Out of equilibrium dynamical properties of Bose-Einstein condensates in ramped up weak disorder

Milan Radonjic[1,2,†], Rodrigo P. A. Lima[3,4,*] and Axel Pelster[4,‡]

[1] I. Institute of Theoretical Physics, Department of Physics, University of Hamburg, Hamburg, Germany
[2] Institute of Physics Belgrade, University of Belgrade, Belgrade, Serbia
[3] GISC and GFTC, Instituto de Física, Universidade Federal de Alagoas, Maceió AL, Brazil
[4] Department of Physics and Research Center OPTIMAS, Rheinland-Pfälzische Technische Universität Kaiserslautern-Landau, Kaiserslautern, Germany

†milan.radonjic@physik.uni-hamburg.de, *rodrigo@fis.ufal.br, ‡axel.pelster@rptu.de

Abstract

We investigate theoretically how the superfluid and the condensate deformation of a weakly interacting ultracold Bose gas evolve during the ramping up of an external weak disorder potential. Both resulting deformations turn out to consist of two distinct contributions, namely a reversible equilibrium one, already predicted by Huang and Meng [1], as well as a non-equilibrium dynamical one, whose magnitude depends on the details of the ramping protocol. For the specific case of the exponential ramping up protocol, we are able to derive analytic time-dependent expressions for the aforementioned quantities. After sufficiently long time, the steady state emerges that is generically out of equilibrium. We make the first step in examining its properties by studying the relaxation dynamics into it. Also, we investigate the two-time correlation function and elucidate its relation to the equilibrium and the dynamical part of the condensate deformation.

System dynamics

Time-dependent Gross-Pitaevskii equation

$$i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + u(\mathbf{x}-\mathbf{v_u}t)f(t) - \mu_0 + g|\Psi(\mathbf{x},t)|^2\right]\Psi(\mathbf{x},t)$$

Moving disorder potential — $u(\mathbf{x}-\mathbf{v_u}t)$; Ramped up protocol — $f(t)$; Chemical potential — μ_0; Isotropic short-range contact interaction — $g|\Psi(\mathbf{x},t)|^2$

Moving frame $\mathbf{r} = \mathbf{x} - \mathbf{v_u}t$ $\quad \Psi(\mathbf{r},t) = e^{i\mathbf{k_s}\cdot\mathbf{r}}\mathrm{Exp}\left[-\frac{i}{\hbar}\left(\frac{\hbar^2k_s^2}{2m}\right)t\right]\psi(\mathbf{r},t)$ with $\mathbf{K} = \mathbf{k_s} - \mathbf{k_u}$

GPE in the moving frame

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} - \frac{i\hbar^2\mathbf{K}}{m}\cdot\nabla + u(\mathbf{r})f(t) - \mu_0 + g|\psi(\mathbf{r},t)|^2\right]\psi(\mathbf{r},t)$$

Pertubation Theory

$$\psi(\mathbf{r},t) = \psi_0 + \psi_1(\mathbf{r},t) + \psi_2(\mathbf{r},t) + \dots$$

Disorder Potential

$$\langle u(\mathbf{x})\rangle = 0 \qquad \langle u(\mathbf{x})u(\mathbf{x}')\rangle = \mathcal{R}(\mathbf{x}-\mathbf{x}')$$
$$\langle u(\mathbf{k})u(\mathbf{k}')\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\mathcal{R}(\mathbf{k})$$

Laplace Transformation

$$i\hbar s\psi_1(\mathbf{k},s) = \left[\hbar\omega_k + \frac{\hbar^2\mathbf{k}\cdot\mathbf{K}}{m} + g\psi_0^2\right]\psi_1(\mathbf{k},s) + g\psi_0^2\psi_1^*(\mathbf{k},s) + u(\mathbf{k})f(s)\psi_0$$
$$-i\hbar s\psi_1^*(\mathbf{k},s) = \left[\hbar\omega_k - \frac{\hbar^2\mathbf{k}\cdot\mathbf{K}}{m} + g\psi_0^2\right]\psi_1^*(\mathbf{k},s) + g\psi_0^2\psi_1(\mathbf{k},s) + u(\mathbf{k})f(s)\psi_0$$

Inverse Laplace Transformation

$$\frac{\omega_k - \frac{\hbar}{m}\mathbf{k}\cdot\mathbf{K} + is}{\Omega_k^2 - \left(\frac{\hbar}{m}\mathbf{k}\cdot\mathbf{K} + is\right)^2} \xrightarrow{\mathcal{L}^{-1}} e^{-i\mathbf{k}\cdot\mathbf{K}\frac{\hbar}{m}t}\left[\frac{\omega_\mathbf{k}}{\Omega_k}\sin(\Omega_k t) + i\cos(\Omega_k t)\right] \equiv e^{-i\mathbf{k}\cdot\mathbf{K}\frac{\hbar}{m}t}\mathcal{K}(\mathbf{k},t)$$

First perturbative correction

$$\psi_1(\mathbf{k},\mathbf{K},t) = -\psi_0 u(\mathbf{k})\int_0^t e^{-i\mathbf{k}\cdot\mathbf{K}\frac{\hbar}{m}(t-t')}\mathcal{K}(\mathbf{k},t-t')f(t')dt'$$
$$\psi_1^*(\mathbf{k},\mathbf{K},t) = -\psi_0 u(\mathbf{k})\int_0^t e^{-i\mathbf{k}\cdot\mathbf{K}\frac{\hbar}{m}(t-t')}\mathcal{K}^*(\mathbf{k},t-t')f(t')dt'$$

Bogoliubov dispersion

$$\hbar\Omega_k = \sqrt{\hbar\omega_k(\hbar\omega_k + 2gn)}$$

Free particle

$$\hbar\omega_k = \frac{\hbar^2k^2}{2m}$$

Superfluid and condensate deformation

Condensate deformation [2]

$$q_\tau(t) = \langle|\psi_1(\mathbf{r},t)|^2\rangle - |\langle\psi_1(\mathbf{r},t)\rangle|^2 = n\int\frac{d\mathbf{k}\mathcal{R}(\mathbf{k})}{(2\pi)^3}\left|\int_0^t dt'\mathcal{K}(\mathbf{k},t-t')f(t')\right|^2$$

Equilibrium Huang-Meng limit [1]

$$\lim_{\substack{t\to\infty\\ \tau\to\infty}} q_\tau(t) = q_{HM} = \frac{m^{3/2}\sqrt{n}R}{4\pi\hbar^3\sqrt{g}}$$

Delta correlated disorder $\mathcal{R}(\mathbf{k}) = R$

Momentum

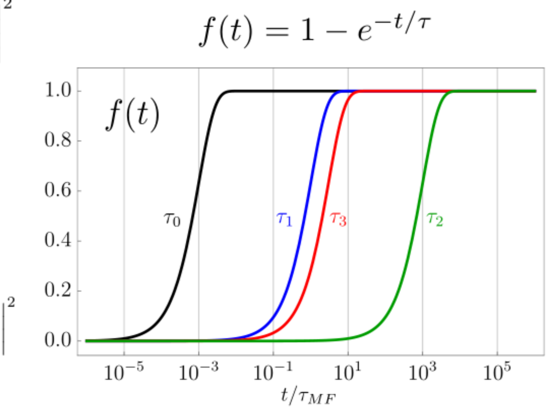
$$\langle\mathbf{p}(t)\rangle = \left\langle\psi_1^*(\mathbf{r},t)\frac{\hbar}{i}\nabla\psi_1(\mathbf{r},t)\right\rangle = n\int\frac{d\mathbf{k}\mathcal{R}(\mathbf{k})\hbar\mathbf{k}}{(2\pi)^3}\left|\int_0^t dt' e^{-i\mathbf{k}\cdot\mathbf{K}\frac{\hbar}{m}t'}\mathcal{K}(\mathbf{k},t-t')f(t')\right|^2$$

Superfluid density [3,4]

$$n_{s,ij} = n\delta_{ij} - \frac{1}{\hbar}\frac{\partial\langle p_i(t)\rangle}{\partial K_j}\bigg|_{K_j=0}$$

Ramped up disorder protocol

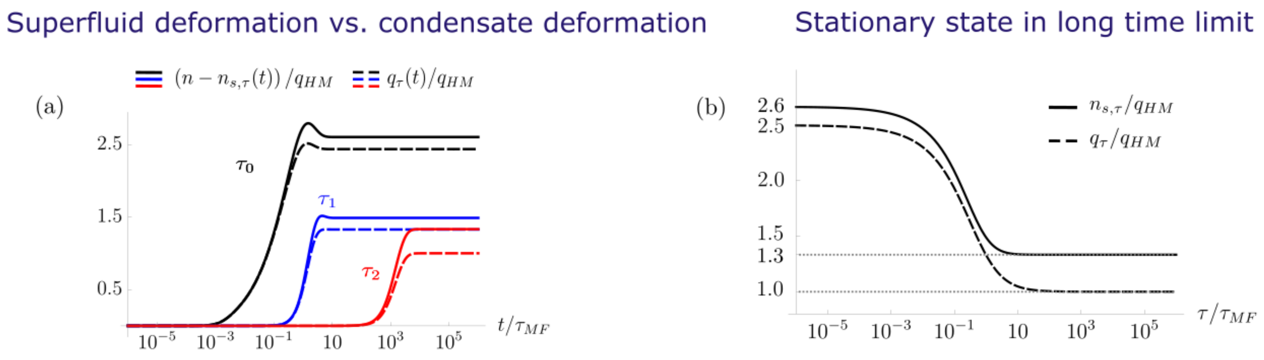
$$f(t) = 1 - e^{-t/\tau}$$



Switch on driving function, for several values of the ramp-up scaled time $10^3\tau_0 = 10^{-3}\tau_2 = \tau_1 = \tau_{MF} \equiv \hbar/(gn)$ and $\tau_3 = 3.0\tau_{MF}$.

Superfluid deformation vs. condensate deformation

Stationary state in long time limit



Superfluid deformation (solid line) and condensate deformation (dashed line) as functions of (a) rescaled time (t/τ_{MF}) and (b) rescaled ramp-up time (τ/τ_{MF}) for three values of disorder ramp-up times $10^3\tau_0 = 10^{-3}\tau_2 = \tau_1 = \tau_{MF} \equiv \hbar/(gn)$. All presented quantities are normalized by equilibrium condensate deformation q_{HM}.

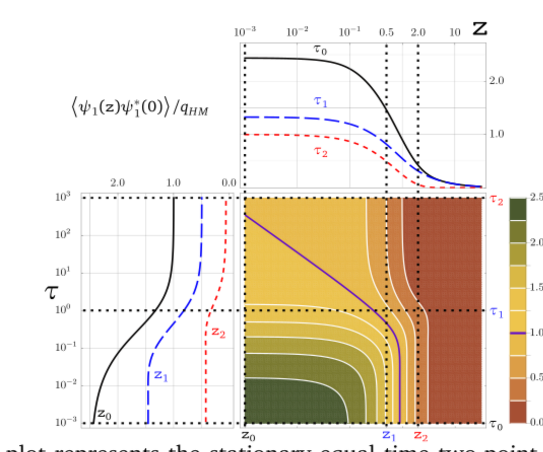
Correlation function at rest

$$\langle\psi(\mathbf{x},t+T)\psi^*(\mathbf{y},t)\rangle_c = \langle\psi(\mathbf{x},t+T)\psi^*(\mathbf{y},t)\rangle - \langle\psi(\mathbf{x},t+T)\rangle\langle\psi^*(\mathbf{y},t)\rangle = \langle\psi_1(\mathbf{x},t+T)\psi_1^*(\mathbf{y},t)\rangle$$

Equal time correlation function

$$\langle\psi_1(\mathbf{x},t)\psi_1^*(\mathbf{y},t)\rangle = \frac{n}{(2\pi)^3}\int d^3\mathbf{k}R(\mathbf{k})e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\left|\int_0^t\mathcal{K}(\mathbf{k},t-t')f(t')dt'\right|^2$$

$$\lim_{|\mathbf{x}-\mathbf{y}|\to\infty}\langle\psi_1(\mathbf{x},t)\psi_1^*(\mathbf{y},t)\rangle = 0$$
$$\lim_{|\mathbf{x}-\mathbf{y}|\to 0}\langle\psi_1(\mathbf{x},t)\psi_1^*(\mathbf{y},t)\rangle = q_\tau(t)$$

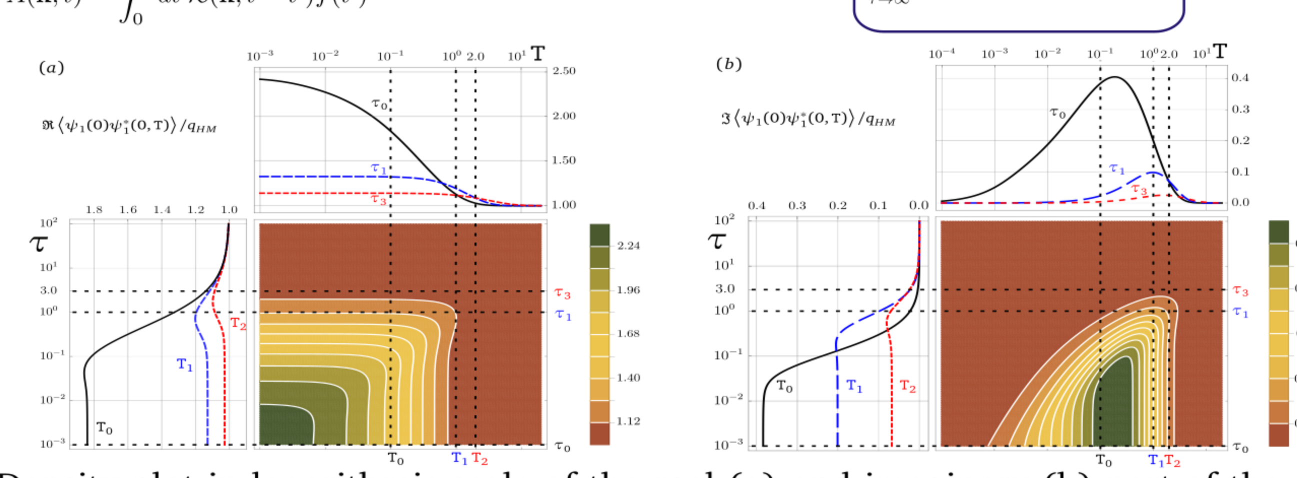


The density plot represents the stationary equal time two-point correlation function normalized by the equilibrium condensate deformation (q_{HM}) in terms of normalized spatial separation $\mathbf{z} = |\mathbf{z}|/\xi$ and the characteristic ramped up scaled time $\tau = t/\tau_{\mathrm{MF}}$, both in logarithmic scale, where $\xi = \hbar/\sqrt{2gnm}$ is the healing length. The equal time two-point correlation (left panel) as a function of the scaled time for three values of the normalized spatial separation $10^3\mathbf{z}_0 = 2\mathbf{z}_1 = \mathbf{z}_2/2 = |\mathbf{z}|/\xi$ (top panel) as a function of normalized spatial separation for three values of the characteristic ramped up scaled time $10^3\tau_0 = \tau_1 = 10^{-3}\tau_2 = t/\tau_{\mathrm{MF}}$.

Equal position correlation function

$$\langle\psi_1(\mathbf{x},t)\psi_1^*(\mathbf{x},t+T)\rangle = \frac{n}{(2\pi)^3}\int d^3\mathbf{k}R(\mathbf{k})A(\mathbf{k},t)A^*(\mathbf{k},t+T)$$
$$A(\mathbf{k},t) = \int_0^t dt'\mathcal{K}(\mathbf{k},t-t')f(t')$$

$$\lim_{T\to 0}\langle\psi_1(\mathbf{x},t)\psi_1^*(\mathbf{x},t+T)\rangle = q_\tau(t)$$
$$\lim_{\substack{t\to\infty\\ \tau\to\infty}}\langle\psi_1(\mathbf{x},t)\psi_1^*(\mathbf{x},t+T)\rangle = q_{HM}$$



Density plot in logarithmic scale of the real (a) and imaginary (b) part of the stationary two-point correlation function normalized by the equilibrium condensate deformation in terms of scaled separation in time T and the characteristic ramped up scaled time τ. Both left panels represent real (a) and imaginary (b) of the stationary two-point correlation function as a function of the scaled time for three values of the normalized temporal separation $10\mathrm{T}_0 = \mathrm{T}_1 = \mathrm{T}_2/2 = t/\tau_{\mathrm{MF}}$ and (top panel) as a function of normalized temporal separation for three values of the characteristic ramped up scaled time $10^3\tau_0 = \tau_1 = \tau_3/3 = t/\tau_{\mathrm{MF}}$.

Outlook

- Analytical time dependent expressions for the superfluid deformation and condensate deformation have been obtained.
- Limiting cases of equal time or equal space correlation function correspond to condensate deformation.
- Stationary superfluid density for delta-correlated disorder, switched on and off, either adiabatically or through a rapid quench.

$n_{\tau_1\tau_2}$	$\tau_2\to\infty$	$\tau_2\to 0$
$\tau_1\to\infty$	0	$\frac{4}{3}q_{HM}$
$\tau_1\to 0$	$\frac{4}{3}q_{HM}$	$\frac{8}{3}q_{HM}$

References

[1] K. Huang and H. F. Meng, Physical Review Letters 69, 644 (1992).
[2] M. Radonjic and A. Pelster, SciPost Physics 10, 008 (2021).
[3] A. Boudjemaa and K. Abbas, Physical Review A 102, 023325 (2020).
[4] M. Ueda, *Fundamentals and New Frontiers of Bose-Einstein Condensation* (2010).

Acknowledgments:

We acknowledge the funding support of the Improvement Coordination of Higher Level Personnel (CAPES) under PROBRAL program Nº 12/2017. R.P.A.L. is grateful to the Technical University of Kaiserslautern and Universidade Federal de Alagoas (were a part of this work has been carried out) for hospitality. A.P. acknowledges financial support by the Deutsche Forschungsgemeinschaft (DFG) via the Collaborative Research Center SFB/TR185 (Project No. 277625399).

Physics School on Ultracold Atoms and Molecules, Bad Honnef, August 06-12, 2023, Germany