

## Abstract

We study harmonically trapped three-dimensional ultracold Bose and Fermi gases in the presence of the short-range isotropic contact and the long-range anisotropic dipole-dipole interaction (DDI). The Hartree-Fock mean-field dynamics of such quantum systems can be described within the framework of the Zaremba-Nikuni-Griffin (ZNG) theory. Usually, the underlying Boltzmann-Vlasov (BV) equation is solved by the relaxation-time approximation for the collision integral, where the relaxation time is treated as a phenomenological parameter. We develop a formalism to determine the relaxation time microscopically for ultracold quantum gases at finite temperature, which allows us to include collision effects self-consistently in the BV formalism.

## Motivation

★ DDI potential:

$$V_{\text{int}}(\mathbf{r}) = \frac{C_{\text{dd}}}{4\pi|\mathbf{r}|^3} (1 - 3\cos^2\vartheta),$$

with  $\vartheta$  being the angle between the polarization direction and  $\mathbf{r}$  relative position of the dipoles

★ For magnetic dipole moments  $m$  DDI is characterized by  $C_{\text{dd}} = \mu_0 m^2$ , e.g. <sup>52</sup>Cr, <sup>53</sup>Cr, <sup>164</sup>Dy, <sup>167</sup>Er

★ For electric dipole moments  $d$  DDI is characterized by  $C_{\text{dd}} = d^2/\varepsilon_0$ , e.g. <sup>40</sup>K<sup>87</sup>Rb, <sup>41</sup>K<sup>87</sup>Rb

★ Second quantized Hamiltonian of quantum gases:

$$\hat{H} = \int d\mathbf{r} \hat{\Psi}^\dagger(\mathbf{r}, t) \left[ -\frac{\hbar^2 \Delta}{2m} + U_{\text{ext}}(\mathbf{r}) \right] \hat{\Psi}(\mathbf{r}, t) + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' V_{\text{int}}(\mathbf{r} - \mathbf{r}') \hat{\Psi}^\dagger(\mathbf{r}, t) \hat{\Psi}^\dagger(\mathbf{r}', t) \hat{\Psi}(\mathbf{r}', t) \hat{\Psi}(\mathbf{r}, t)$$

## ZNG equations for Bose Gases

★ In case of Bose gas we apply Bogoliubov decomposition [1]:

$$\hat{\Psi}(\mathbf{r}, t) = \Phi(\mathbf{r}, t) + \hat{\psi}(\mathbf{r}, t), \text{ where } \langle \hat{\psi}(\mathbf{r}, t) \rangle = 0$$

• Condensate described by:

$$\Phi(\mathbf{r}, t) = \langle \hat{\Psi}(\mathbf{r}, t) \rangle = \sqrt{n_c(\mathbf{r}, t)}$$

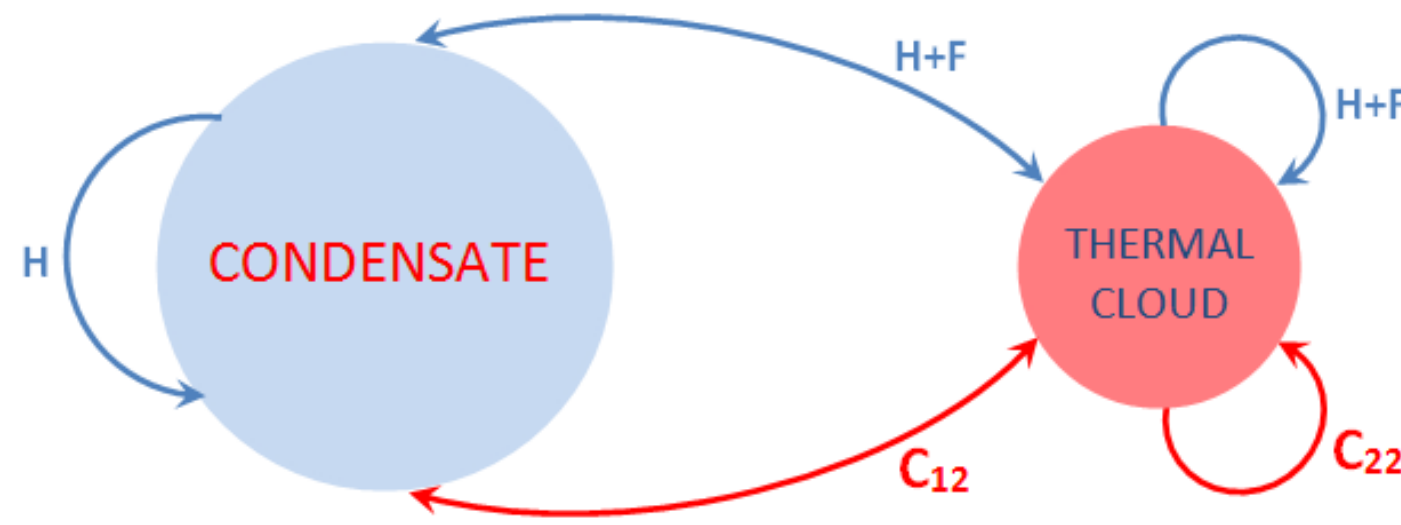
• Non-condensate described by Wigner function:

$$f(\mathbf{r}, \mathbf{p}, t) = \int d\mathbf{s} e^{i\mathbf{p}\cdot\mathbf{s}} \left\langle \hat{\psi}^\dagger\left(\mathbf{r} + \frac{\mathbf{s}}{2}, t\right) \hat{\psi}\left(\mathbf{r} - \frac{\mathbf{s}}{2}, t\right) \right\rangle$$

with spatial density given by:

$$n_{nc}(\mathbf{r}, t) = \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} f(\mathbf{r}, \mathbf{p}, t)$$

★ Schematic overview of self-consistent description of condensate wave function and Wigner function:



★ Dynamics of condensate wave function:

$$i\hbar \frac{\partial \Phi(\mathbf{r}, t)}{\partial t} = \left\{ -\frac{\hbar^2 \Delta}{2m} + U_{\text{ext}}(\mathbf{r}) + \int d\mathbf{r}' V_{\text{int}}(\mathbf{r} - \mathbf{r}') [n_c(\mathbf{r}', t) + n_{nc}(\mathbf{r}', t)] \right\} \Phi(\mathbf{r}, t) \\ + \int d\mathbf{r}' V_{\text{int}}(\mathbf{r} - \mathbf{r}') \langle \hat{\psi}^\dagger(\mathbf{r}', t) \hat{\psi}(\mathbf{r}, t) \rangle \Phi(\mathbf{r}', t) - \frac{i\hbar}{\Phi^*(\mathbf{r}, t)} \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} C_{12}[f]$$

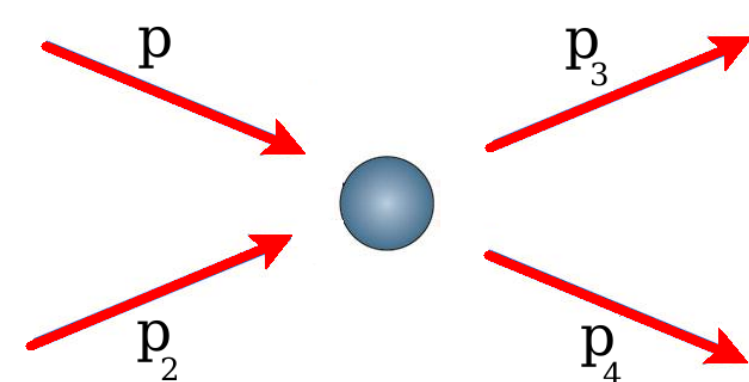
★ Time evolution of  $f(\mathbf{r}, \mathbf{p}, t)$  described by applying time dependent perturbation theory with respect to interaction up to second order [1, 3], yielding BV equation:

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \nabla_{\mathbf{r}} f + \nabla_{\mathbf{p}} U \nabla_{\mathbf{r}} f - \nabla_{\mathbf{r}} U \nabla_{\mathbf{p}} f = C_{12}[f] + C_{22}[f] = I_{\text{coll}}[f],$$

where  $U(\mathbf{r}, t) = U_{\text{ext}}(\mathbf{r}) + \int d\mathbf{r}' V_{\text{int}}(\mathbf{r} - \mathbf{r}') [n_c(\mathbf{r}', t) + n_{nc}(\mathbf{r}', t) + \langle \hat{\psi}^\dagger(\mathbf{r}', t) \hat{\psi}(\mathbf{r}, t) \rangle + \Phi^*(\mathbf{r}', t) \Phi(\mathbf{r}, t)]$  includes self-consistent Hartree-Fock (HF) dynamic mean field.

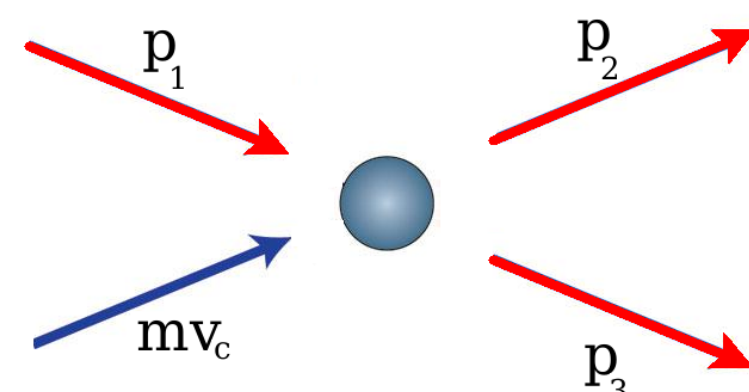
★ RHS of BV Eq. represents effect of collisions between atoms:

• Collisions between excited atoms:



$$C_{22}[f] = \frac{1}{2(2\pi)^5 \hbar^7} \int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 [\tilde{V}_{\text{int}}(\mathbf{p} - \mathbf{p}_3) + \tilde{V}_{\text{int}}(\mathbf{p} - \mathbf{p}_4)]^2 \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \\ \times \delta(\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3} - \varepsilon_{\mathbf{p}_4}) [(1+f)(1+f_2)f_3f_4 - ff_2(1+f_3)(1+f_4)]$$

• Collisions between condensate and non-condensate atoms:



$$C_{12}[f] = \frac{n_c}{2(2\pi)^2 \hbar^4} \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 [\tilde{V}_{\text{int}}(\mathbf{p}_1 - \mathbf{p}_2) + \tilde{V}_{\text{int}}(\mathbf{p}_1 - \mathbf{p}_3)]^2 \delta(m\mathbf{v}_c + \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) \\ \times \delta(\varepsilon_c + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3}) [\delta(\mathbf{p} - \mathbf{p}_1) - \delta(\mathbf{p} - \mathbf{p}_2) - \delta(\mathbf{p} - \mathbf{p}_3)] [(1+f_1)f_2f_3 - f_1(1+f_2)(1+f_3)]$$

where  $f \equiv f(\mathbf{r}, \mathbf{p}, t)$ ,  $f_i \equiv f(\mathbf{r}, \mathbf{p}_i, t)$  and variables  $\mathbf{v}_c$ ,  $n_c$ ,  $\varepsilon_c$  and  $\varepsilon_{\mathbf{p}} = \mathbf{p}^2/2m + U(\mathbf{r}, t)$  are functions of  $\mathbf{r}$  and  $t$ .

## BV equation for Fermi Gases

★ BV approach can be used for Fermi gases

★ There is no condensate, so only  $C_{22}$  collision integral remains

★ The form of BV Eq. is the same as for Bose gas with:

- HF mean field [4, 5]:

$$U(\mathbf{r}, \mathbf{p}, t) = U_{\text{ext}}(\mathbf{r}) + \int d\mathbf{r}' n(\mathbf{r}', t) V_{\text{int}}(\mathbf{r} - \mathbf{r}') - \int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} f(\mathbf{r}, \mathbf{p}', t) \tilde{V}_{\text{int}}(\mathbf{p} - \mathbf{p}')$$

- Collisions between atoms:

$$C_{22}[f] = \frac{1}{2(2\pi)^5 \hbar^7} \int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 [\tilde{V}_{\text{int}}(\mathbf{p} - \mathbf{p}_3) - \tilde{V}_{\text{int}}(\mathbf{p} - \mathbf{p}_4)]^2 \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \\ \times \delta(\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3} - \varepsilon_{\mathbf{p}_4}) [(1-f)(1-f_2)f_3f_4 - ff_2(1-f_3)(1-f_4)]$$

## Relaxation time

★ Description of collision integral in terms of relaxation time approximation [3]:

$$I_{\text{coll}}[f] = -\frac{f - f^{\text{le}}}{\tau}$$

with relaxation into local equilibrium  $I_{\text{coll}}[f^{\text{le}}] = 0$ .

★ In [6, 7] relaxation time is defined as:

$$\frac{1}{\tau_l} = \frac{\int d\mathbf{p}_l P_l(\cos\theta_1) \delta I}{\int d\mathbf{p}_l P_l(\cos\theta_1) \delta f}$$

★ Assuming isotropic scattering with cross section  $\sigma$  and case  $l = 1$  relaxation time for fermions is:

$$\tau_1 = \frac{9\hbar^2}{16m\sigma g k_B^2 T^2}$$

## Conclusions and Outlook

- ★ We generalized ZNG result for Bose gas including dipolar interaction instead of contact interaction
- ★ In a case of Bose gas with contact interaction  $V(\mathbf{r} - \mathbf{r}') = g\delta(\mathbf{r} - \mathbf{r}')$  all results are in agreement with ZNG theory [1–3].
- ★ Collision integral difference for excited bosons and fermions are in agreement with Born approximation
- ★ We plan to apply scaling approach:
  - to study dynamics of anisotropic trapped BEC
  - to show that momentum distribution is stretched along the orientation of dipoles, arising dominantly from Fermi surface anisotropy
- ★ Time-of-flight expansion all the way from collisionless to hydrodynamic regime
- ★ Relaxation time including anisotropic scattering due to dipolar interaction

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## Acknowledgments

We gratefully acknowledge support by DAAD - German Academic and Exchange Service under project NAI-DBEC and the Ministry of Education, Science, and Technological Development of the Republic of Serbia under projects No. ON171017 and NAI-DBEC.

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and Technological Development  
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