Stationary and Transient Properties of Photon Condensates

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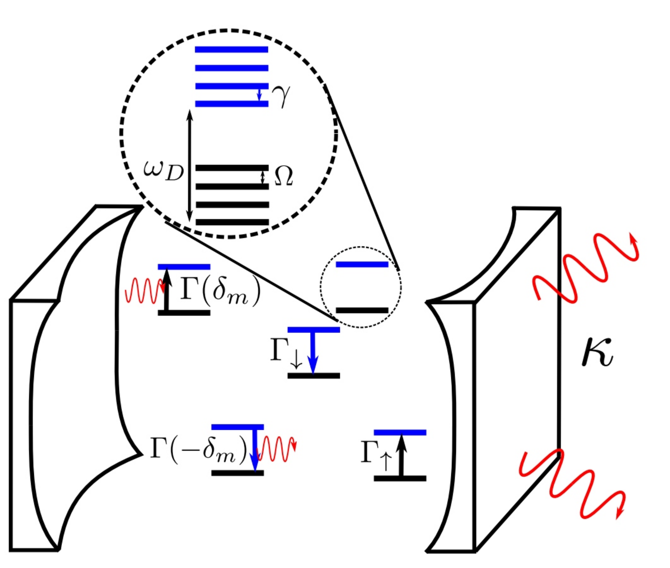
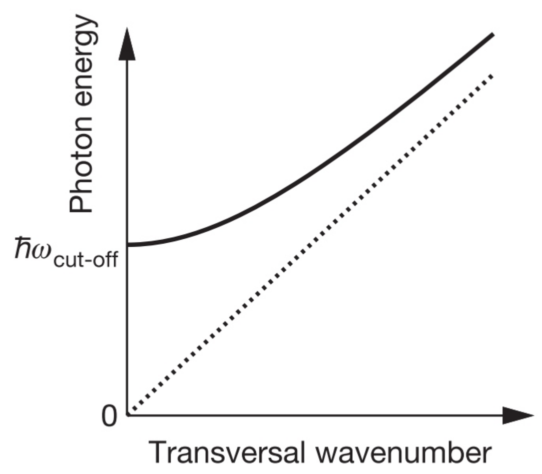
Abstract: A seminal experiment in Bonn has presented convincing evidence of both a Bose-Einstein distribution and a macroscopic occupation of the lowest mode for a gas of photons confined in a dye-filled optical microcavity [1]. Furthermore, these equilibrium properties could recently be understood within the framework of a non-equilibrium model of photons in terms of a steady state solution [2]. Here we utilize this non-equilibrium model in order to study in detail the stationary photon distribution in the respective mode of the dye-filled cavity. It turns out that, depending on the dye pumping rate and the cavity decay rate, different modes become macroscopically occupied. In particular, we present the corresponding phase diagrams and describe the transitions between the phases analytically. Furthermore, we obtain a moment generating function for the photon statistics and calculate the stationary equal-time autocorrelation function. Using a linear stability analysis we show that the stationary states are always unconditionally stable. Finally, we examine how the relaxation times toward equilibrium depend on the respective system parameters and compare them with the thermalization times obtained experimentally from the corresponding transient dynamics [3].

- Photon BEC in a dye-filled optical microcavity [1] -



Schematic of the physical processes in a dye-filled optical microcavity



Quadratic dispersion relation of cavity photons

- Non-equilibrium model of photon condensation [2] -

Hamiltonian of the photon-dye system:

$$H = \sum_m \omega_m a_m^\dagger a_m + \sum_i \frac{\omega_D}{2}\sigma_i^z + \Omega\big[b_i^\dagger b_i + \sqrt{S}(b_i^\dagger + b_i)\sigma_i^z\big] + \sum_{m,i} g\big(a_m^\dagger \sigma_i^- + a_m \sigma_i^+\big)$$

Master equation of the system:

$$\dot\rho = -i[H_0,\rho] - \Big\{\sum_m \frac{\kappa}{2}\mathcal{L}[a_m] + \sum_i \frac{\Gamma_\uparrow}{2}\mathcal{L}[\sigma_i^+] + \sum_i \frac{\Gamma_\downarrow}{2}\mathcal{L}[\sigma_i^-] + \sum_{m,i}\frac{\Gamma(\delta_m)}{2}\mathcal{L}[a_m\sigma_i^+] + \frac{\Gamma(-\delta_m)}{2}\mathcal{L}[a_m^\dagger\sigma_i^-]\Big\}\rho$$

$$\omega_m = \omega_{\text{cut-off}} + m\epsilon, \quad \delta_m = \omega_m - \omega_D, \quad \mathcal{L}[X]\rho = \{X^\dagger X, \rho\} - 2X\rho X^\dagger, \quad \Gamma(\delta) = 2\,\mathrm{Re}[K(\delta)]$$

$$K(\delta) = g^2\int_0^\infty f(t) e^{-(\Gamma_\uparrow+\Gamma_\downarrow)|t|/2} e^{i\delta t} dt, \quad f(t) = \exp\left[-\frac{2S\gamma}{\pi}\int_{-\infty}^{\infty}\frac{2\sin^2\frac{\nu t}{2}\coth\frac{\beta\nu}{2} + i\sin\nu t}{(\Omega-\nu)^2+\gamma^2/4}d\nu\right]$$

Semiclassical rate equations:

$$n_m(t) = \mathrm{Tr}\big[\rho(t) a_m^\dagger a_m\big], \quad P_g(t) = \mathrm{Tr}\Big[\rho(t)\sum_{i=1}^N |g_i\rangle\langle g_i|\Big]$$

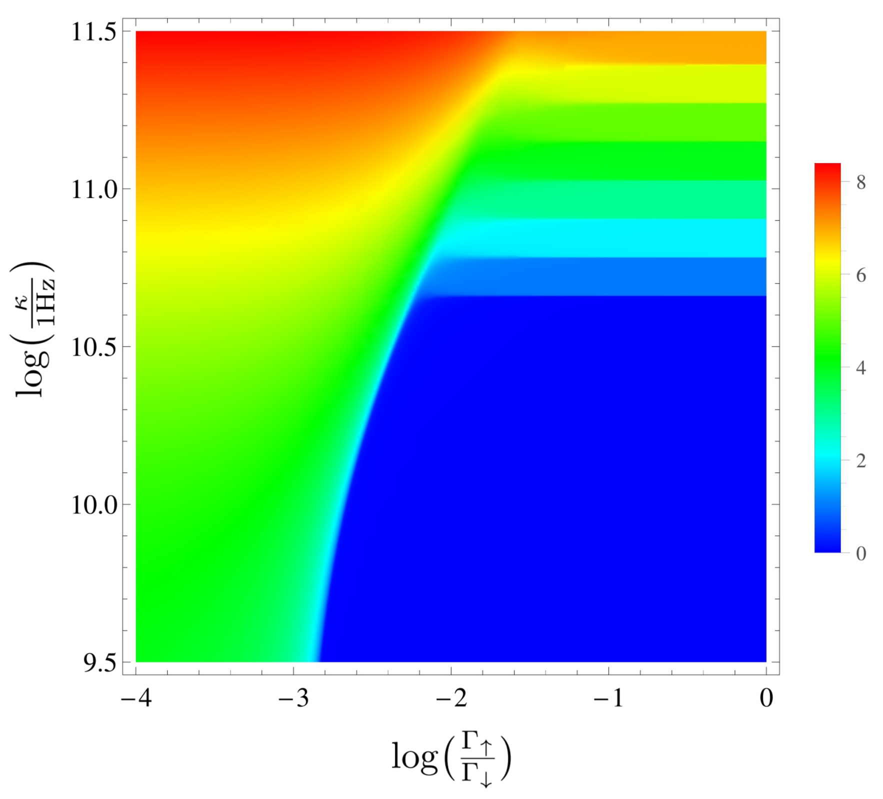
$$\dot n_m(t) = -\kappa n_m(t) + \Gamma(-\delta_m)[n_m(t)+1][N-P_g(t)] - \Gamma(\delta_m) n_m(t) P_g(t)$$

$$\dot P_g(t) = N\widetilde\Gamma_\downarrow(t) - \big[\widetilde\Gamma_\uparrow(t) + \widetilde\Gamma_\downarrow(t)\big]P_g(t)$$

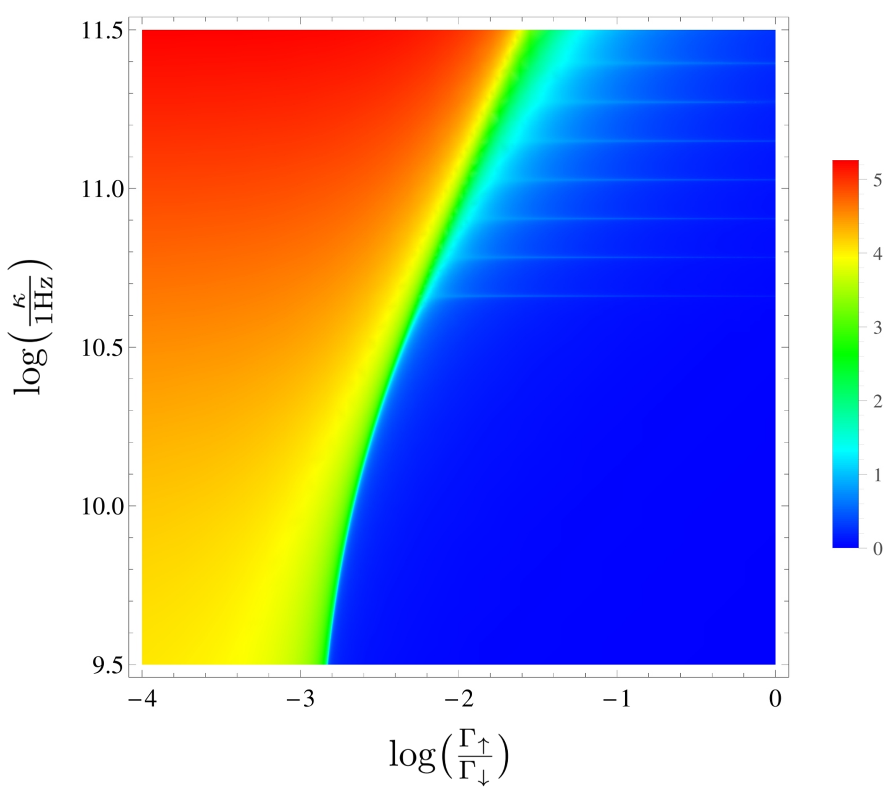
$$\widetilde\Gamma_\uparrow = \Gamma_\uparrow + \sum_m g_m n_m \Gamma(\delta_m), \quad \widetilde\Gamma_\downarrow = \Gamma_\downarrow + \sum_m g_m (n_m+1)\Gamma(-\delta_m)$$

$\omega_D = 2\pi\times 555\,\mathrm{THz}$
$N = 10^8, \; S = 0.5$
$\Omega = 50\,\mathrm{THz}$
$\gamma = 150\,\mathrm{THz}$
$\Gamma_\uparrow = 10\,\mathrm{MHz}$
$\Gamma_\downarrow = 1\,\mathrm{GHz}$
$\lambda_{\text{cut-off}} = 585\,\mathrm{nm}$
$g = 5\,\mathrm{GHz}, \; T = 300\,\mathrm{K}$

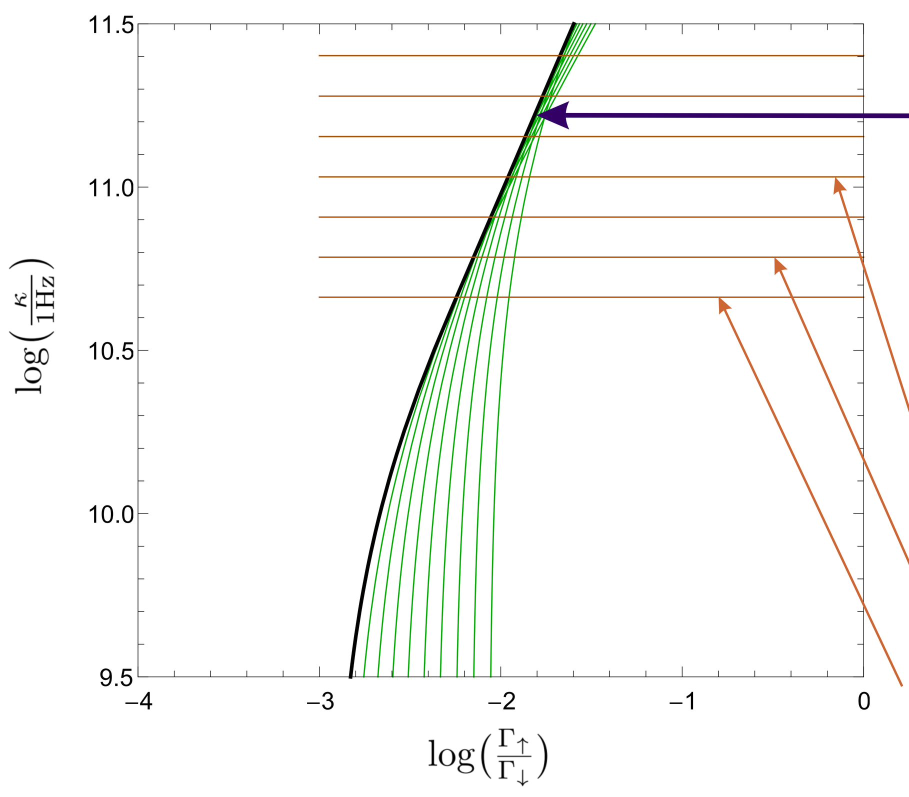
- Properties of stationary states -



Mean mode number $\langle m\rangle = \dfrac{\sum_m m\cdot n_m}{\sum_m n_m}$



Standard deviation $\sigma(m) = \sqrt{\dfrac{\sum_m (m-\langle m\rangle)^2\cdot n_m}{\sum_m n_m}}$



Threshold line as an envelope:

$$\kappa < \max_m N\frac{\Gamma_\uparrow\Gamma(-\delta_m) - \Gamma_\downarrow\Gamma(\delta_m)}{\Gamma_\uparrow + \Gamma_\downarrow}$$

$$\Gamma_\uparrow/\Gamma_\downarrow > \min_m \Gamma(\delta_m)/\Gamma(-\delta_m)$$

Crossovers between different macroscopically occupied cavity modes

Analytical characterization of the phase diagram

$$\kappa_{m,m+1} \approx N\frac{\Gamma(-\delta_{m+1})\Gamma(\delta_m) - \Gamma(\delta_{m+1})\Gamma(-\delta_m)}{\Gamma(-\delta_m) + \Gamma(\delta_m) - \Gamma(-\delta_{m+1}) - \Gamma(\delta_{m+1})}$$

- Transient properties -

Linear stability equations:

$$\Big\{\omega + \frac{N\Gamma(-\delta_m)}{(1+R)n_m}\Big\}\delta n_m = -\frac{(1+R)n_m}{N}\big[\kappa + N\Gamma(\delta_m)\big]\delta P_g$$

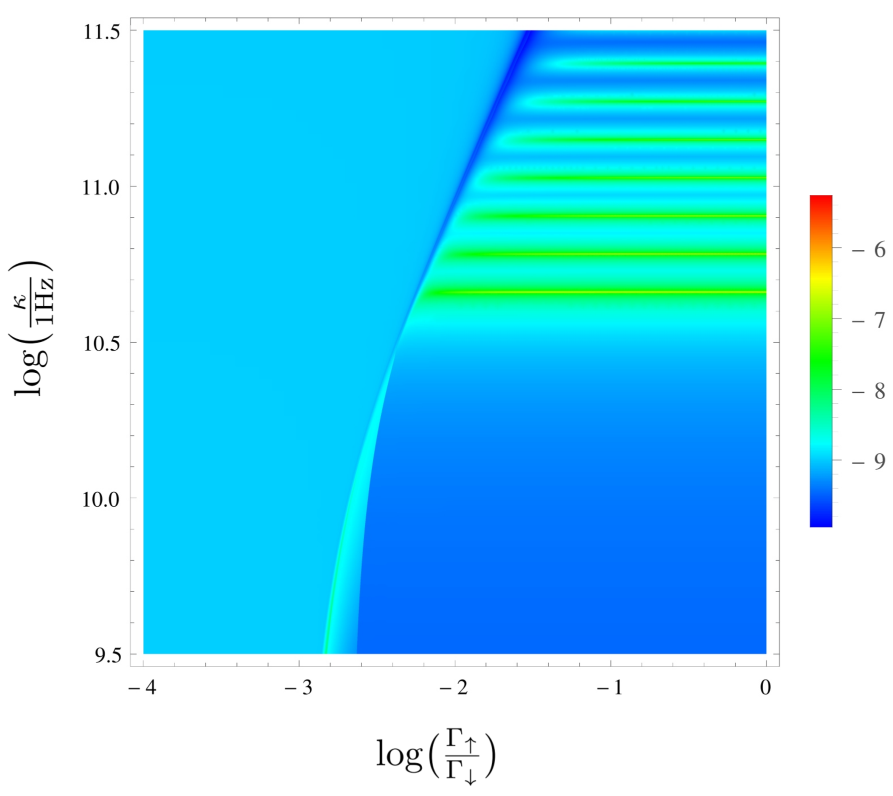
$$\Big\{\omega + \Gamma_\downarrow + \Gamma_\uparrow + \frac{1+R}{N}\sum_m g_m n_m\big[\kappa + N\Gamma(\delta_m)\big]\Big\}\delta P_g = \frac{N}{1+R}\sum_m g_m\big[\Gamma(-\delta_m) - R\Gamma(\delta_m)\big]\delta n_m$$

Characteristic equation:

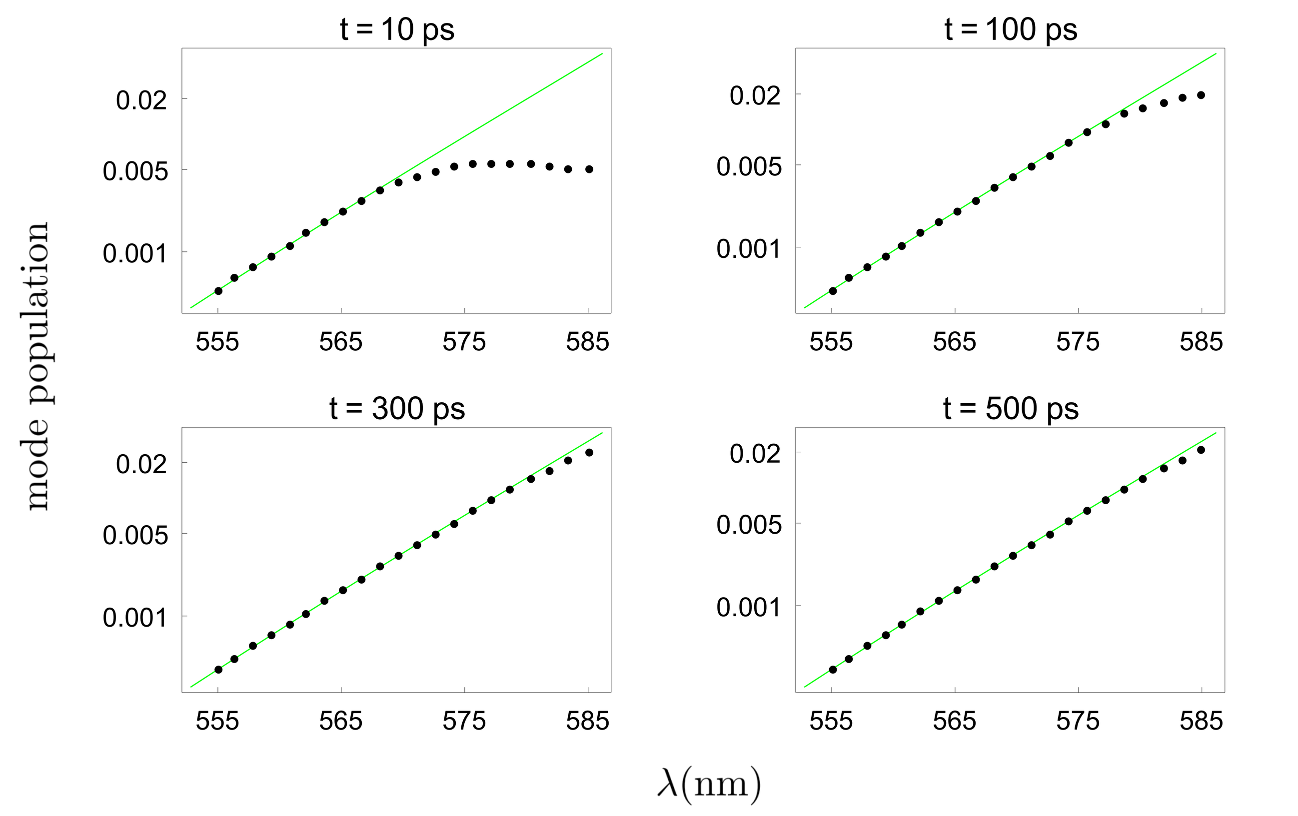
$$\frac{N(\omega+\Gamma_\downarrow+\Gamma_\uparrow)}{1+R} + (\kappa+\omega)\sum_m \frac{g_m n_m[\kappa + N\Gamma(\delta_m)]}{\frac{N\Gamma(-\delta_m)}{(1+R)n_m} + \omega} = 0, \quad R = \widetilde\Gamma_\downarrow/\widetilde\Gamma_\uparrow$$

Stationary states are unconditionally stable because all characteristic frequencies have negative real parts

Maximal relaxation times (log-scale)



- Thermalization times of ~ 500 ps agree with Ref. [3] -



Maximal relaxation time is ~ 1000 ps

- Moment generating function -

$$M(z,t) = \mathrm{Tr}\big[\rho(t)\exp(z a_m^\dagger a_m)\big]$$

Evolution equation:

$$\frac{\partial}{\partial t}M(z,t) = (1-e^{-z})\Big\{\Gamma(-\delta_m)P_e(t)e^z M(z,t) + \big[\Gamma(-\delta_m)P_e(t)e^z - \kappa - \Gamma(\delta_m)P_g(t)\big]\frac{\partial}{\partial z}M(z,t)\Big\}$$

Stationary generating function:

$$\lim_{t\to\infty} M(z,t) \equiv M(z) = \frac{1}{n_m + 1 - n_m\exp(z)}, \quad n_m = \frac{\Gamma(-\delta_m)P_e}{\kappa + \Gamma(\delta_m)P_g - \Gamma(-\delta_m)P_e}$$

Second order moment and zero-delay autocorrelation function:

$$\overline{n_m^2} = 2n_m^2 + n_m \quad \Longrightarrow \quad g^{(2)}(0) = \frac{\overline{n_m^2} - n_m}{n_m^2} = 2$$

- References -

[1] J. Klaers, J. Schmitt, F. Vewinger, and M. Weitz, Nature (London) 468, 545 (2010)
[2] P. Kirton and J. Keeling, Phys. Rev. Lett. 111, 100404 (2013)
[3] J. Schmitt, T. Damm, D. Dung, F. Vewinger, J. Klaers, and M. Weitz, eprint arXiv: 1410.5713 (2014)

- Acknowledgments -

We gratefully acknowledge support by the Ministry of Education, Science, and Technological Development of the Republic of Serbia under projects ON171017, III45016 and NAI-DBEC, by DAAD - German Academic and Exchange Service under project NAI-DBEC, by DFG - German Research Foundation within the Collaborative Research Centers SFB/TR 49 and SFB 910.

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