B6

Dimensional Crossover in Trapped Photon Gases

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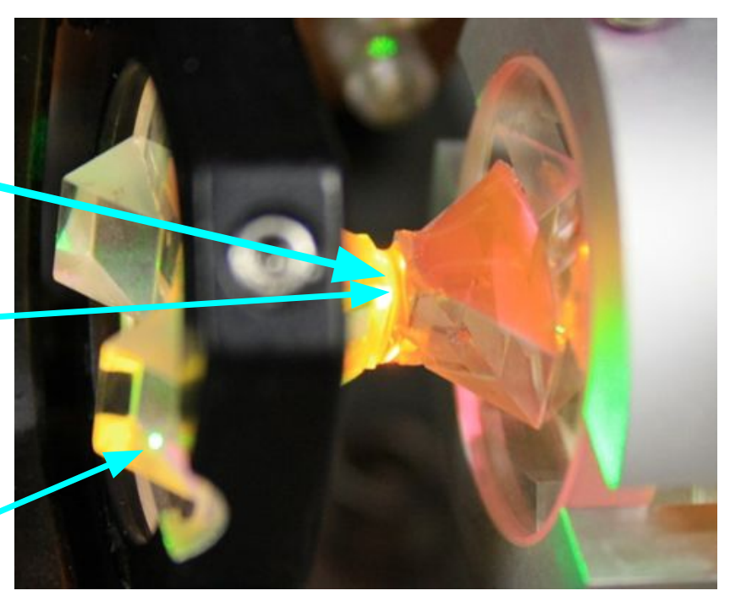
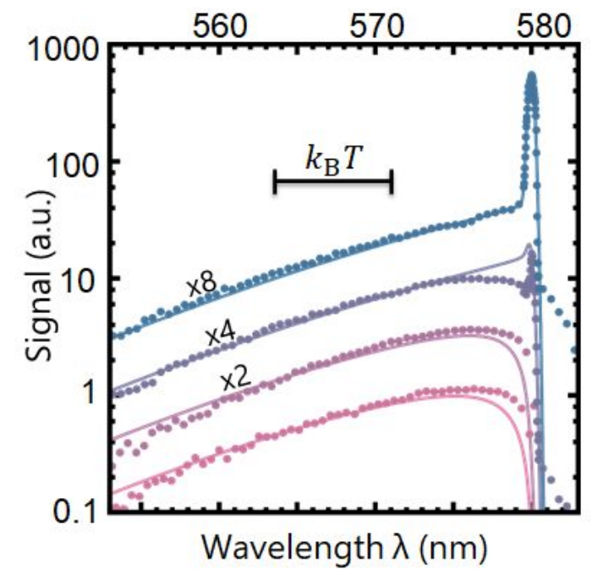
Introduction

Bose-Einstein condensation of photons

- High finesse cavity: Provides energy cutoff
- Dye solution: Heat and particle reservoir
- Pump radiation: Provides chemical potential



Energy distribution
⇒ Thermodynamic properties



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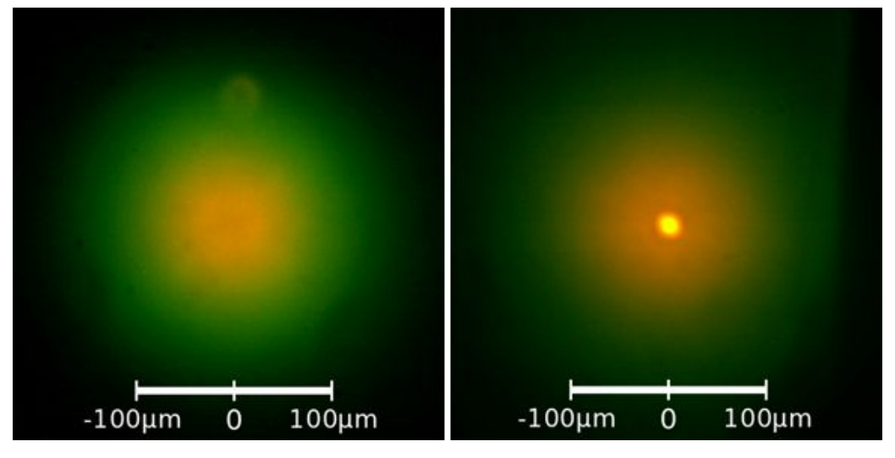
Spatial information



[1][2]

[1] J. Klaers et al., Nature **468**, 545 (2010)
[2] J. Klaers et al., Nat. Phys. **6**, 512 (2010)
[3] T. Damm et al., Nature Commun. **7**, 11340 (2016)

Ideal Bose Gas: Dimensional Crossover (arxiv: 2011.06339)

Aim: Theoretical prediction of effective system dimension at dimensional crossover in harmonic trapping potential

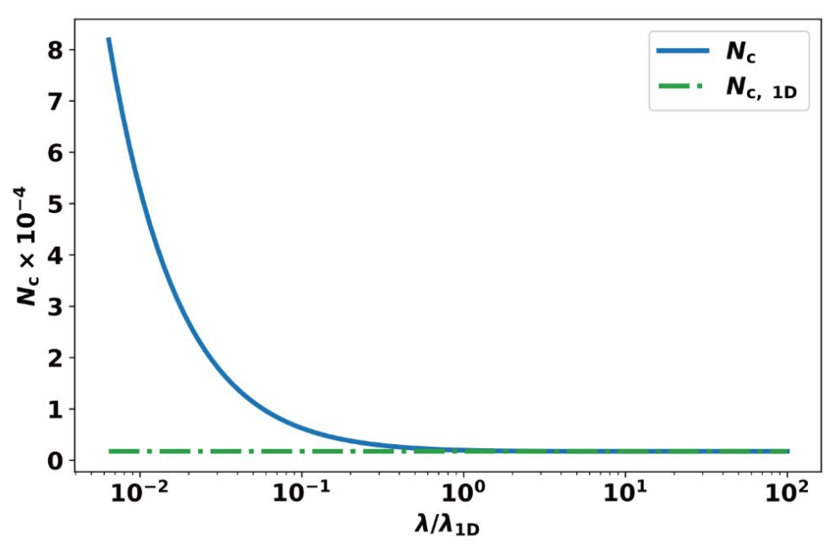
Energy levels: $E_{jn}(\lambda) = \hbar\Omega\left(j + \lambda n + \frac{1+\lambda}{2}\right)$ **, trap-aspect ratio:** $\lambda = \frac{\Omega_y}{\Omega}$

Grand-Canonical Partition Function: $\Pi = \Pi_{1\mathrm{D}} + \Delta\Pi$

$$\Big\Downarrow \lambda \to \infty$$

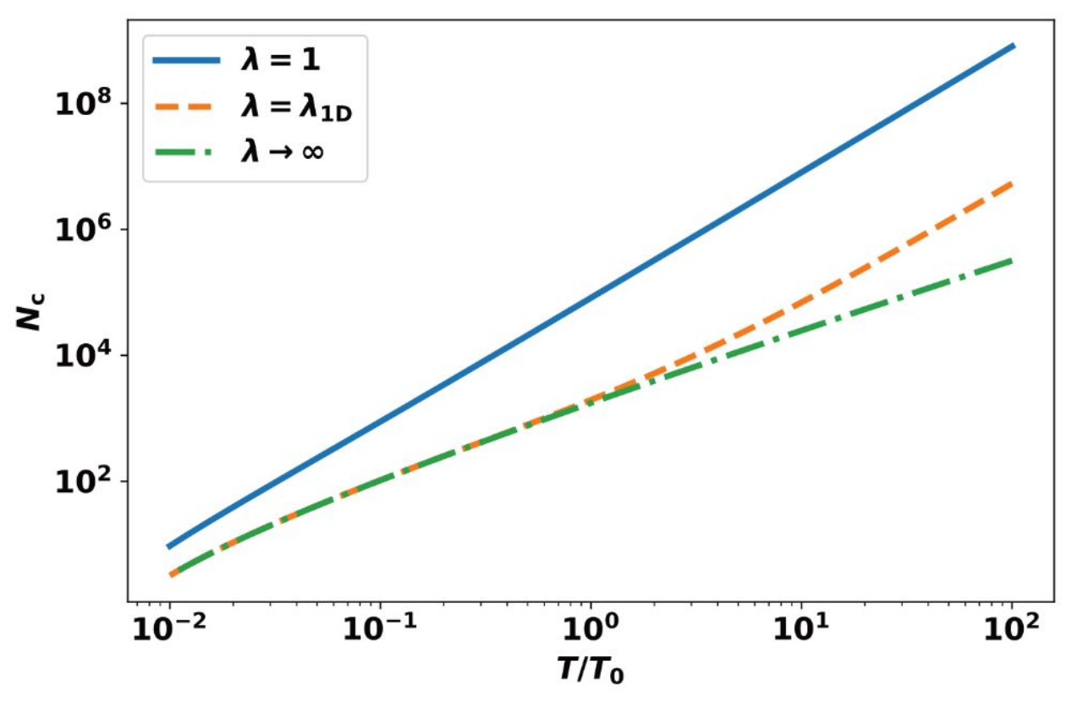
Effective 1D Limit: $\Pi_{1\mathrm{D}}$

Example: Critical Particle Number at Crossover

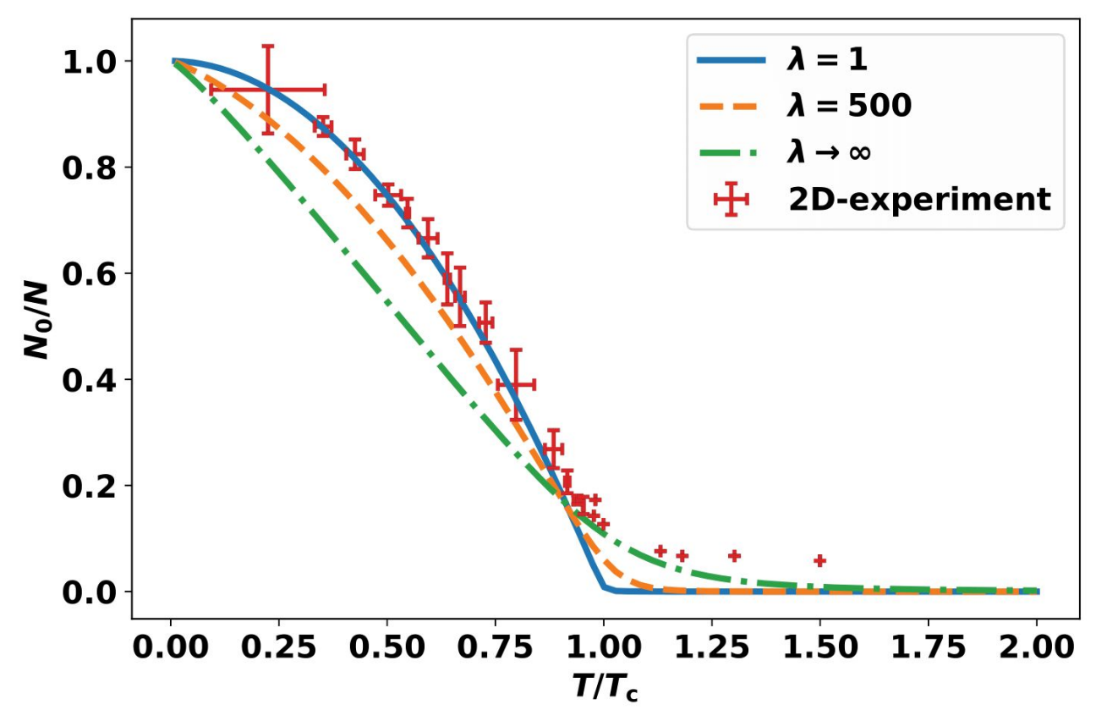


Thermodynamic Quantities:

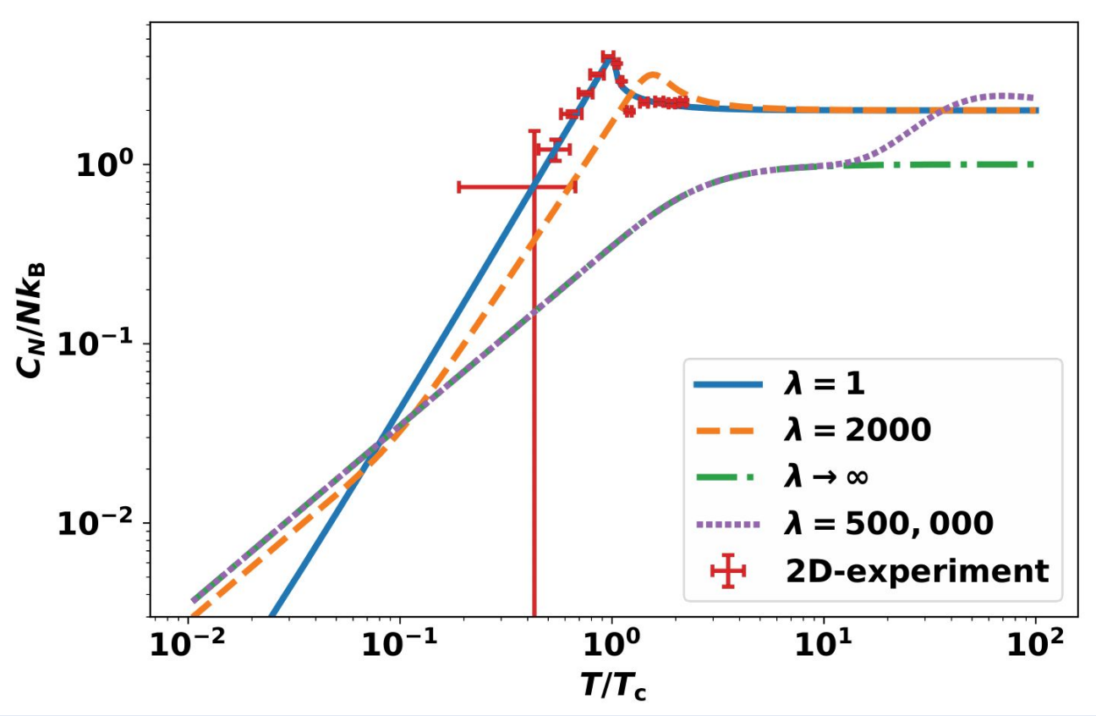
Critical Particle Number



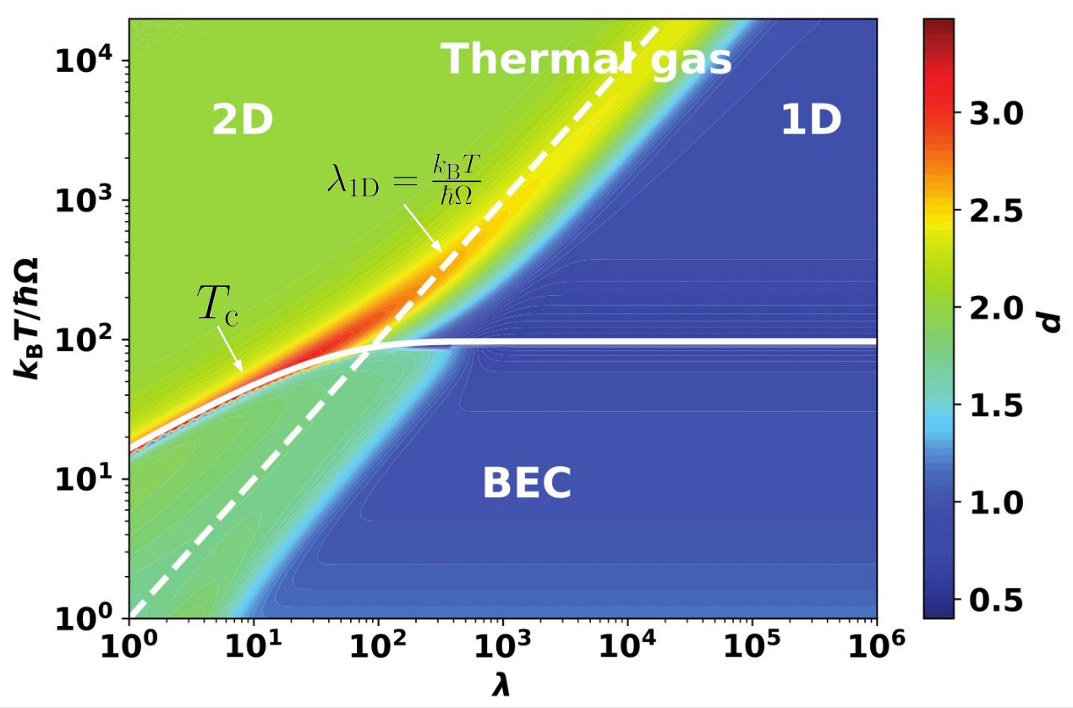
Condensate Fraction



Specific Heat

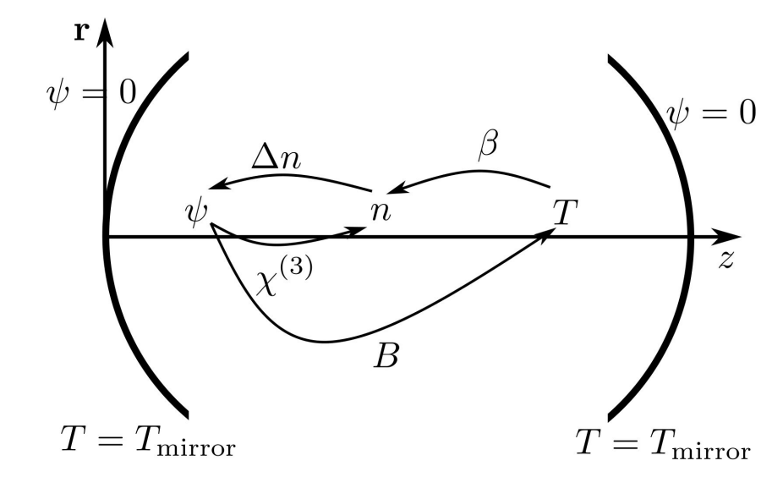


Effective Dimension: $d = \frac{1}{Nk_\mathrm{B}}\begin{cases} C_N, & \text{thermal,} \\ \frac{\partial \ln C_N}{\partial \ln b}, & \text{BEC} \end{cases}$



Photon BEC Ground State with Interaction

Aim: Behaviour of ground state of photon BEC at dimensional crossover including interaction effects



Stein, Vewinger, Pelster, NJP **21**, 103044 (2019)

Photon-Energy Functional:

$$E[\psi,\psi^*] = \int d^2x \left[\frac{\hbar^2}{2m}|\nabla\psi|^2 + \frac{m\Omega^2}{2}\left(x^2+\lambda^4 y^2\right)|\psi|^2 + \underbrace{\frac{g_K}{2}|\psi|^4}_{\text{Kerr effect}} + \underbrace{\frac{g_T}{2}\int d^2x'\,\mathcal{G}(\mathbf{x}-\mathbf{x}')|\psi(\mathbf{x})|^2|\psi(\mathbf{x}')|^2}_{\text{Thermo-optic effect}}\right]$$

Minimisation with Gaussian Ansatz:

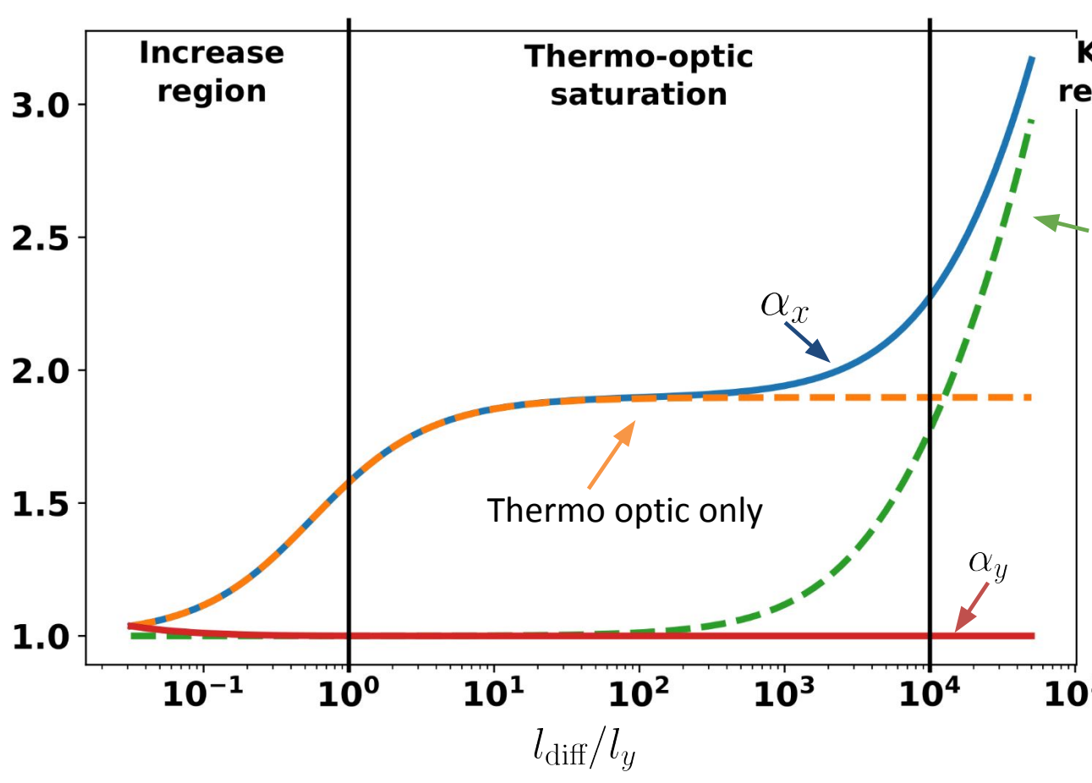
$$\psi = \sqrt{\frac{\lambda N}{\alpha_x\alpha_y\pi l_x^2}}\exp\left[-\frac{1}{2l_x^2}\left(\frac{x^2}{\alpha_x^2} + \lambda^2\frac{y^2}{\alpha_y^2}\right)\right]$$

Oscillator length: $l_x = \sqrt{\frac{\hbar}{m\Omega}}$

Variational parameters: α_x , α_y

Results:

Widths

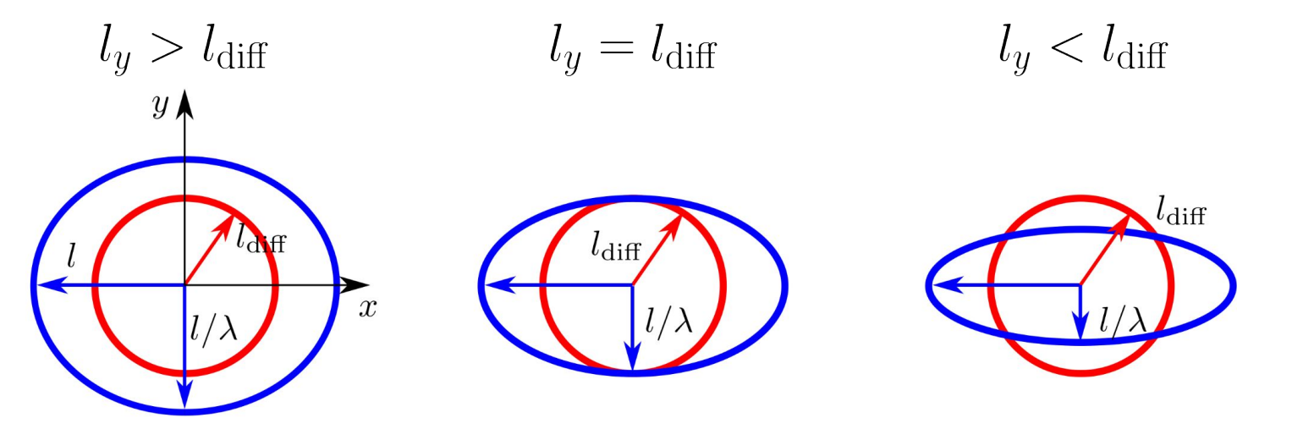


Diffusion length: $l_{\mathrm{diff}} = \sqrt{\tau\mathcal{D}}$

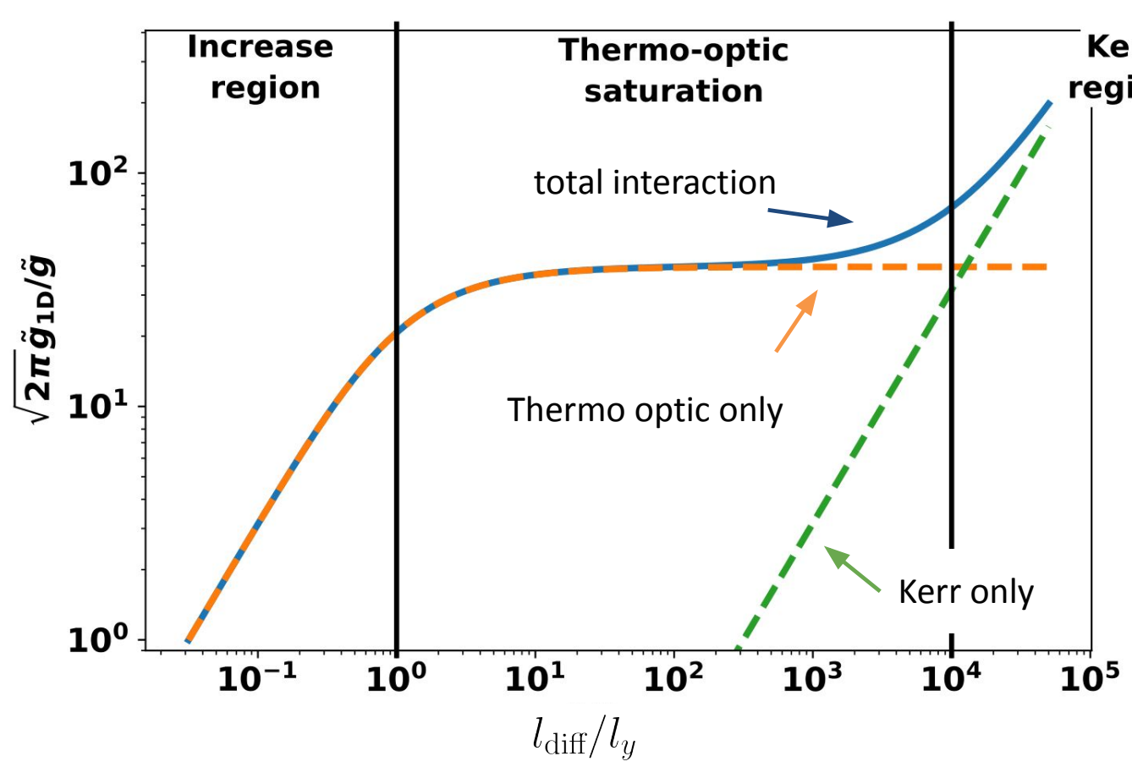
relaxation time

diffusion coefficient

Competition between diffusion and thermo-optic interaction:



Effective 1D Interaction Strength



Dimensionless interaction strength: $\tilde{g}_{.} = \frac{mg_{.}}{\hbar^2}$

$$\tilde{g}_{1\mathrm{D}}(\lambda) = \frac{1}{\sqrt{2\pi}}\left[\tilde{g}_K\lambda + \frac{\tilde{g}_T l_x}{l_{\mathrm{diff}}}\sqrt{\frac{\pi}{2}}e^{l_y^2/2l_{\mathrm{diff}}^2}\operatorname{erfc}\left(\frac{l_y}{\sqrt{2}l_{\mathrm{diff}}}\right)\right]$$

Large trap anisotropy:

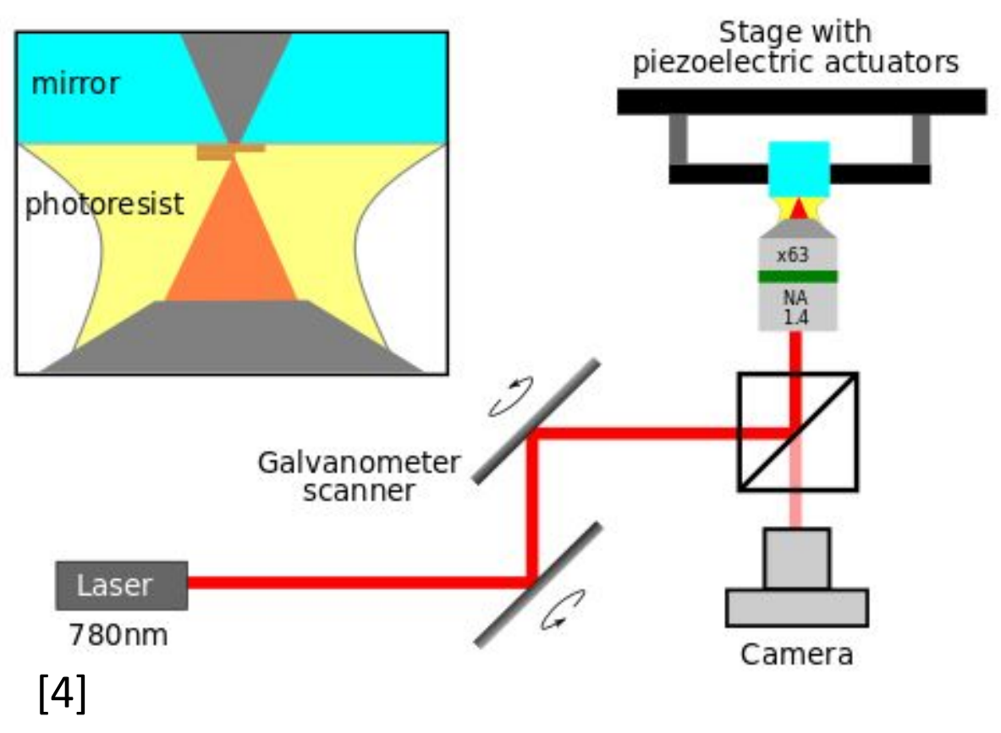
$$\lim_{\lambda\to\infty}\tilde{g}_{1\mathrm{D}}(\lambda) = \frac{1}{\sqrt{2\pi}}\left(\tilde{g}_K\lambda + \frac{\tilde{g}_T l_x}{l_{\mathrm{diff}}}\right)$$

Objectives

1: Potential landscapes for the photon gas ✅
2: Semiclassical mean-field theory ✅
3: Steady-state properties of a 1d photon gas
4: Superfluidity in anisotropic box traps
5: Correlations in one and two dimensions

Direct Laser Writing for Realizing 1D Potentials for Photon Gases

Microstructure mirror surface using DLW for producing dimple traps



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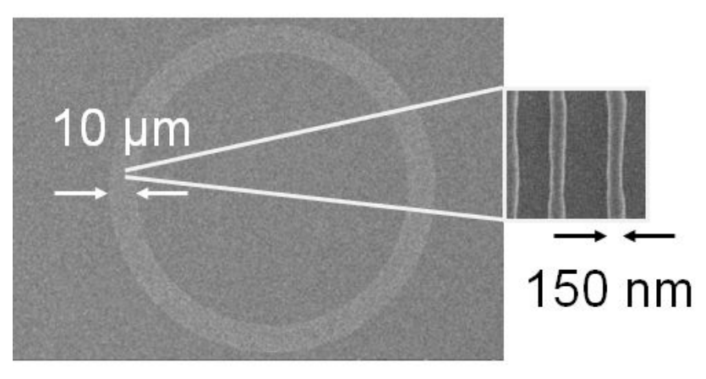
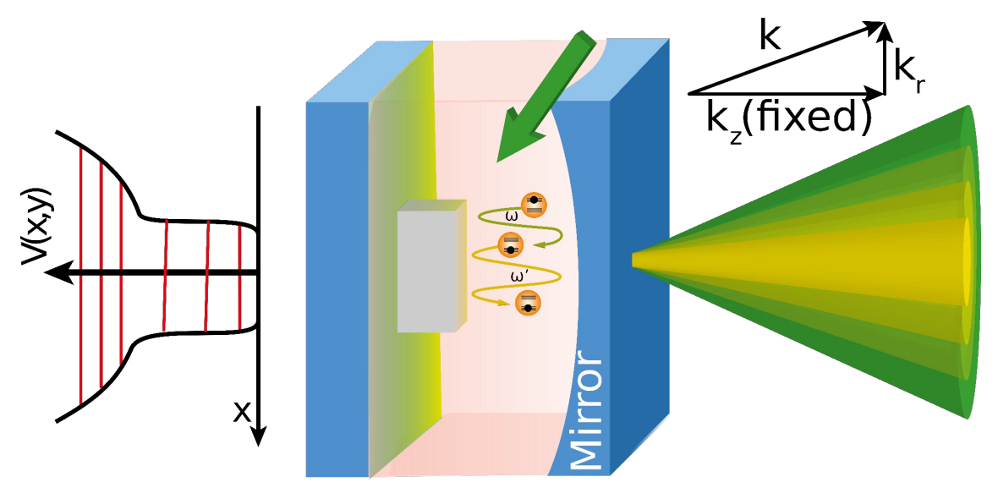
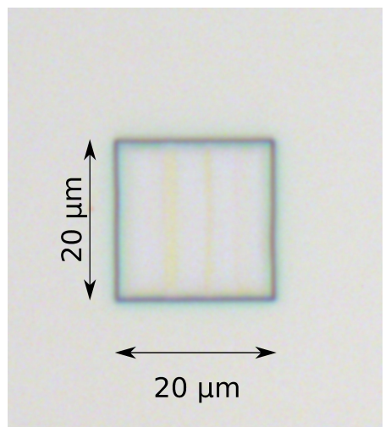
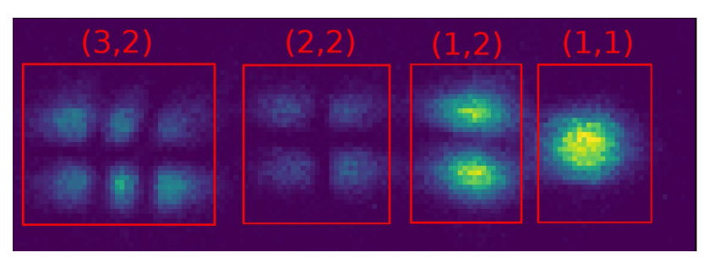




Feasibility test of direct laser writing of structures in a dye microcavity

- 3D printed box potential
- Structural and chemical stability in a dye micro cavity
- Thermalized photon gas





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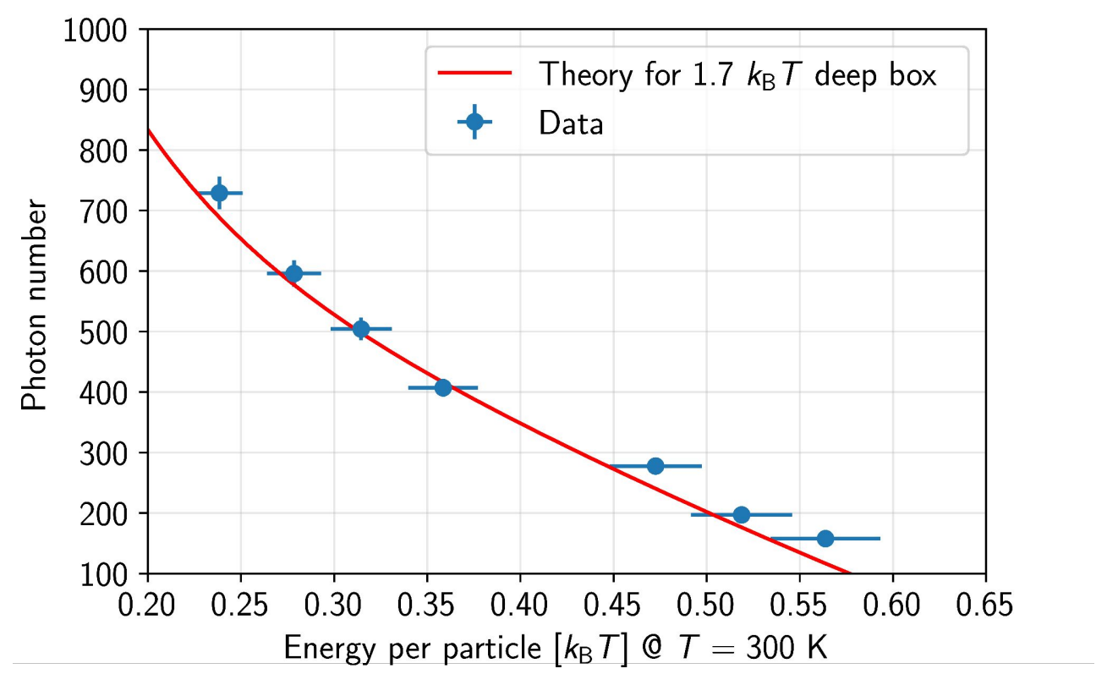

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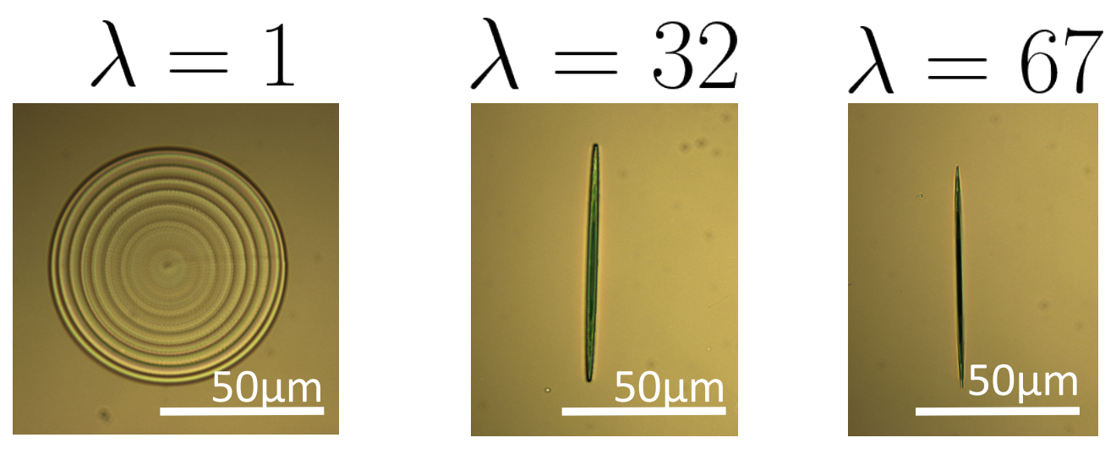
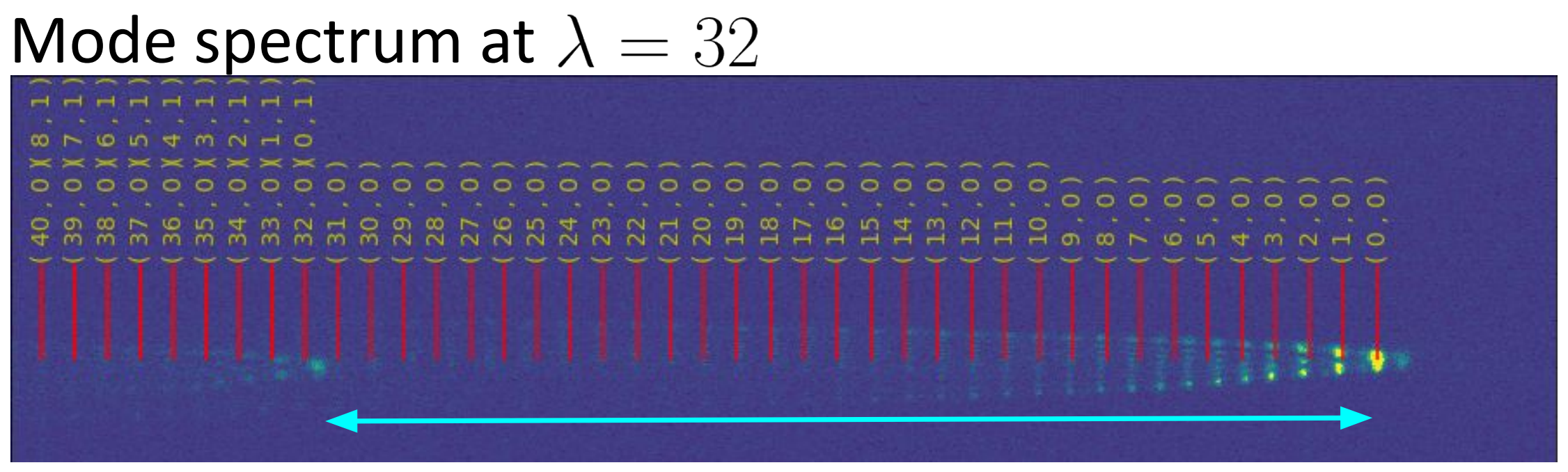
Parabolic structures for dimensional crossover study



Good agreement of mode spectrum with designed aspect ratio of printed harmonic trap

Mode spectrum at $\lambda = 32$



$\omega_x = 2\pi \times 0.21$ THz

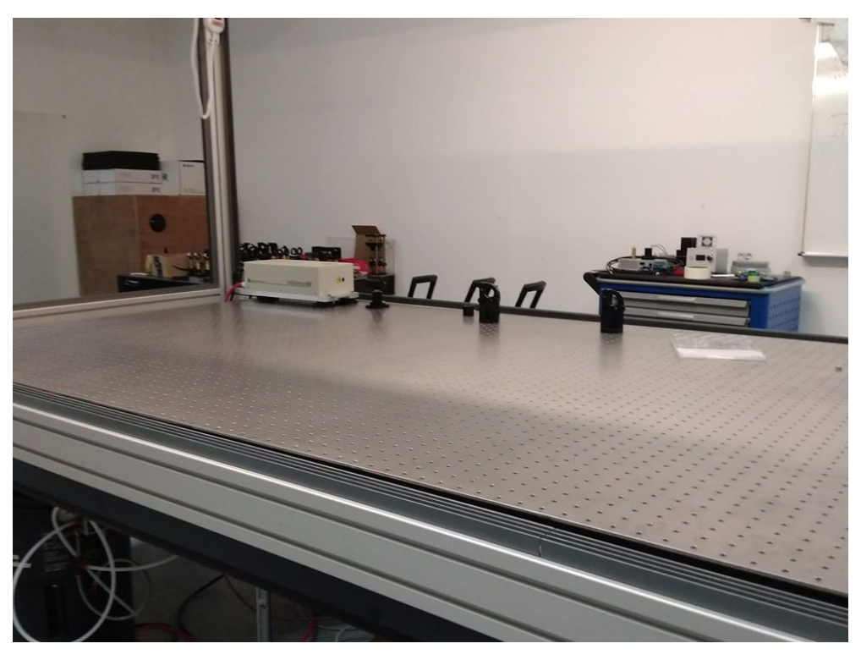
$\omega_y = 32 \times \omega_x$

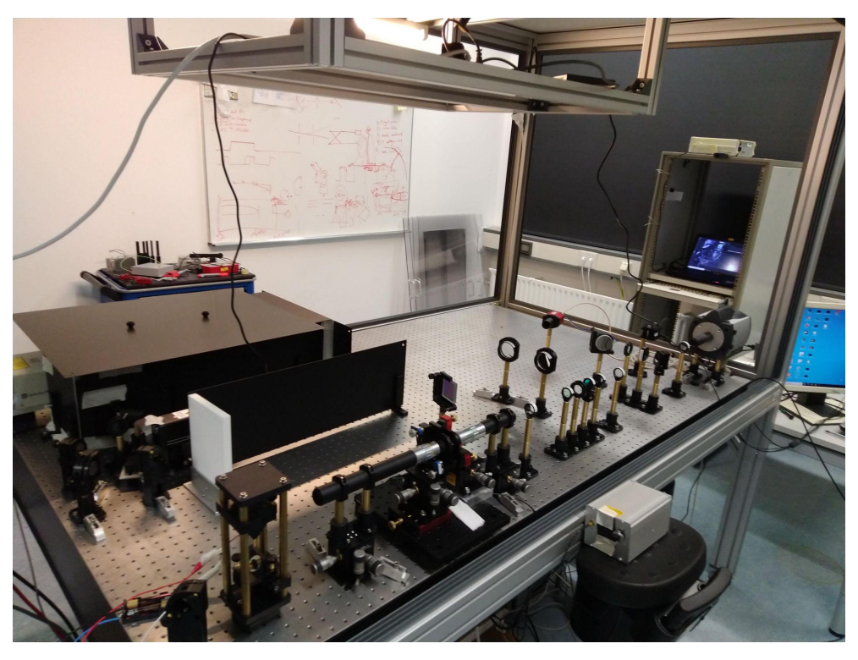
[5]

$k_\mathrm{B}T$

[4] Nanoscribe GmbH Photonic Professional (GT): User Manual
[5] Kirankumar K U, Investigation of 3D-printed potentials for photon gases (Master Thesis)

New Photon BEC Setup @ TU Kaiserslautern





QR code of timelapse

$N < N_\mathrm{c}$

$N > N_\mathrm{c}$

Outlook

Theory
- Dimensional crossover in dimple trap to close up with experiment
- Hartree-Fock theory with thermo-optic interaction at dimensional crossover

Experiment
- Fabricate and study photon gas in highly anisotropic parabolic structures
- Study thermodynamic properties of dimensional crossover 2D ⟷ 1D