + q\cos(2t') - \varepsilon\left(f_s\left(\frac{u_\rho}{u_z}\right) - \frac{u_\rho}{2u_z}f_s'\left(\frac{u_\rho}{u_z}\right)\right)\right]$$
$$\frac{\Omega^2}{4\omega_\rho^2}\ddot{u}_z + \lambda^2 u_z = \frac{1}{u_z^3} + \frac{p}{u_\rho^2 u_z^2}\left[1 + q\cos(2t') - \varepsilon\left(f_s\left(\frac{u_\rho}{u_z}\right) + \frac{u_\rho}{u_z}f_s'\left(\frac{u_\rho}{u_z}\right)\right)\right]$$

$$p = \sqrt{\frac{2}{\pi}}\frac{Na_s}{a_{\mathrm{ho}}}\,, \quad a_s = \frac{gm}{4\pi\hbar^2}\,, \quad f_s(\kappa) = \frac{1+2\kappa^2}{1-\kappa^2} - 3\frac{\kappa^2\,\mathrm{artanh}\sqrt{1-\kappa^2}}{(1-\kappa^2)^{\frac{3}{2}}}\,, \quad \kappa = \frac{u_\rho}{u_z}$$

- **Linearization** $u_{\rho,z}(t') = u_{0\rho,z} + \delta u_{\rho,z}(t')$, $\delta\mathbf{u} = (\delta u_\rho, \delta u_z)^T$

$$\delta\ddot{\mathbf{u}} + (\hat{A} + \hat{Q}\cos 2t')\delta\mathbf{u} = \mathbf{f}_0 + \mathbf{f}\cos 2t'\,, \quad \text{in equilibrium } \mathbf{f}_0 = \mathbf{0}$$

$$\text{Floquet theory:} \quad \delta\mathbf{u}(t') = e^{\beta t'}\sum_{n=-\infty}^{\infty}\mathbf{u}_{2n}e^{2int'}$$

f plays no rule for stability [5]
Floquet exponent β follows from continued fraction [6, 7]

$$\mathrm{Det}\Big[\beta^2\hat{I} + \hat{A} - \frac{1}{2}\hat{Q}\left(2(\beta+2i)^2\hat{I} + 2\hat{A} - \hat{Q}\left(2(\beta+4i)^2\hat{I} + 2\hat{A} - \ldots\right)^{-1}\hat{Q}\right)^{-1}\hat{Q}$$
$$-\frac{1}{2}\hat{Q}\left(2(\beta-2i)^2\hat{I} + 2\hat{A} - \hat{Q}\left(2(\beta-4i)^2\hat{I} + 2\hat{A} - \ldots\right)^{-1}\hat{Q}\right)^{-1}\hat{Q}\Big] = 0$$

Equilibrium

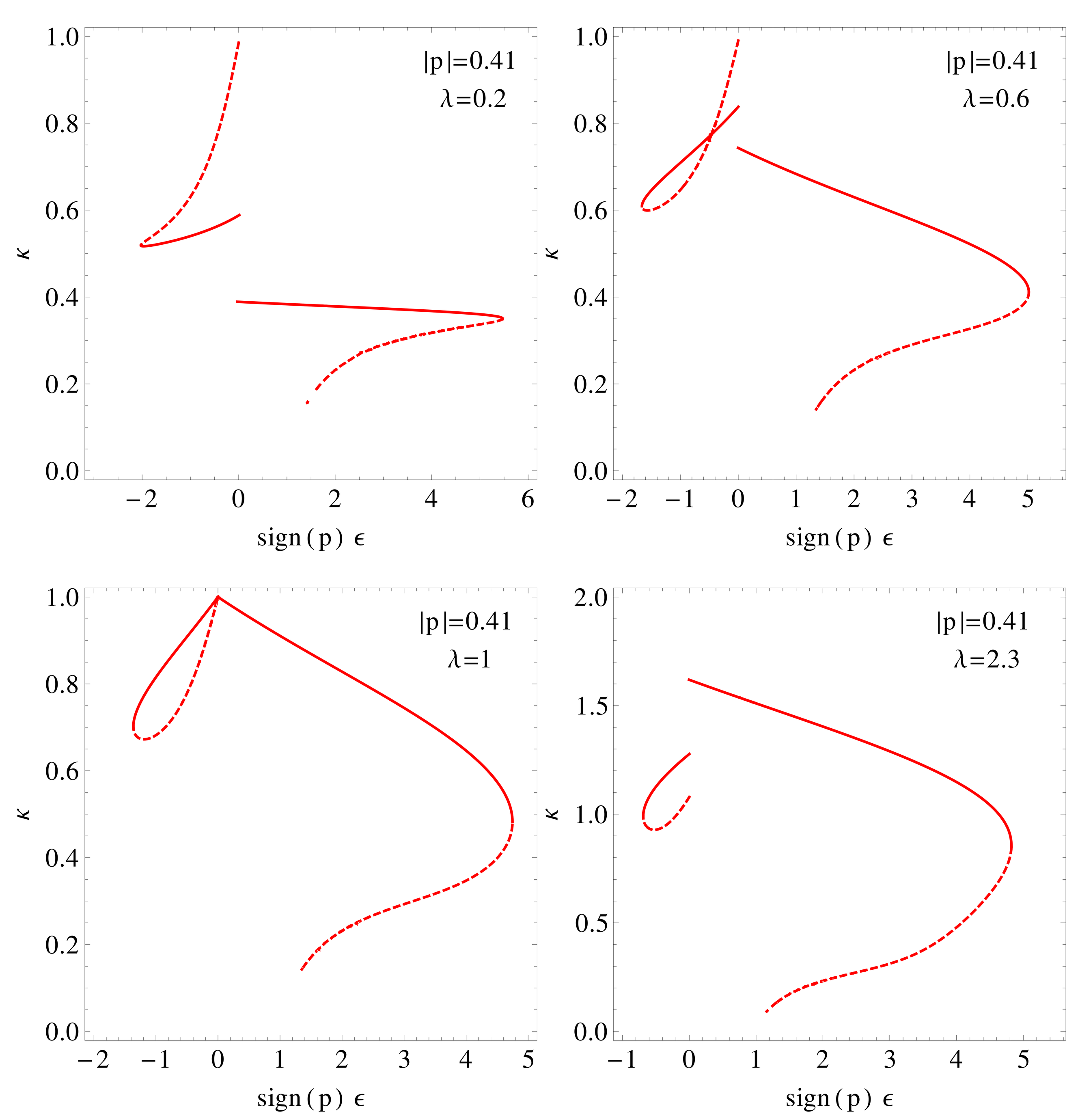

Fig. 1: Equilibrium dependence of the condensate aspect ratio κ on the relative dipole-dipole interaction strength ε calculated from $\mathbf{f}_0(\kappa,\lambda,p,\varepsilon) = \mathbf{0}$. Solid lines represent stable and dashed unstable equilibria.

Negative scattering length

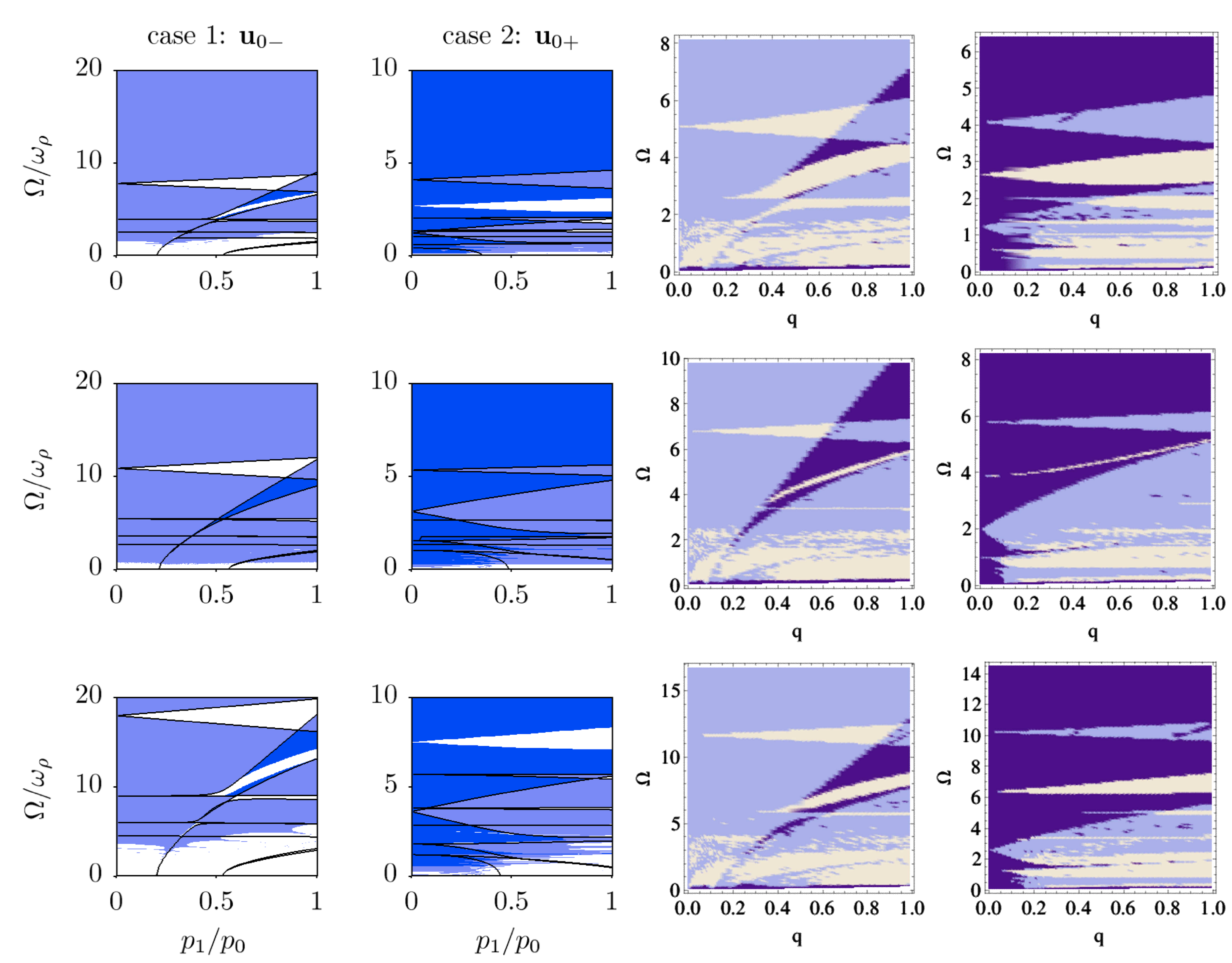

Fig. 2: Stability diagram for contact interaction ε = 0 (left) [1] and ε = 0.7 (right), for λ = 0.2, 1, 2.3 from top to the bottom. Dark-blue, light-blue, and white color represent regions with two, one, and zero stable degrees of freedom.

Conclusions and outlook

- Unstable equilibria can be stabilized using parametric resonance: experimental realization?
- What happens with unstable equilibria at positive scattering length?
- Searching for bistable configurations and dynamics
- In future: including quantum fluctuations [8, 9]
- Investigating dipolar Fermi gases [10, 11]
- Stronger driving ampliutde: linearization about nonlinear periodic solution [12]
- Comparison of linear analytic and nonlinear numeric stability analysis needed [1]

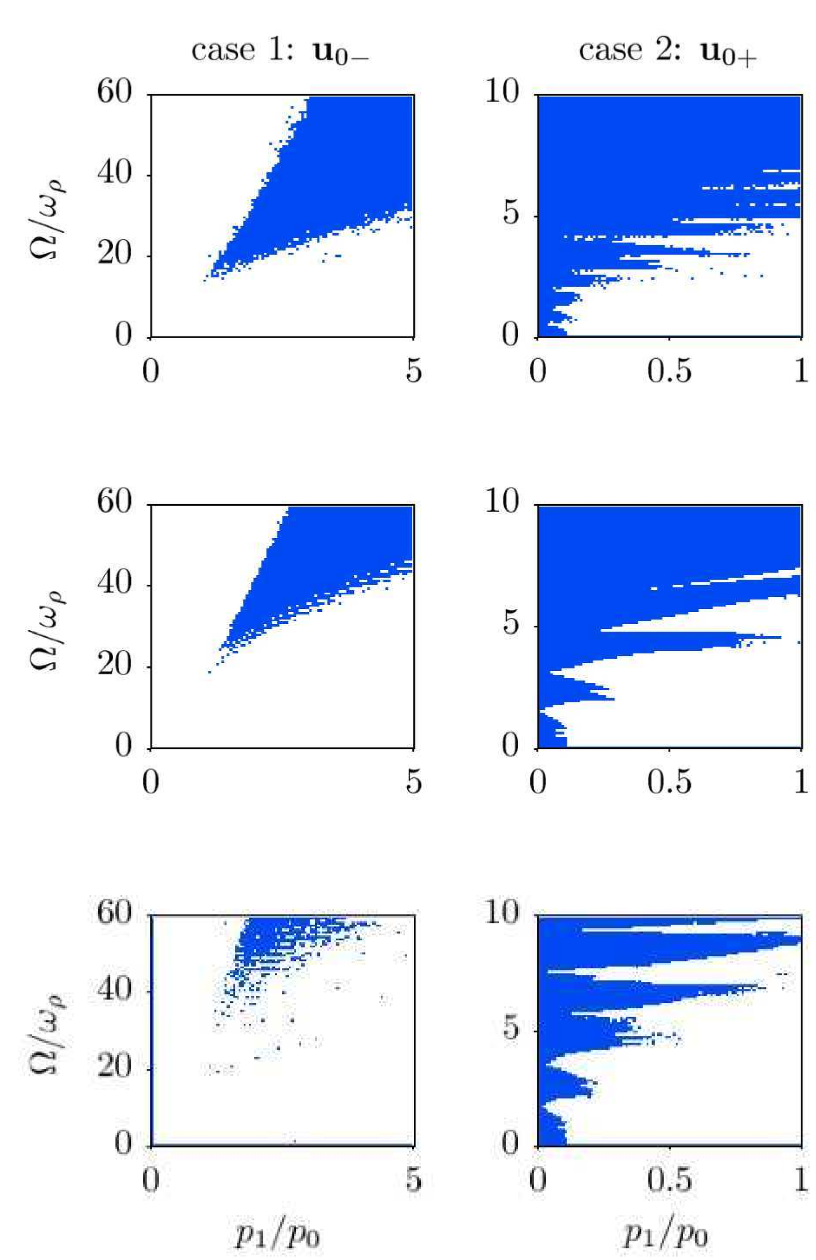

Fig. 3: Stability diagram for contact interaction ε = 0 [1] for λ = 0.2, 1, 2.3 from top to the bottom. The results are obtained by numerically solving the nonlinear Euler-Lagrange equations.

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Acknowledgments

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