TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Theory of Condensed Matter and Many Body Systems

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Two Intriguing Examples for Topological Effects in Ultra-Cold Atoms

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1. Ground-State Properties of Anyons in a One-Dimensional Lattice [1]

- Identical quantum particles in $D=3$: bosons and fermions
- $D=2$: anyons obey fractional statistics, phase factor $e^{i\theta}$ with $0 \leq \theta \leq \pi$ for exchanging two anyons, e.g. electrons in fractional quantum Hall effect
- Interpolation between Bose-Einstein and Fermi-Dirac statistics: e.g. Haldane [2], Polychronakos [3], ..., but virial expansion shows that those fractional statistics disagree with anyons, so anyon statistics is not yet known [4]
- $D=1$: Anyons with quantum gases realizable via photon-assisted tunneling [5], Raman scheme [6], lattice-shaking-induced resonant tunneling [7]
- Anyon-Hubbard Hamiltonian: $\hat{H}^a = -J\sum_{j=1}^{L}(\hat{a}_j^\dagger \hat{a}_{j+1} + \text{h.c.}) + \frac{U}{2}\sum_{j=1}^{L}\hat{n}_j(\hat{n}_j - 1)$

$$\hat{a}_j\hat{a}_k^\dagger - e^{-i\theta \text{sgn}(j-k)}\hat{a}_k^\dagger\hat{a}_j = \delta_{jk}\,, \quad \hat{a}_j\hat{a}_k - e^{i\theta\text{sgn}(j-k)}\hat{a}_k\hat{a}_j = 0$$

- Generalized Jordan-Wigner transformation [5]: $\hat{a}_j = \hat{b}_j \exp\left(i\theta\sum_{i=1}^{j-1}\hat{n}_i\right)$
- Bose-Hubbard model: $\hat{H}^b = -J\sum_{j=1}^{L}(\hat{b}_j^\dagger\hat{b}_{j+1}e^{i\theta\hat{n}_j} + \text{h.c.}) + \frac{U}{2}\sum_{j=1}^{L}\hat{n}_j(\hat{n}_j-1)$

$$\hat{b}_j\hat{b}_k^\dagger - \hat{b}_k^\dagger\hat{b}_j = \delta_{jk}\,, \quad \hat{b}_j\hat{b}_k - \hat{b}_k\hat{b}_j = 0$$

- Gutzwiller mean-field theory: $E\left(\left\{f_n^{(j)}\right\}\right) = \langle G|\hat{H}^b|G\rangle\ , |G\rangle = \prod_j\left(\sum_{n=0}^{n_{\max}} f_n^{(j)}|n\rangle\right)\ , \ \sum_{n=0}^{n_{\max}}|f_n^{(j)}|^2 = 1$

Classical GW: $f_n^{(j)} = f_n$,

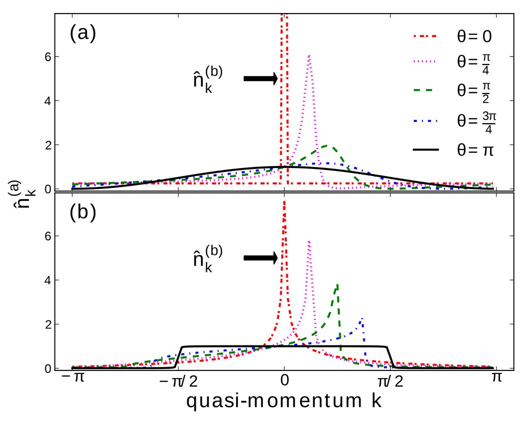
Modified GW 1 [1]: $f_n^{(j)} = F_n e^{i\alpha_n^{(j)}}$, Modified GW 2 [8]: $f_n^{(j)} = F_n e^{i(\alpha_n + j\beta_n)}$

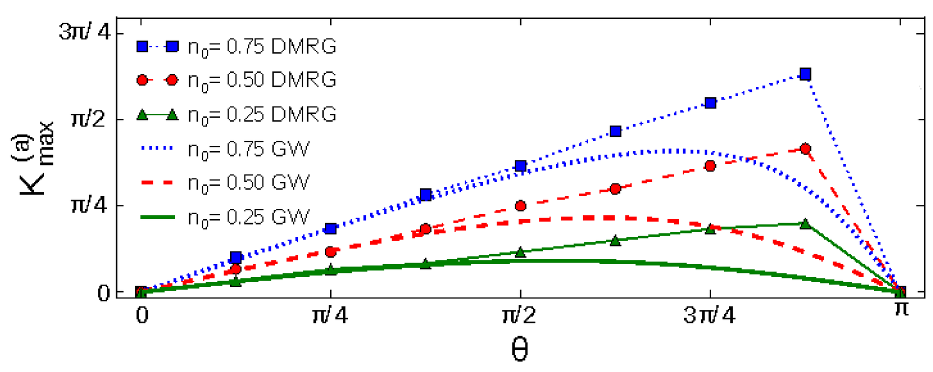
$\Longrightarrow$ Energy extremization yields Gutzwiller amplitudes and phases

- Quasi-momentum distributions depend on anyon statistical parameter θ:

Bosonic version: $\langle\hat{n}_k^{(b)}\rangle = \frac{1}{L}\sum_{ij}e^{ik(x_i - x_j)}\langle\hat{b}_i^\dagger\hat{b}_j\rangle$, anyonic version: $\langle\hat{n}_k^{(a)}\rangle = \frac{1}{L}\sum_{ij}e^{ik(x_i-x_j)}\langle\hat{a}_i^\dagger\hat{a}_j\rangle$

- Hard-core anyons: quasi-momentum distributions ($n_0 = N/L = 0.5\,, L=120$)

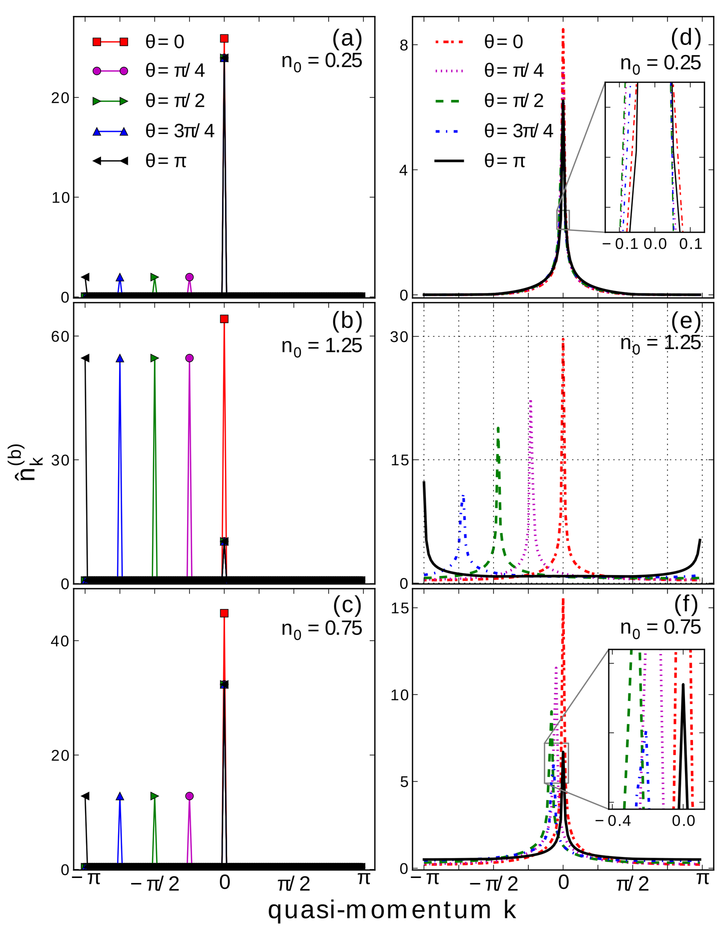


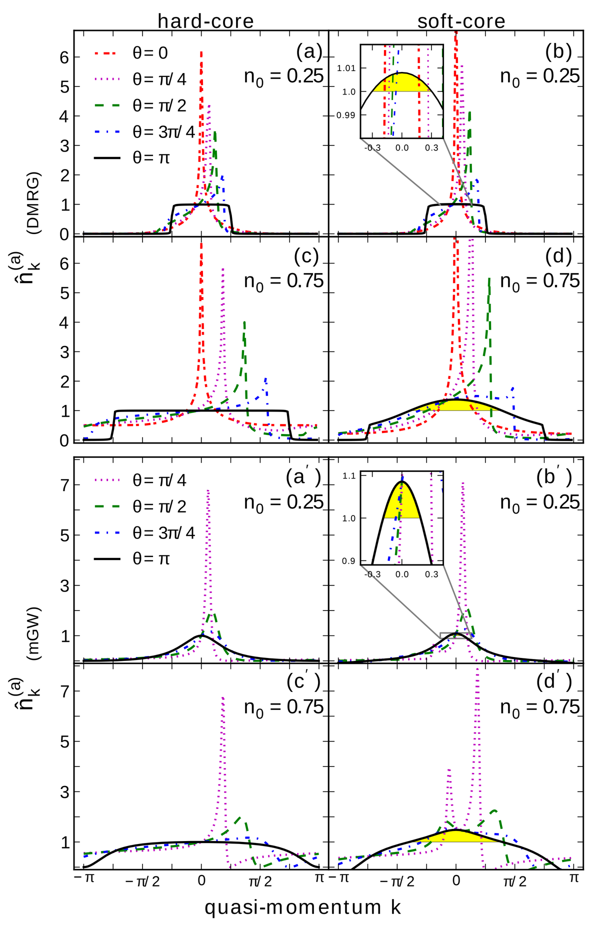


(a) Classical GW approach: $n_{\max} = 1$

(b) DMRG: matrix product states, open boundary conditions

- Soft-core anyons: quasi-momentum distributions ($J/U = 0.1\,, L = 120$)





- Modified Gutzwiller ($n_{\max} = 2$): $\langle\hat{n}_k^{(b)}\rangle \xrightarrow{L\to\infty} n_0 - C + A\delta(k) + B\delta(k+\theta)$

  A, B, C determined via energy extremization depend on n_0:

  low density $n_0 \ll 1$: peak at $k=0$, high density $n_0 \gg 1$: peak at $k = -\theta$
- Low density $n_0 \ll 1$: soft-core reproduces hard-core
- Yellow region $\langle\hat{n}_k^{(a)}\rangle > 1$: pseudofermions
- Distribution maxima shift: depends on Θ and n_0

2. Outlook

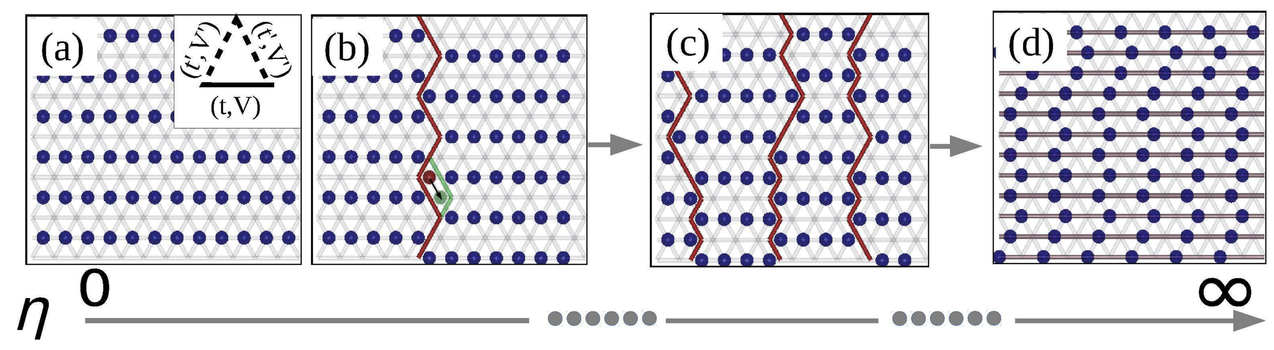
- Density: Friedel oscillations (open boundary conditions) [7]
- Quantum phase diagram: two types of superfluids possible [9]
- Nearest neighbor interactions: induce nontrivial topological phases [10]
- Jordan-Wigner Mapping of Anyon-Hubbard to Hubbard model: independence of exclusion principle and exchange statistics [11]
- Finite temperature: interpolation between Bose-Einstein and Fermi-Dirac statistics

3. Quantum Domain Walls Induce Incommensurate Supersolid Phase [12]

- Hard-core bosons on anisotropic triangular lattice, anisotropy parameter $\eta = t/t' = V/V'$ [13]:

$$\hat{H} = \sum_{\langle i,j\rangle_x}\left[-t\left(\hat{b}_i^\dagger\hat{b}_j + \text{h.c.}\right) + V(\hat{n}_i - 1/2)(\hat{n}_j - 1/2)\right] + \sum_{\langle i,j\rangle_\wedge}\left[-t'\left(\hat{b}_i^\dagger\hat{b}_j + \text{h.c.}\right) + V'(\hat{n}_i - 1/2)(\hat{n}_j - 1/2)\right]$$

- Transition from 2D solid phase ($\eta = 0$) to decoupled 1D chains ($\eta = \infty$):



- Quantum domains walls for $\eta < 1$ (see [14]): $E(N_D) = N_D L_y\left[\frac{V'-V}{2} - \frac{2}{\pi t'} + f\left(\frac{N_D}{L_x}\right)V'\right]$

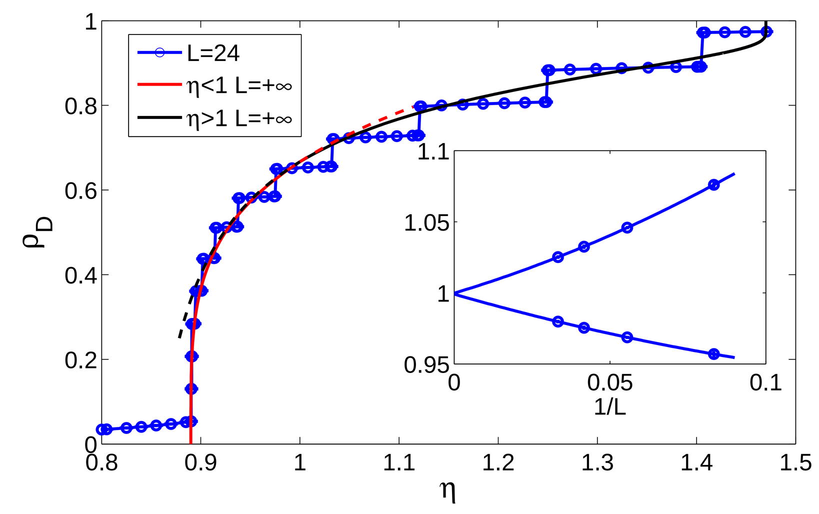
Interaction energy $f\left(\frac{2M}{L_x}\right) = \sum_{i=1}^{M}\frac{\eta_i}{2M} - \frac{\eta_{c1}}{2}$ expressed in terms of jump points between plateaus

$$\eta_M = \eta_{c1} + 2Mf\left(\frac{2M}{L_x}\right) - 2(M-1)f\left(\frac{2(M-1)}{L_x}\right)\ ; \quad M = 1, 2, \ldots\,, \quad \eta_{c1} = 1 - \frac{4t'}{\pi V'}$$

- Quantum Monte Carlo: stochastic series expansion [15], thermalization steps: 5×10^5, measuring steps: 10^6, $L_x = L_y = 24$
  - Topological, difficult to change number of domain walls with ordinary QMC updates
  - Extension of parallel tempering method [16]

Domain wall density with $\bar{n} = 1 - n$

$$\rho_D = \sum_{i_y=1}^{L_y}\sum_{i_x=1}^{L_x}\frac{n_{(i_x,i_y)}\bar{n}_{(i_x+1,i_y)} + n_{(i_x+1,i_y)}\bar{n}_{(i_x,i_y)}}{L_x L_y}$$



- Quantum domain wall model for $\eta < 1$ continued: fit result $f(\rho_D) \sim \rho_D^\alpha$, $\alpha = 4 \pm 0.1$

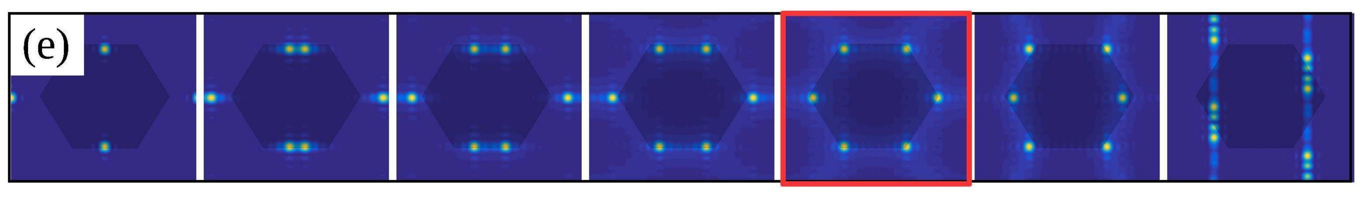
$$\rho_D(\eta) = \frac{2}{3}\left(\frac{\eta - \eta_{c1}}{1 - \eta_{c1}}\right)^{1/4}\ , \quad 1 - \frac{4t'}{\pi V'} = \eta_{c1} \leq \eta < 1$$

- Density pair excitations for $\eta > 1$: $E(N_D) = (L_x - N_D)L_y V'\left[\frac{\eta-1}{2} - 2\frac{t}{V'} + g\left(\frac{L_x - N_D}{L_x}\right)\right]$

$$\rho_D(\eta) = 1 + \frac{1}{3}\left\{1 + W_{-1}\left[-\frac{2(\eta_{c2}-\eta)}{e^2(\eta_{c2}-1)}\right]\right\}^{-1}\ , \quad 1 \leq \eta < \eta_{c2} = \frac{1}{1 - 4t'/V'}$$

W_{-1}: branch -1 of Lambert W function, $z = W(z)e^{W(z)}$

- Structure factor:

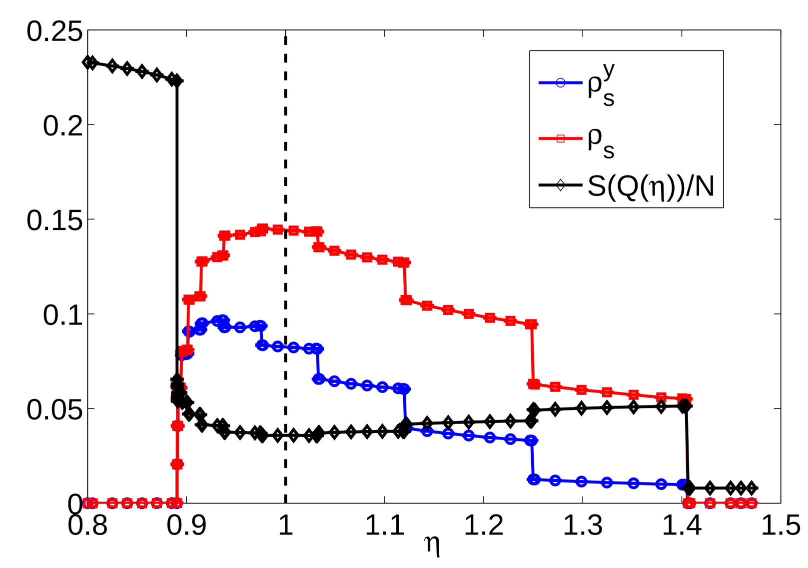


  - Each domain wall removes half of density oscillation, structure factor peaks shift by $\pm\pi/L_x$
  - microscopic origin of observed incommensurable order $\mathbf{Q} = \pi(\pm 2 - \rho_D, 0)\,, \pi(\pm\rho_D, \pm 2/\sqrt{3})$

- Anisotropic superfluid density: determined via winding numbers [17]

  $\rho_s^{x(y)} = W_{x(y)}^2/[4\beta t(t')]\,, \rho_s = \rho_s^x + \rho_s^y$
  - $\eta < 1$: ρ_s^y dominates
  - $\eta = 1$: ρ_s maximal
  - $\eta > 1$: ρ_s^x dominates

  $\Longrightarrow$ Superfluid density behaves opposite to structure factor



4. Outlook

- Linear response theory [18]: $p_i = VM\left(n_{nij}v_{nj} + n_{sij}v_{sj}\right) + \ldots$
- Dipolar interaction at zero temperature [19, 20]: no anisotropic superfluidity
- Dipolar interaction at finite temperature [21]: anisotropic first/second sound velocity
- Dipolar interaction and isotropic disorder at zero temperature [22, 23]
- Spin-orbit coupling: Elliptic vortices [24]
- Proposed Kagome superlattice: tunable anisotropic superfluidity [25]
- Josephson sum rule [26]: linear response theory (isotropic case)
  - Consequence for critical exponents [27]: $\beta_s = \beta_0 - \eta\nu$
  - Experimental verfication?
  - Anisotropic case?

$$n_s = \frac{m^2 n_0}{\lim_{\mathbf{k}\to 0}\hbar\mathbf{k}^2\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}A(\mathbf{k},\omega)}$$

$A(\mathbf{k},\omega)$: spectral function, i.e. Fourier transformed Green's function

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