Green's function approach to the Bose-Hubbard model with disorder

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Abstract

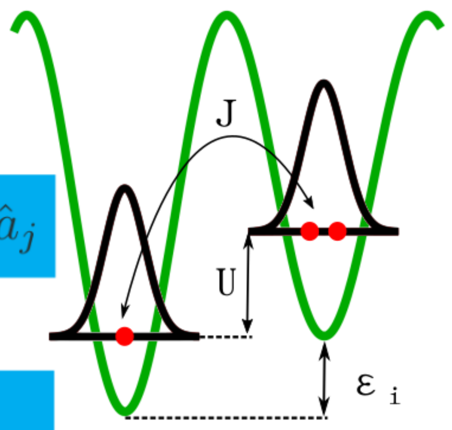
We present the findings obtained in [1], where we investigated the criteria for identifying the different ground states of the disordered Bose-Hubbard model at finite temperatures and for small values of the tunneling energy. To this end we constructed a perturbative expansion to the single-particle Green's function. By summing tree-level contributions of the approximation, we obtained the condition to the long-range correlations which leads to the phase boundary between superfluid and insulating phases. We then obtained a renormalized expression to the local density of states, which unambiguously distinguishes the Mott-insulator and Bose-glass phases. As a result, we constructed the phase diagram considering bounded on-site disorder where the region occupied by each ground state can be identified.

Introduction

Disordered Bose-Hubbard model (DBHM)

$$\hat{H}_{BH} = \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i-1) + \sum_i(\varepsilon_i-\mu)\hat{n}_i - J\sum_{\langle ij\rangle}\hat{a}_i^\dagger\hat{a}_j$$

On-site interaction energy | Local chemical potential | Hopping energy

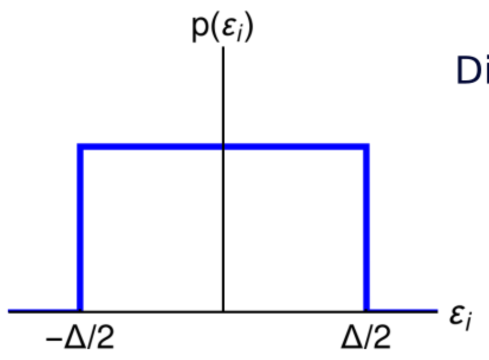


Randomly distributed ε_i

Uniform distribution

$$p(\varepsilon_i) = \frac{1}{\Delta}\left[\Theta\left(\varepsilon_i+\frac{\Delta}{2}\right)-\Theta\left(\varepsilon_i-\frac{\Delta}{2}\right)\right]$$

Δ→ Disorder strength

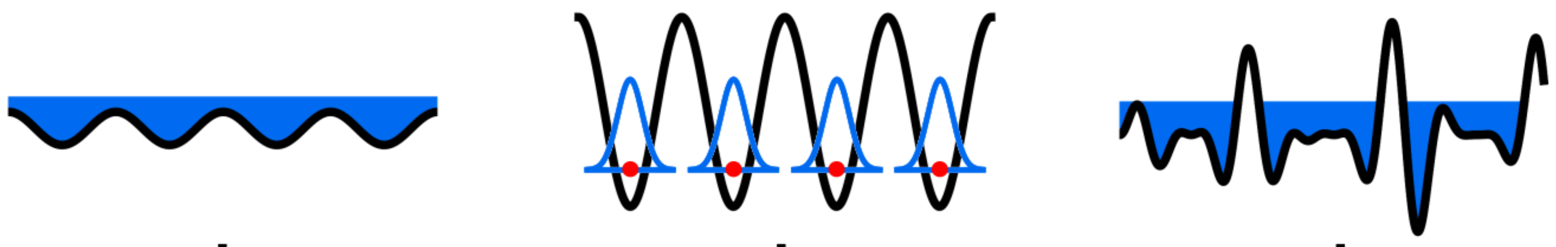


Disorder ensemble average

$$\overline{G} = \prod_i \int_{-\infty}^{\infty} d\varepsilon_i G_i p(\varepsilon_i) d\varepsilon_i$$

$$G_i \equiv G(\varepsilon_i)$$

Ground states



Superfluid (SF): Presence of long-range order. Divergence of long-range correlations. $\overline{G}(\mathbf{k}=\mathbf{0},\omega=0)\to\infty$

Mott insulator (MI): **Vanishing** local density of states for zero-energy excitations. $\overline{\rho}(\omega=0)=0$

Bose glass (BG): **Finite** local density of states for zero-energy excitations. $\overline{\rho}(\omega=0)\neq 0$

Theory

Diagrammatic expansion of the Green's function [2]

$$G_{ij}(\omega_1;\omega_2) = \delta_{ij} \;(\omega_2 \to i \to \omega_1) + (\omega_2 \to i \to j \to \omega_1) + (\omega_2 \to i \to \cdots \to j \to \omega_1) + \cdots$$

Diagrammatic rules: $(\omega_2 \to i \to \omega_1) = \delta_{\omega_1,\omega_2} g_i(\omega_1)$, $i \longrightarrow j = J_{ij}$.

SF phase boundary

Summation of all tree-level diagrams in Fourier space

$$\overline{G}(\mathbf{k},\omega) = \sum_{n=0}^{\infty}[\overline{g}(\omega)]^{n+1}J(\mathbf{k})^n \;\to\; \overline{G}(\mathbf{k},\omega) = \left[\frac{1}{\overline{g}(\omega)}-J(\mathbf{k})\right]^{-1} \;\to\; J(\mathbf{0}) = [\overline{g}(0)]^{-1}$$

$$J(\mathbf{k}) = 2J\sum_{\alpha=1}^{D}\cos(k_\alpha a)$$

MI-BG phase boundary

First contribution to the local Green's function

$$G_{ii}(\omega_1;\omega_2) = (\omega_2 \to i \to \omega_1) + (\omega_2 \to i \to \omega_1 \text{ with loop}) + \cdots$$

Where: $(\omega_2 \to i \to \omega_1) = \delta_{\omega_1,\omega_2}[g_i(\omega_1)]^2 \longrightarrow$ Double poles

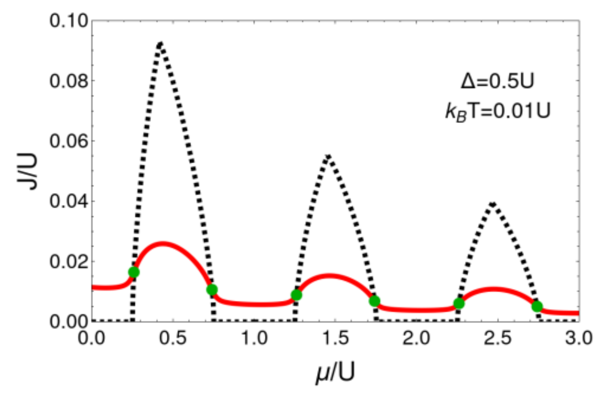
Poincaré-Lindstedt method [3] to the renormalization of double poles

Renormalized density of states $\longrightarrow \overline{\rho}(\omega) \equiv \overline{\rho}(\omega,J^2) = -\frac{1}{\pi}\mathrm{Im}\,\overline{G_{ii}}(\omega,J^2) \longrightarrow \overline{\rho}(0)\begin{cases}=0 & \to \mathrm{MI}\\ \neq 0 & \to \mathrm{BG}\end{cases}$

Results

Phase boundaries for a cubic lattice

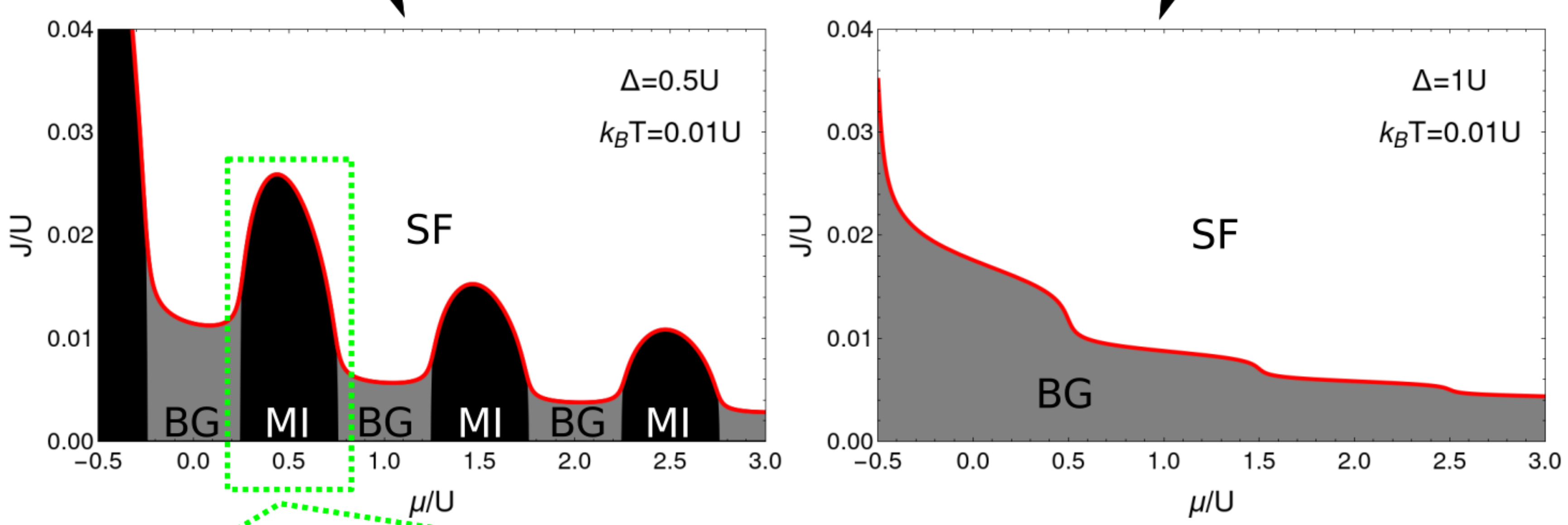
SF phase boundary · MI-BG phase boundary · Tri-critical points

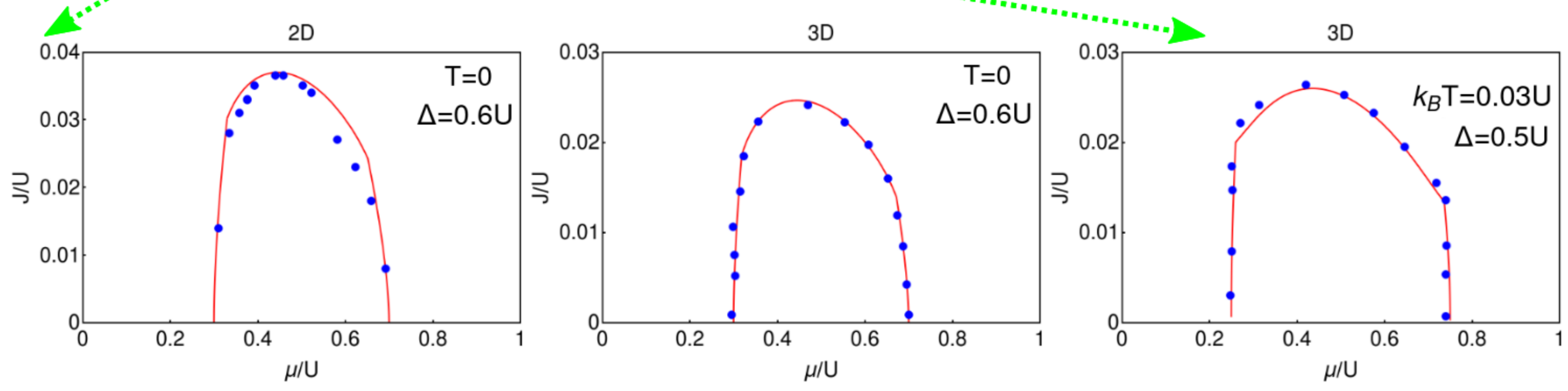


Δ < U: MI and BG coexist — $\overline{\rho}(\omega=0)=0$

Δ ≥ U: BG suppresses MI — $\overline{\rho}(\omega=0)\neq 0$

Phase diagram for a cubic lattice





Theoretical result · Numerical points

Comparison with numerical results of Refs. [4, 5, 6] for the phase boundary of the first Mott lobe

Outlook

- We have constructed a perturbative Green's function approach to the DBHM for finite temperatures and for small values of the hopping energy.
- We constructed a phase diagram where each ground state can be identified.
- By using the renormalized expression to the density of states we obtained the phase boundary between MI and BG.
- Our results are in excellent agreement with numerical calculations from the literature.

References

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