Multi-Component Bose-Einstein Condensates

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Spinor Bose-Einstein Condensates

Abstract

Due to the combination of spin degrees of freedom and the conservation of the total magnetization, the interaction-free $F = 1$ spinor gas possesses three different phases: a gas, a ferromagnetic, and an antiferromagnetic phase [1]. The latter phase is distinct because of the occurrence of a double condensation. We calculate the critical temperatures of the phase transitions and their dependence on the total magnetization. We determine the occupation number of the three different Zeeman states of both the excited and the Bose-Einstein condensed particles and calculate the heat capacity as a function of temperature and magnetization.

The treatment is generalized to the case of a weakly interacting $F = 1$ spinor gas, which is characterized by the two s-wave scattering lengths a_0 and a_2. Within first-order perturbation theory, we derive an analytical expression for the shift of the first critical temperature as a function of magnetization. Our results agree well with a numerical solution of the Hartree-Fock-Popov approximation [2].

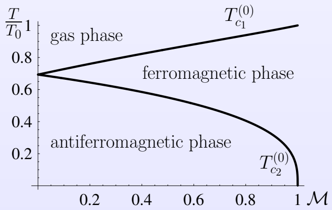
Non-Interacting Spinor Gas [3,4]: $U(\boldsymbol{x}) = \frac{M}{2}\omega_i^2 x_i^2, \quad E_0 = \frac{\hbar}{2}(\omega_1+\omega_2+\omega_3)$

$$\mathcal{A}^{(0)} = \int_0^{\hbar\beta} d\tau \int d^3x\, \psi_i^*(\boldsymbol{x},\tau)\left[\left(\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2}{2M}\Delta + U(\boldsymbol{x}) - \mu\right)\delta_{ij} - \eta F_{ij}^{(z)}\right]\psi_j(\boldsymbol{x},\tau)$$

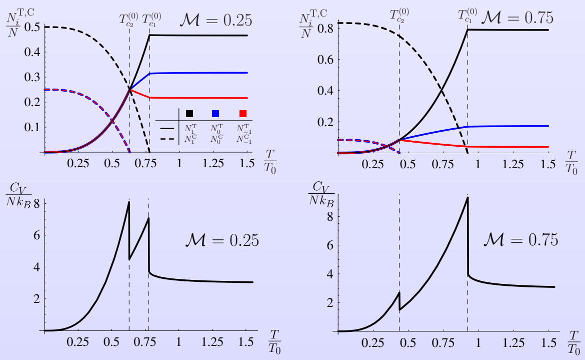
- Dependence of critical temperatures on magnetization $\mathcal{M}$ [5]:

Gross-Pitaevskii equations:

$$(E_0 - \mu - \eta)\Psi_1(\boldsymbol{x}) = 0$$
$$(E_0 - \mu)\Psi_0(\boldsymbol{x}) = 0$$
$$(E_0 - \mu + \eta)\Psi_{-1}(\boldsymbol{x}) = 0$$



- Particle number and heat capacity [5]:



Interacting spinor gas: $\mathcal{A} = \mathcal{A}^{(0)} + \mathcal{A}^{(\text{int})}, \quad c_0 \propto a_0 + 2a_2, \quad c_2 \propto a_2 - a_0$

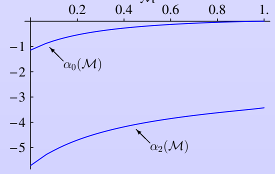
$$\mathcal{A}^{(\text{int})} = \frac{1}{2}\int_0^{\hbar\beta} d\tau \int d^3x \left\{ c_0\left[\psi_i^*(\boldsymbol{x},\tau)\,\psi_i(\boldsymbol{x},\tau)\right]^2 + c_2 \sum_{a=x,y,z}\left[\psi_i^*(\boldsymbol{x},\tau) F_{ij}^a\, \psi_j(\boldsymbol{x},\tau)\right]^2 \right\}$$

- First-order T_{c_1}-shift [5]:

$$\frac{\Delta T_{c_1}}{T_{c_1}^{(0)}} = \alpha_0(\mathcal{M})\frac{a_0}{\lambda_0} + \alpha_2(\mathcal{M})\frac{a_2}{\lambda_0}$$

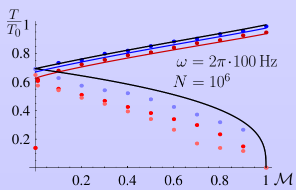
$$\lim_{\mathcal{M}\to 1}\frac{\Delta T_{c_1}}{T_{c_1}^{(0)}} = -3.426\,\frac{a_2}{\lambda_0}$$

$$\lim_{\mathcal{M}\to 1}\frac{\Delta T_{c_1}}{T_{c_1}^{(0)}} = -1.371\left(\frac{a_0}{\lambda_0} + 5\,\frac{a_2}{\lambda_0}\right)$$



- Examples: ^{87}Rb / ^{23}Na [2,5]:

	T_{c_1}	T_{c_2}	T_{c_3}	$T_{c_1}^{\text{analytic}}$
^{87}Rb	●	●	●	—
^{23}Na	●	●		—



Boson-Fermion Mixtures

- Trapped interacting boson-fermion gas: $U_i(\boldsymbol{x}) = \frac{M_i}{2}\left(\omega_{i,r}^2 r^2 + \omega_{i,z}^2 z^2\right), \quad i = B, F$

$$g_{BB} = \frac{4\pi\hbar^2 a_{BB}}{M_B}, \qquad g_{BF} = 2\pi\hbar^2 a_{BF}\frac{M_B + M_F}{M_B\, M_F}$$

- Euclidean action: $\mathcal{A} = \mathcal{A}_B + \mathcal{A}_F + \mathcal{A}_{BF}$

Bose action:

$$\mathcal{A}_B = \int_0^{\hbar\beta} d\tau \int d^3x\, \psi_B^*(\boldsymbol{x},\tau)\left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2}{2M_B}\Delta + U_B(\boldsymbol{x}) - \mu_B + \frac{1}{2}g_{BB}\,|\psi_B(\boldsymbol{x},\tau)|^2\right]\psi_B(\boldsymbol{x},\tau)$$

Fermi action: $\mathcal{A}_F = \int_0^{\hbar\beta} d\tau \int d^3x\, \psi_F^*(\boldsymbol{x},\tau)\left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2}{2M_F}\Delta - U_F(\boldsymbol{x}) - \mu_F\right]\psi_F(\boldsymbol{x},\tau)$

Mixed action: $\mathcal{A}_{BF} = g_{BF}\int_0^{\hbar\beta} d\tau^3 x\, |\psi_B(\boldsymbol{x},\tau)|^2\, |\psi_F(\boldsymbol{x},\tau)|^2$

- Density profiles for a ^{87}Rb–^{40}K–mixture [6]:

Gross-Pitaevskii equation in the Thomas-Fermi approximation:

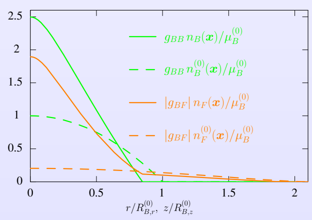
$$V_B(\boldsymbol{x}) - \mu_B + g_{BB}\, n_B(\boldsymbol{x}) + g_{BF}\, n_F(\boldsymbol{x}) = 0$$

Particle density of fermions:

$$n_F(\boldsymbol{x}) = \kappa\,\Theta\left(\mu_F - V_F(\boldsymbol{x}) - g_{BF}\, n_B(\boldsymbol{x})\right)\left[\mu_F - V_F(\boldsymbol{x}) - g_{BF}\, n_B(\boldsymbol{x})\right]^{3/2}, \quad \kappa = \frac{(2M_F)^{3/2}}{6\pi^2\hbar^3}$$

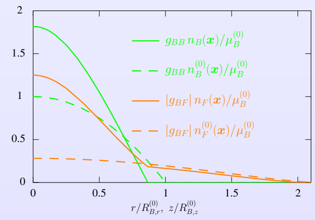
Florence experiment (2002) [7]

$N_B = 80700 \quad N_F = 17900$

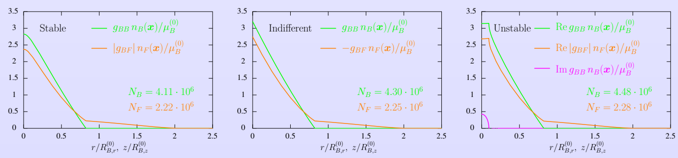


Hamburg experiment (2005) [8]

$N_B = 1.74\cdot 10^6 \quad N_F = 7.83\cdot 10^5$



- On the road from stability to instability (Hamburg experiment) [6]:



- Stability against collapsing [8,9]:

Grand-canonical free energy [10,11]: $\tilde{\mu}_F(\boldsymbol{x}) = \mu_F - V_F(\boldsymbol{x}) - g_{BF}\,|\Psi(\boldsymbol{x})|^2$

$$\mathcal{F} = \int d^3x \left[\frac{\hbar^2}{2M_B}|\nabla\Psi(\boldsymbol{x})|^2 + (V_B(\boldsymbol{x}) - \mu_B)|\Psi(\boldsymbol{x})|^2 + \frac{g_{BB}}{2}|\Psi(\boldsymbol{x})|^4 - \frac{2\kappa}{5}\Theta(\tilde{\mu}_F(\boldsymbol{x}))\,\tilde{\mu}_F(\boldsymbol{x})^{5/2}\right]$$

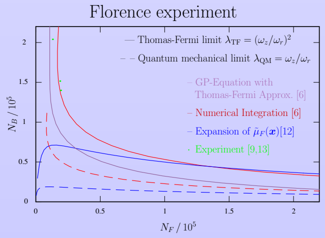
Ansatz with variational widths $\alpha L_{B,k}$:

$$\Psi(\boldsymbol{x}) = \sqrt{\frac{N_B\lambda^{1/2}}{\pi^{3/2}\alpha^3 L_{B,r}^3}}\exp\left\{-\frac{r^2 + \lambda z^2}{2\alpha^2 L_{B,r}^2}\right\}, \qquad \lambda = \left(\frac{L_{B,r}}{L_{B,z}}\right)^2, \qquad L_{B,k} = \sqrt{\frac{\hbar}{M_B\,\omega_{B,k}}}$$

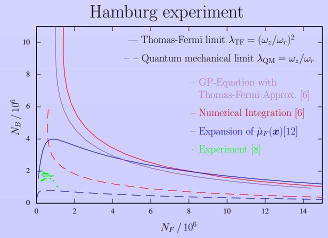
Expansion of $\tilde{\mu}_F(\boldsymbol{x})$ up to the 3rd order in g_{BF} [12]:

$$\mathcal{F} \approx \int d^3x \left[\frac{\hbar^2}{2M_B}|\nabla\Psi(\boldsymbol{x})|^2 + V_{\text{eff}}(\boldsymbol{x})\,|\Psi(\boldsymbol{x})|^2 + \frac{g_{\text{eff}}}{2}|\Psi(\boldsymbol{x})|^4 + \frac{\kappa g_{BF}^3}{8\mu_F^{1/2}}|\Psi(\boldsymbol{x})|^6\right]$$

Florence experiment



Hamburg experiment



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