BECs with $1/r$ Interatomic Interaction

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Experimental Motivation

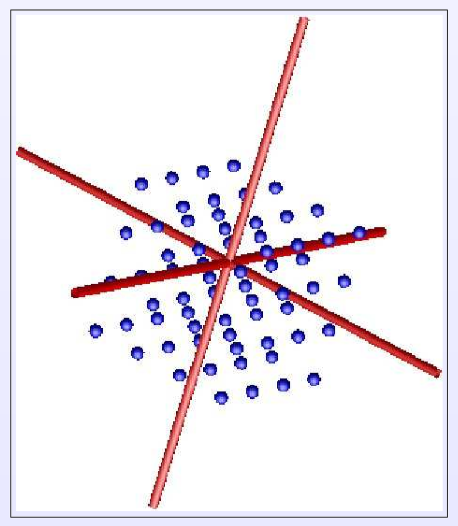
1) Gravity-like interaction [1,2]:

- Interaction potential of radiation field and atoms [3–5]:

$$U(\mathbf{r}) = \frac{I\alpha^2(k)}{4\pi c\varepsilon_0{}^2}\,\hat{e}_i{}^*\,\hat{e}_j\,\frac{1}{r^3}\Big[(\delta_{ij} - 3\,\hat{r}_i\hat{r}_j)\,(\cos kr + kr\sin kr) - (\delta_{ij} - \hat{r}_i\hat{r}_j)\,k^2r^2\cos kr\Big]\cos(\mathbf{k}\cdot\mathbf{r})$$

- Near-zone potential, $kr \ll 1$:

Setup with three crossed static lasers:



Typical numbers

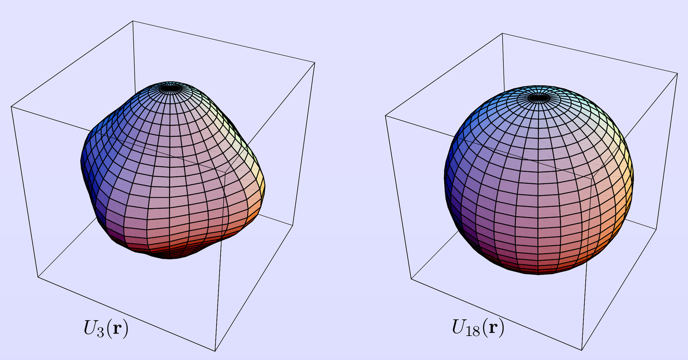
static polarizability α	^{23}Na: $24.08 \cdot 10^{-24}\,\mathrm{cm}^3$ ^{87}Rb: $46.3 \cdot 10^{-24}\,\mathrm{cm}^3$
wavelength of a CO_2-laser	$\lambda = 10.6\,\mu\mathrm{m}$
intensity	$I = 10^8\,\mathrm{W/cm}^2$
gravitational-like length a_G	^{23}Na: $0.1\,\mathrm{m}$ ^{87}Rb: $0.027\,\mathrm{m}$

$$U_3(\mathbf{r}) = -\frac{3Ik^2\alpha^2}{16\pi c\varepsilon_0^2}\frac{1}{r}\left[\frac{7}{3} + (\sin\vartheta\,\cos\varphi)^4 + (\sin\vartheta\,\sin\varphi)^4 + \cos^4\vartheta\right]$$

18 crossed lasers:

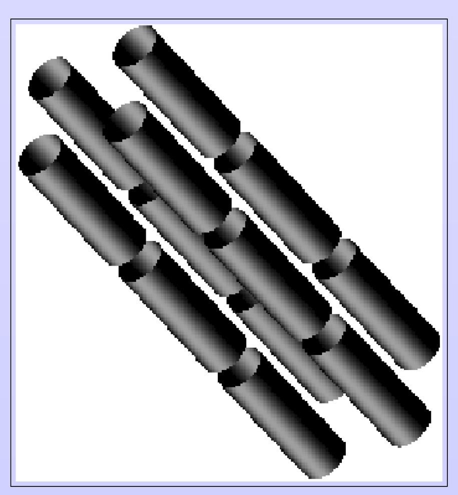
$$U_{18}(\mathbf{r}) = -\frac{11Ik^2\alpha^2}{4\pi c\varepsilon_0{}^2}\frac{1}{r} = -\frac{u}{r}$$

$$u = \frac{4\pi^2\hbar^2}{ma_G}$$



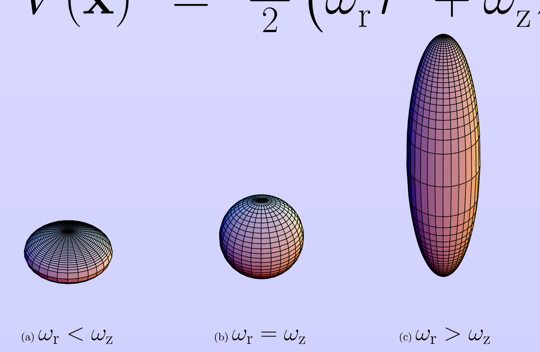
2) Coulomb interaction of trapped ions:

- Linear quadrupole trap [6]:



Effective harmonic trap

$$V(\mathbf{x}) = \frac{M}{2}\left(\omega_r^2r^2 + \omega_z^2z^2\right):$$



- Cooling by mutual interactions of $^4He^+$ and $^9Be^+$
- Extension to BEC?

Gross-Pitaevskii Theory

- Gross-Pitaevskii equation in Thomas-Fermi approximation:

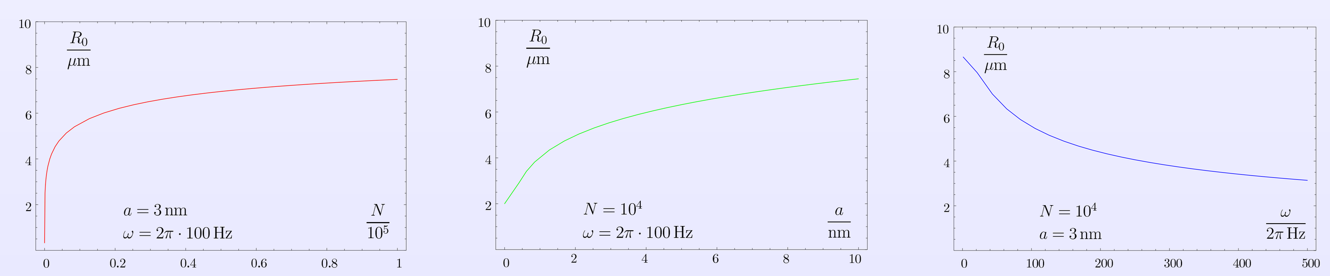
$$\mu\Psi(\mathbf{r}) = \left[g|\Psi(\mathbf{r})|^2 + \frac{m\omega^2r^2}{2} - u\int\frac{|\Psi(\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}d^3r'\right]\Psi(\mathbf{r})$$

- Solution ($k = \sqrt{\frac{4\pi u}{g}}$): $|\Psi(r)|^2 = \left[\frac{A\sin kr}{r} - \frac{3m\omega^2}{4\pi u}\right]\Theta(R_0 - r)$

$$A = -\frac{gk^2N + 4\pi R_0{}^3m\omega^2}{4\pi g\,[kR_0\cos(kR_0) - \sin(kR_0)]}$$

$$(kR_0)^2\cot(kR_0) = -\frac{Ngk^5}{12\pi m\omega^2} - \frac{(kR_0)^3}{3} + kR_0$$

- Thomas-Fermi radius of ^{23}Na:



- Self-binding without a trap ($\omega = 0$) [1]:

$$\Psi_{TF}(r) = \frac{\sqrt{N}}{2R_0}\sqrt{\frac{\sin(\pi r/R_0)}{r}}\,\Theta(R_0 - r)$$

- Dynamics: Time-dependent variational approach [7]:

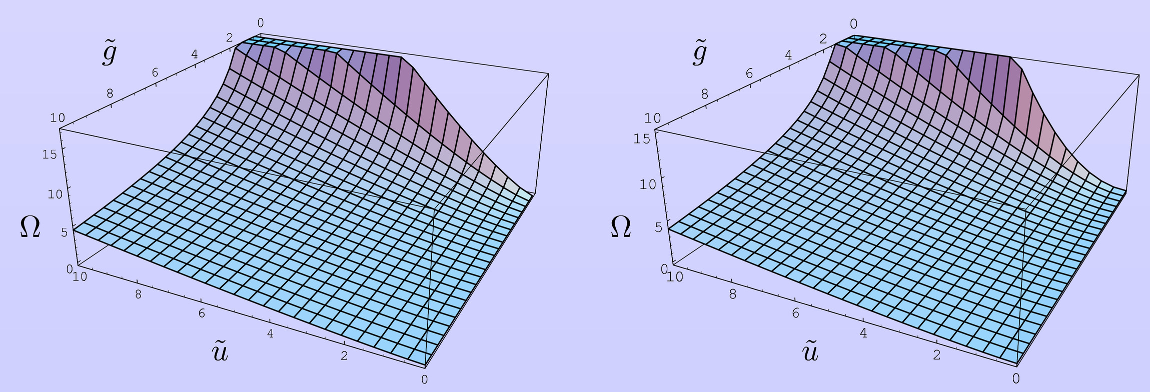
$$\Psi(r,t) = \frac{\sqrt{N}}{\pi^{3/4}\lambda(t)^{3/2}l^{3/2}}\exp\left\{-r^2\left[\frac{1}{2\lambda(t)^2l^2} + \imath B(t)\right]\right\}$$

Frequency Ω of collective excitations:

	TF-Grav $k=0,\ \omega=0$	TF-Hard $k=0,\ \tilde{u}=0$	Ideal $\tilde{g}=0,\ \tilde{u}=0$	Gravitation $\tilde{g}=0,\ \omega=0$	Grav-trapless $\omega=0$
Ω	$\tilde{u}^2\sqrt{\frac{2}{(\tilde{u}\tilde{g})^{3/2}}}$	$\sqrt{5}$	2	$\tilde{u}^2$	$4\tilde{u}^2\sqrt{\frac{1+4\tilde{u}\tilde{g}+\sqrt{1+4\tilde{u}\tilde{g}}}{(1+\sqrt{1+4\tilde{u}\tilde{g}})^5}}$

$$l = \sqrt{\frac{\hbar}{m\omega}}, \qquad \tilde{u} = \sqrt{\frac{2}{\pi}}\frac{Nlm}{3\hbar^2}u\,, \qquad \tilde{g} = \sqrt{\frac{1}{8\pi^3}}\frac{Nm}{l\hbar^2}g$$

- Frequencies:



(a) TF-Grav (b) TF-trapless

Critical Temperature

- Bose gas in harmonic trap

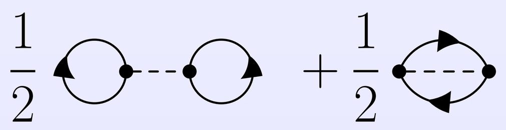
$$\mathcal{Z} = \oint\mathcal{D}\psi^*\,\mathcal{D}\psi\ e^{-\left(\mathcal{A}^{(0)}[\psi^*,\psi]+\mathcal{A}^{(\mathrm{int})}[\psi^*,\psi]\right)/\hbar}$$

$$\mathcal{A}^{(0)}[\psi^*,\psi] = \int_0^{\hbar\beta}d\tau\int d^3x\,\psi^*(\mathbf{x},\tau)\times\left\{\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2}{2M}\mathbf{\Delta} + V(\mathbf{x}) - \mu\right\}\psi(\mathbf{x},\tau)$$

$$\mathcal{A}^{(\mathrm{int})}[\psi^*,\psi] = \frac{1}{2}\int_0^{\hbar\beta}d\tau\int d^3x\int d^3x'\left[g\,\delta(\mathbf{x}-\mathbf{x}') \mp \frac{u}{|\mathbf{x}-\mathbf{x}'|}\right]\times\psi^*(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\psi(\mathbf{x},\tau)\psi(\mathbf{x}',\tau)$$

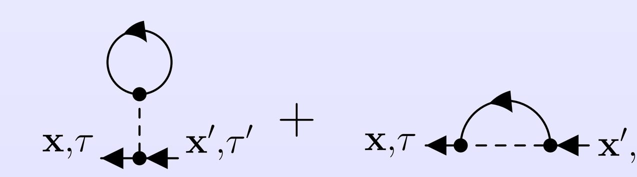
- Free energy:

$$\mathcal{F} = \mathcal{F}^{(0)} - \frac{1}{\beta}\left\{\frac{1}{2}\ [\text{diagram}]\ + \frac{1}{2}\ [\text{diagram}]\ + \ldots\right\}$$



- Self energy:

$$\Sigma(\mathbf{x},\tau;\mathbf{x}',\tau') = \ [\text{diagram}]\ + \ [\text{diagram}]\ + \ldots$$



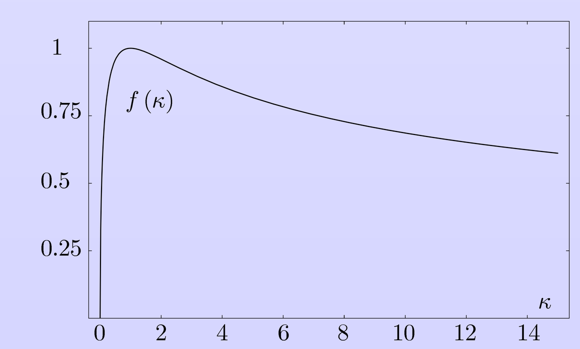
- First-order analytical result in semiclassical approximation [8]:

$$\left(\frac{\Delta T_c}{T_c^{(0)}}\right)_{G,C} = -c_\delta\frac{a}{\lambda_{T_c^{(0)}}} \pm c_E\frac{\lambda_{T_c^{(0)}}}{a_{G,C}} \pm c_D\frac{\lambda_{T_c^{(0)}}}{a_{G,C}}\frac{f(\frac{\omega_z}{\omega_r})}{(\hbar\beta_c^{(0)}\tilde{\omega})^2}$$

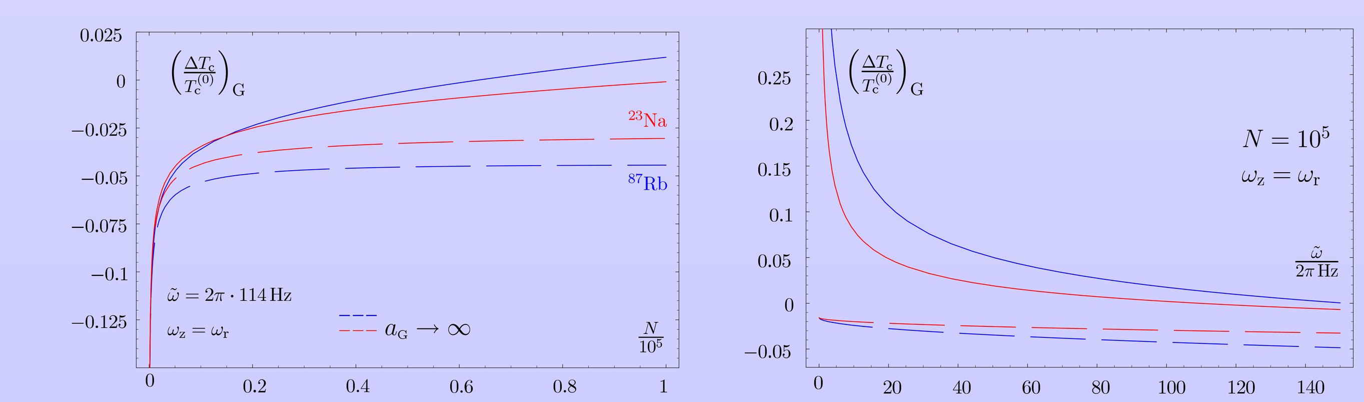
$$c_\delta \approx 3.42603, \qquad c_E \approx -13.839, \qquad c_D \approx 2.0951$$

- Anisotropy function, $\kappa = \omega_z/\omega_r$:

$$f(\kappa) = \frac{\kappa^{-1/3}}{\sqrt{1-\kappa^{-2}}}\arccos\left(\frac{1}{\kappa}\right)$$



- Graphical results:



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References

[1] D. O'Dell, S. Giovanazzi, G. Kuritzki, and V. M. Akulin, *Bose-Einstein Condensates with $1/r$ Interatomic Attraction: Electromagnetically Induced Gravity*, Phys. Rev. Lett. **84**, 5687 (2000)

[2] J. Anglin, *Why trapped atoms are attractive*, Nature (London) **406**, 29 (2000)

[3] D. P. Craig and T. Thirunamachandran, *Molecular Quantum Electrodynamics* (Academic Press, London, 1984)

[4] T. Thirunamachandran, *Intermolecular Interactions in the Presence of an Intense Radiation Field*, Mol. Phys. **40**, 393 (1980)

[5] D. L. Andrews and T. Thirunamachandran, *A quantum electrodynamical theory of differential scattering baesd on a mode with two chromophores*, Proc. Roy. Soc. A **358**, 297 (1978)

[6] B. Roth, U. Fröhlich, and S. Schiller, *Sympathetic Cooling of $^4He^+$ Ions in a Radio-Frequency Trap*, Phys. Rev. Lett. **94**, 053001 (2005)

[7] V. M. Pérez-García, J. I. Cirac, M. Lewenstein, and P. Zoller, *Low Energy Excitations of a Bose-Einstein Condensate: A Time-Dependent Variational Analysis*, Phys. Rev. Lett. **77**, 5320 (1996)

[8] H. Kleinert, *Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*, Forth Edition (World Scientific, Singapore, 2006)