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Critical Properties of Homogeneous Bose-Einstein Condensates

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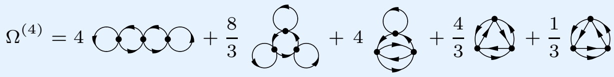
1. Variational Perturbation Theory

- Bose gas with weak repulsive two-particle interaction:

$$\mathcal{A}[\psi^*,\psi] = \int_0^{\hbar\beta} d\tau \int d^3x \left\{ \psi^*(\mathbf{x},\tau)\left(\hbar\,\partial_\tau - \frac{\hbar^2\nabla^2}{2M}\right)\psi(\mathbf{x},\tau) - \mu\,|\psi(\mathbf{x},\tau)|^2 + \frac{2\pi\hbar^2 a_s}{M}\,|\psi(\mathbf{x},\tau)|^4 \right\}$$

1.1 Dimension $D = 3+1$

- **Goal:** Extension of three-loop calculation for grand-canonical potential [1] to four-loops [2,3]



- **Goal:** Variational calculation of critical temperature [4]

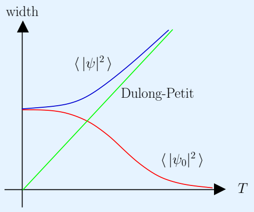
$$\frac{\Delta T_c}{T_c^{(0)}} = c_1 a_s n^{1/3} + [c_2' \ln(a_s n^{1/3}) + c_2] a_s^2 n^{2/3} + \mathcal{O}(a_s^3 n)$$

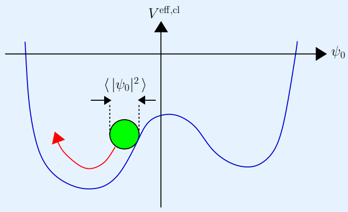
- **Goal:** Arbitrary interaction potential [3]

$$\frac{\Delta T_c}{T_c^{(0)}} = \frac{M n^{1/3}}{3\pi\hbar^2 \zeta^{4/3}(3/2)} \sum_{k=1}^{\infty} \left[\frac{-\pi n^{2/3}}{\zeta^{2/3}(3/2)}\right]^k \times \int d^3x V^{(\mathrm{int})}(\mathbf{x})(\mathbf{x}^2)^k \sum_{l=1}^{k} \frac{\zeta(l+1/2)\zeta(3/2-k-l)}{l!(k-l)!}$$

- **Goal:** Develop effective classical field theory [5,6]

$$Z = \int \mathcal{D}\psi_0^*(\mathbf{x}) \int \mathcal{D}\psi_0(\mathbf{x})\, e^{-\mathcal{A}^{\mathrm{eff,cl}}[\psi_0^*,\psi_0]/\hbar}$$





- **Goal:** Justification of intuitive matching procedure [7]
- **Goal:** Improvement of effective action calculation [8,9]
- **Goal:** Condensate density, superfluid density, specific heat

1.2 Dimension $D = 3$

- Shift of critical temperature in dilute limit:

$$\frac{\Delta T_c}{T_c^{(0)}} = -\frac{2}{3}\frac{\Delta n}{n} = c_1\, a_s\, n^{1/3}$$

- Calculation of Δn from classical limit of field theory:

$$\mathcal{A}[\Psi] = \int d^3x \left[\frac{1}{2}|\nabla\Psi|^2 + \frac{1}{2}m^2|\Psi|^2 + \frac{u}{4!}|\Psi|^4\right]$$

$$\Psi_1 + i\Psi_2 = \sqrt{\frac{\hbar^3}{M k_B T}}\,\psi_0\,, \quad m^2 = -\frac{2M\mu}{\hbar^2}\,, \quad u = \frac{48\pi a_s M k_B T}{\hbar^3}$$

- Five-loop perturbation expansion [10]: $c_1 = \sum_{l=-1}^{3} c_1^{(l)} \left(\frac{u}{m_r}\right)^l$

  critical limit $m_r \to 0$ $\iff$ strong-coupling problem

- Applying variational perturbation theory [6,11] yields resummation of c_1-series [12]: $c_1 \approx 0.91$

  $1/N$-expansion [13]: $c_1 \approx 1.71$, Monte-Carlo result [14]: $c_1 \approx 1.29$

- **Goal:** Six-loop variational calculation

2. Mott Insulator Transition in Optical Lattices

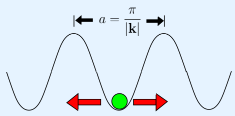
- Periodic potential: $V(\mathbf{x}) = V_0 \sum_{i=1}^{3} \sin^2(k_i x_i)\,, \qquad E_r = \frac{\hbar^2\mathbf{k}^2}{2M}$

  Experimentally [15], V_0 can be raised to $22E_r$

- **Goal:** Effective $|\psi|^4$-field theory for lowest energy band in Bose-Hubbard model [16]

$$\frac{\hbar^2\mathbf{q}^2}{2M_{\mathrm{eff}}} \;\widehat{=}\; 2J\sum_{i=1}^{3}(1-\cos q_i a)$$

$$\frac{4\pi\hbar^2 a_s^H}{M_{\mathrm{eff}}} \;\widehat{=}\; \sqrt{\frac{8}{\pi}}|\mathbf{k}|a_s E_r \left(\frac{V_0}{E_r}\right)^{3/4}$$



2.1 Goldstone Boson Model

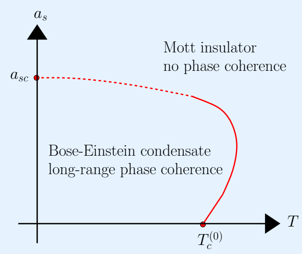
- Decompose field $\psi(\mathbf{x},\tau) = [\sqrt{n_0} + \rho(\mathbf{x},\tau)]\, e^{i\gamma(\mathbf{x},\tau)}$ and integrate out density fluctuations ρ
- **Goal:** Study phase fluctuations in hydrodynamic limit

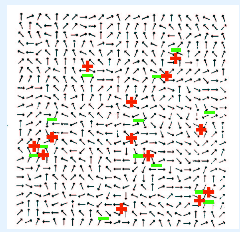
$$\mathcal{L}^{\mathrm{hydr}} \approx \frac{\hbar^2 n_0}{2M_{\mathrm{eff}}}\left[(\nabla\gamma)^2 + \frac{1}{c^2}\partial_\tau\gamma\left(1 - \frac{\hbar^2\nabla^2}{4M_{\mathrm{eff}}^2 c^2}\right)^{-1}\partial_\tau\gamma\right]$$

  with Bogoliubov spectrum

$$\epsilon(\mathbf{p}) = \sqrt{c^2\mathbf{p}^2 + \left(\frac{\mathbf{p}^2}{2M_{\mathrm{eff}}}\right)^2}\,, \qquad c = \sqrt{\frac{4\pi\hbar^2 a_s n}{M_{\mathrm{eff}}^2}}$$

- **Goal:** Determine whole phase diagram





Phase decoherence or vortex-antivortex pairs

2.2 Phase Decoherence

- **Goal:** Study new O(N)-symmetric nonlinear σ-model

$$\mathcal{L}^{\sigma} = \frac{\hbar^2 n_0}{2M_{\mathrm{eff}}}\left[(\nabla\mathbf{m})^2 + \frac{1}{c^2}\partial_\tau\mathbf{m}\left(1 - \frac{\hbar^2\nabla^2}{4M_{\mathrm{eff}}^2 c^2}\right)^{-1}\partial_\tau\mathbf{m}\right] + \lambda(\mathbf{m}^2 - 1)$$

- **Goal:** Extend classical criterion for dephasing transition [17] to quantum regime
- **Goal:** Characterize Mott insulator transition by energy gap [18]

$$\epsilon(\mathbf{p}) = \sqrt{c^2\mathbf{p}^2 + \left(\frac{\mathbf{p}^2}{2M_{\mathrm{eff}}}\right)^2 + \Delta^2}$$

2.3 Disorder Gauge Field Theory

- **Goal:** Describe ensembles of vortex lines by disorder field ϕ
- **Goal:** Introduce auxiliary vector potential $\mathbf{a}$ by duality transformation
- **Goal:** Develop Meissner-Higgs theory [19,20] whose Feynman diagrams represent pictures of vortex lines

$$\mathcal{L}^{\mathrm{GL}} = \frac{1}{2}\left[(\nabla\times\mathbf{a})_{\parallel}\left(1 - \frac{\hbar^2\nabla^2}{4M_{\mathrm{eff}}^2 c^2}\right)(\nabla\times\mathbf{a})_{\parallel} + (\nabla\times\mathbf{a})_{\perp}^2\right] + \left|\left(\nabla - \frac{2\pi i}{\hbar}\sqrt{\frac{n_0}{M_{\mathrm{eff}}}}\,\mathbf{a}\right)\phi\right|^2 + m^2|\phi|^2 + g|\phi|^4$$

- **Goal:** Characterize quantum phase transition by proliferation of vortex lines whose cores are Mott insulators [21]

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