Quantum Fluctuations in Dipolar Bose Gases

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Abstract

We investigate the influence of quantum fluctuations upon dipolar Bose gases by means of the Bogoliubov-de Gennes theory. Thereby, we make use of the local density approximation to evaluate the dipolar exchange interaction between the condensate and the excited particles. This allows to derive the Bogoliubov spectrum analytically in the limit of large particle numbers. After discussing the condensate depletion and the ground-state energy correction, we derive quantum corrected equations of motion for harmonically trapped dipolar Bose gases by using superfluid hydrodynamics. These equations are subsequently applied to investigate the equilibrium configuration, the low-lying oscillation frequencies, and the time-of-flight dynamics. We find that both atomic magnetic [1–3] and molecular electric [4–6] dipolar systems offer promising scenarios for detecting beyond mean-field effects.

Bogoliubov-de Gennes theory [7,8]

- Hamilton operator (creation and annihilation)

$$\hat{H} = \int d^3x \hat{\Psi}^\dagger(\mathbf{x}) \left[h_0 + \int \frac{d^3y}{2} \hat{\Psi}^\dagger(\mathbf{y}) V_{\rm int}(\mathbf{x}-\mathbf{y}) \hat{\Psi}(\mathbf{y}) \right] \hat{\Psi}(\mathbf{x})$$

- Free Hamiltonian and trapping potential

$$h_0 = -\frac{\hbar^2\nabla^2}{2M} + U_{\rm tr}(\mathbf{x}), \qquad U_{\rm tr}(\mathbf{x}) = \frac{M}{2}\omega_x^2(x^2+y^2+\lambda^2 z^2)$$

- Interaction (s-wave, dipolar and relative interaction strength)

$$V_{\rm int}(\mathbf{x}) = \frac{4\pi\hbar^2 a_s}{M}\left[\delta(\mathbf{x}) + \frac{3a_{\rm dd}}{4\pi a_s |\mathbf{x}|^3}\left(1 - 3\frac{z^2}{|\mathbf{x}|^2}\right)\right], \qquad \epsilon_{\rm dd} = \frac{a_{\rm dd}}{a_s}$$

- Bogoliubov prescription (mean-field and quantum fluctuations)

$$\hat{\Psi}(\mathbf{x}) = \Psi(\mathbf{x}) + \delta\hat{\psi}(\mathbf{x})$$

- Fluctuation decomposition (Bogoliubov amplitudes)

$$\delta\hat{\psi}(\mathbf{x}) = \sum_\nu{}' \left[\mathcal{U}_\nu(\mathbf{x})\hat{\alpha}_\nu + \mathcal{V}_\nu^*(\mathbf{x})\hat{\alpha}_\nu^\dagger\right]$$

- Bogoliubov-de Gennes equation

$$\int d^3y \begin{pmatrix} H_{\mathcal{U}\mathcal{U}}(\mathbf{x},\mathbf{y}) & H_{\mathcal{U}\mathcal{V}}(\mathbf{x},\mathbf{y}) \\ H^*_{\mathcal{U}\mathcal{V}}(\mathbf{x},\mathbf{y}) & H^*_{\mathcal{U}\mathcal{U}}(\mathbf{x},\mathbf{y}) \end{pmatrix} \begin{pmatrix} \mathcal{U}_\nu(\mathbf{y}) \\ \mathcal{V}_\nu(\mathbf{y}) \end{pmatrix} = \varepsilon_\nu \begin{pmatrix} \mathcal{U}_\nu(\mathbf{x}) \\ -\mathcal{V}_\nu(\mathbf{x}) \end{pmatrix}$$

with

$$H_{\mathcal{U}\mathcal{U}}(\mathbf{x},\mathbf{y}) = \delta(\mathbf{x}-\mathbf{y})H_{\rm Fl}(\mathbf{y}) + \Psi^*(\mathbf{y})V_{\rm int}(\mathbf{x}-\mathbf{y})\Psi(\mathbf{x})$$
$$H_{\mathcal{U}\mathcal{V}}(\mathbf{x},\mathbf{y}) = \Psi(\mathbf{y})V_{\rm int}(\mathbf{x}-\mathbf{y})\Psi(\mathbf{x})$$

- Fluctuation Hamiltonian

$$H_{\rm Fl}(\mathbf{x}) = h_0 - \mu + \int d^3x' \Psi^*(\mathbf{x}')V_{\rm int}(\mathbf{x}-\mathbf{x}')\Psi(\mathbf{x}')$$

- Semiclassical approximation [9]

$$\varepsilon_\nu \to \varepsilon(\mathbf{x},\mathbf{k}), \qquad \mathcal{U}_\nu \to \mathcal{U}(\mathbf{x},\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}, \qquad \mathcal{V}_\nu \to \mathcal{V}(\mathbf{x},\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}$$

- Local density approximation for the exchange interaction [10,11]

$$I_{\rm Ex} \equiv \int d^3y H_{\mathcal{U}\mathcal{V}}(\mathbf{x},\mathbf{y})\, q_\nu(\mathbf{y})$$
$$\approx q(\mathbf{x},\mathbf{k})\,\xi(\mathbf{x},\mathbf{k}) = q(\mathbf{x},\mathbf{k})\, g n_0(\mathbf{x})\left[1+\epsilon_{\rm dd}\left(3\cos^2\theta - 1\right)\right]$$

- Semiclassical Bogoliubov spectrum

$$\varepsilon^2(\mathbf{x},\mathbf{k}) = \varepsilon^2_{\rm LDA}(\mathbf{x},\mathbf{k}) - \xi^2(\mathbf{x},\mathbf{k})$$

with the abbreviation $\varepsilon_{\rm LDA}(\mathbf{x},\mathbf{k}) = \frac{\hbar^2\mathbf{k}^2}{2M} + \xi(\mathbf{x},\mathbf{k})$ and the Bogoliubov amplitudes

$$\mathcal{U}(\mathbf{x},\mathbf{k})^2 - 1 = \mathcal{V}(\mathbf{x},\mathbf{k})^2 = \frac{1}{2}\left[\frac{\varepsilon_{\rm LDA}(\mathbf{x},\mathbf{k})}{\varepsilon(\mathbf{x},\mathbf{k})} - 1\right]$$

- Condensate depletion

$$n(\mathbf{x}) - n_0(\mathbf{x}) = \frac{8n(\mathbf{x})}{3}\mathcal{Q}_3(\epsilon_{\rm dd})\sqrt{\frac{n(\mathbf{x})a_s^3}{\pi}}$$

- Ground-state energy correction

$$\frac{\Delta E(\mathbf{x})}{n(\mathbf{x})} = \frac{64}{15} g n(\mathbf{x})\mathcal{Q}_5(\epsilon_{\rm dd})\sqrt{\frac{n(\mathbf{x})a_s^3}{\pi}}$$

- Auxiliary functions

$$\mathcal{Q}_l(x) = \int_0^1 du\,\left(1 - x + 3\,x\,u^2\right)^{l/2}$$

Variational superfluid hydrodynamics [7,8,12]

- Action

$$\mathcal{A}[n,\chi] = -\int dt d^3x\, n\left\{M\left[\dot{\chi} + \frac{1}{2}\nabla\chi^2\right] + e_{\rm MF}[n] + \frac{64}{15}gn\mathcal{Q}_5(\epsilon_{\rm dd})\sqrt{\frac{na_s^3}{\pi}}\right\}$$

- Mean-field energy density

$$e_{\rm MF}[n] = U_{\rm tr}(\mathbf{x}) + \frac{g}{2}n(\mathbf{x},t) + \int d^3x' \frac{V_{\rm dd}(\mathbf{x}-\mathbf{x}')}{2} n(\mathbf{x}',t)$$

- Ansatz

$$n(\mathbf{x},t) = \frac{15N}{8\pi\overline{R}^3(t)}\left[1 - \sum_{i=x,y,z}\frac{x_i^2}{R_i^2(t)}\right], \qquad \overline{R}^3 = R_x^2 R_z$$
$$\chi(\mathbf{x},t) = \frac{1}{2}[\alpha_x(t)x^2 + \alpha_y(t)y^2 + \alpha_z(t)z^2]$$

- Equations of motion with quantum corrections

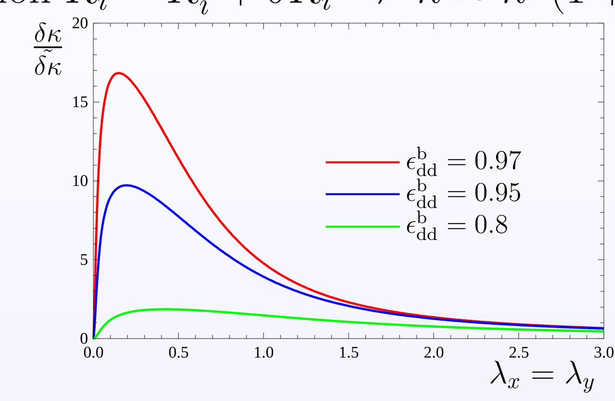
$$\ddot{R}_x = -\omega_x^2 R_x + \frac{15gN}{4\pi M R_x \overline{R}^3}\left[1 - \epsilon_{\rm dd}A\left(\frac{R_x}{R_z}\right) + \frac{\beta}{\overline{R}^{\frac{3}{2}}}\right]$$
$$\ddot{R}_z = -\omega_z^2 R_z + \frac{15gN}{4\pi M R_z \overline{R}^3}\left[1 + 2\epsilon_{\rm dd}B\left(\frac{R_x}{R_z}\right) + \frac{\beta}{\overline{R}^{\frac{3}{2}}}\right]$$

- Auxiliary functions $A(x) = 1 + \frac{3}{2}\frac{x^2 f_s(x)}{x^2-1}, \quad B(x) = 1 + \frac{3}{2}\frac{f_s(x)}{x^2-1}$

- Anisotropy function [13–18] $\quad f_s(x) = \frac{1+2x^2}{1-x^2} - \frac{3x^2\tanh^{-1}\sqrt{1-x^2}}{(1-x^2)^{3/2}}$

Beyond mean-field aspect ratio [7,8]

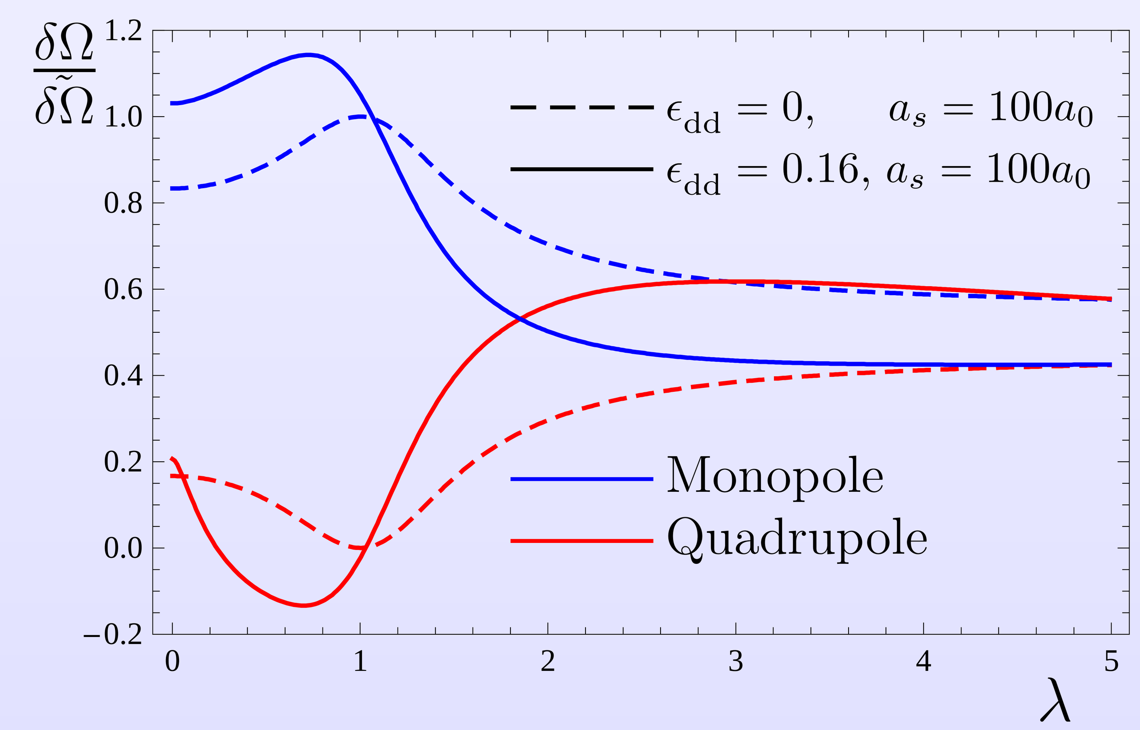
- First order correction $R_i = R_i^0 + \delta R_i \;\Rightarrow\; \kappa \approx \kappa^0(1+\delta\kappa)$



with the abbreviation $\tilde{\delta\kappa} = \frac{105\sqrt{\pi}}{32}\sqrt{a_s^3 n(\mathbf{0})}$

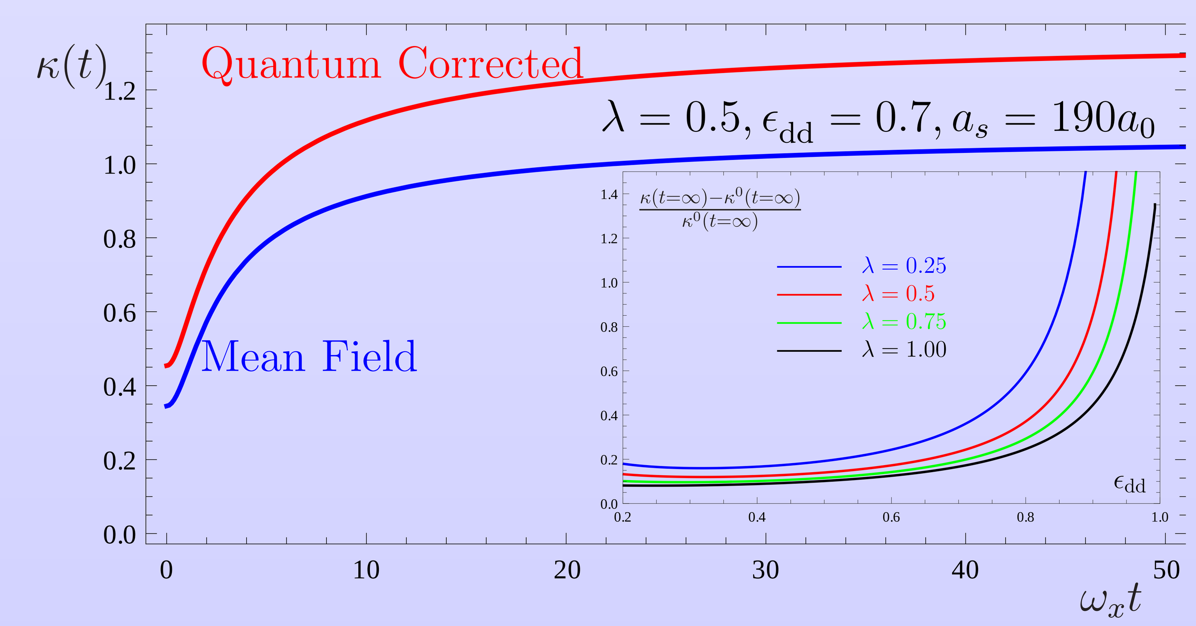
Low-Lying excitations [7,8]

- Linearize the equations of motion according to $R_i(t) = R_i(0) + \eta_i \sin(\Omega t + \varphi)$



Time-of-flight dynamics [7]

- Set $\omega_i = 0$ in the equations of motion



Acknowledgements

This work was financially supported by the German Research Foundation (DFG) within project KL256/53-1.

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