Time-of-flight Expansion for Trapped Dipolar Fermi Gases: From Collisionless to Hydrodynamic Regime

Vladimir Veljić[1], Antun Balaž[1], and Axel Pelster[2]

[1]Scientific Computing Laboratory, Institute of Physics Belgrade, University of Belgrade, Serbia
[2] Department of Physics and Research Center Optimas, Technical University of Kaiserslautern, Germany

Abstract

Some time ago it was predicted that the momentum distribution of Fermi gas is deformed from spherical to cylindrical, provided a dipole-dipole interaction is present [1]. A recent time-of-flight expansion experiment has now unambiguously detected such a Fermi surface deformation in a dipolar quantum gas of fermionic erbium atoms in the collisionless regime [2]. Here we follow Ref. [3] and perform a systematic study of time-of-flight expansions for trapped dipolar Fermi gases ranging from the collisionless to the hydrodynamic regime at zero temperature. To this end we solve analytically the underlying Boltzmann-Vlasov equation in the vicinity of equilibrium by using a suitable rescaling of the equilibrium distribution [4], where the collision integral is simplified within a relaxation-time approximation. The resulting ordinary differential equations for the scaling parameters are then solved numerically for experimentally realistic parameters for increasing relaxation times. Our analysis is, thus, useful for future time-of-flight experiments in order to determine the value of the underlying relaxation time from expansion data.

Motivation

- Dipole-dipole interaction potential:

$$V_{\text{int}}(\mathbf{r}) = \frac{C_{\text{dd}}}{4\pi\,|\mathbf{r}|^3}\left(1 - 3\cos^2\vartheta\right),$$

where ϑ is the angle between the polarization direction and relative position of the dipoles and $C_{\text{dd}} = \mu_0 m^2$ for magnetic dipole moments m or $C_{\text{dd}} = d^2/\varepsilon_0$ for electric dipole moments d

- The sample in experiment [2] contains $N = 7\cdot 10^4$ fermionic erbium atoms confined into harmonic trap with frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi(579, 91, 611)\,\text{s}^{-1}$
- Relative interaction strength:

$$\epsilon_{\text{dd}} = \frac{C_{\text{dd}}}{4\pi}\sqrt{\frac{M^3\omega}{\hbar^5}}N^{1/6},$$

where $\omega = (\omega_x\omega_y\omega_z)^{1/3}$ is the geometric averaged trap frequency

- Dipolar Fermi gases in current cold-atom experiments are:

gas	^{53}Cr	^{167}Er	^{161}Dy	^{40}K^{87}Rb
dipole moment	$6\mu_B$	$7\mu_B$	$10\mu_B$	0.2 D
ϵ_{dd}	0.02	0.15	0.30	0.97

Boltzmann-Vlasov Equation for Fermi Gases

- Dynamics of the Fermi gas can be described by Boltzmann-Vlasov equation:

$$\frac{\partial f(\mathbf{r},\mathbf{p},t)}{\partial t} + \frac{\mathbf{p}}{m}\nabla_{\mathbf{r}} f + \nabla_{\mathbf{p}}U(\mathbf{r},\mathbf{p},t)\nabla_{\mathbf{r}}f(\mathbf{r},\mathbf{p},t) - \nabla_{\mathbf{r}}U(\mathbf{r},\mathbf{p},t)\nabla_{\mathbf{p}}f(\mathbf{r},\mathbf{p},t) = I_{\text{coll}}[f](\mathbf{r},\mathbf{p},t),$$

where f is the Wigner function and $U = \int d\mathbf{r}' n(\mathbf{r}',t)V_{\text{int}}(\mathbf{r}-\mathbf{r}') - \int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} f(\mathbf{r},\mathbf{p}',t)\tilde{V}_{\text{int}}(\mathbf{p}-\mathbf{p}') + U_{\text{ext}}(\mathbf{r})$ includes self-consistent Hartree-Fock (HF) dynamic mean field and trap potential

- Description of the collision integral I_{coll} in terms of a relaxation time approximation [4]:

$$I_{\text{coll}}[f] = -\frac{f - f^{\text{le}}}{\tau},$$

with relaxation into the local equilibrium defined by $I_{\text{coll}}[f^{\text{le}}] = 0$

Global Equilibrium and Scaling Ansatz

- Ansatz for the global equilibrium distribution function f^0:

$$f^0(\mathbf{r},\mathbf{k}) = \Theta\left(1 - \sum_i \frac{r_i^2}{R_i^2} - \sum_i \frac{k_i^2}{K_i^2}\right),$$

where the variational parameters R_i and K_i represent the Thomas-Fermi radii and momenta, which are determined by minimizing the total energy of the system

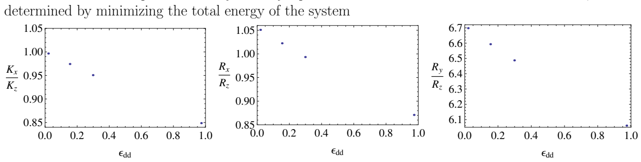

Figure 1. Aspect ratios in momentum and real space in equilibrium.

- Performing the scaling ansatz using the global equilibrium distribution function f^0 [4]:

$$f(\mathbf{x},\mathbf{q},t) \to \Gamma(t) f^0(\mathbf{r}(t),\mathbf{k}(t)),$$

with rescaled variables $r_i(t) = \frac{x_i}{b_i(t)}$ and $k_i(t) = \frac{1}{\sqrt{\theta_i(t)}}\left[q_i - \frac{m\dot{b}_i(t)}{\hbar b_i(t)}\right]$

- Normalization factor: $\Gamma(t)^{-1} = \prod_i b_i(t)\sqrt{\theta_i(t)}$
- Equations of motion for scaling parameters with assumption that collisions do not change spatial distribution:

$$\ddot{b}_i + \omega_i^2 b_i - \frac{\hbar^2 K_i^2\theta_i}{m^2 b_i R_i^2} + \frac{48Nc_0}{m b_i R_i^2\prod_j b_j R_j}\left[F\left(\frac{b_x R_x}{b_z R_z}, \frac{b_y R_y}{b_z R_z}\right) - b_i R_i \frac{\partial}{\partial b_i R_i} F\left(\frac{b_x R_x}{b_z R_z}, \frac{b_y R_y}{b_z R_z}\right)\right]$$

$$-\frac{48Nc_0}{m b_i R_i^2\prod_j b_j R_j}\left[F\left(\frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_x}K_x}, \frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_y}K_y}\right) + \sqrt{\theta_i}K_i\frac{\partial}{\partial\sqrt{\theta_i}K_i}F\left(\frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_x}K_x}, \frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_y}K_y}\right)\right] = 0,$$

$$\dot{\theta}_i + 2\frac{\dot{b}_i}{b_i}\theta_i = \frac{1}{\tau}(\theta_i - \theta_i^{\text{le}}),$$

where F denotes the anisotropy function [3]

- Coupling constant c_0 measures the strength of the dipolar interaction

$$c_0 = \frac{2^{10}C_{\text{dd}}}{3^4\cdot 5\cdot 7\cdot \pi^3}$$

- Collective oscillations all the way from the collisionless to the hydrodynamic regime are discussed in Ref. [3]

Local Equilibrium

- Minimization of the energy in local equilubrium leads to:

$$\theta_x^{\text{le}} = \theta_y^{\text{le}},$$

$$\frac{\hbar^2\theta_z^{\text{le}}K_z^2}{2m} - \frac{\hbar^2\theta_x^{\text{le}}K_x^2}{2m} = \frac{72Nc_0}{\prod_j b_j^{\text{le}}R_j}\left[1 + \frac{\left(2\theta_x^{\text{le}}K_x^2 + \theta_z^{\text{le}}K_z^2\right)F_s\left(\frac{\sqrt{\theta_z^{\text{le}}}K_z}{\sqrt{\theta_x^{\text{le}}}K_x}\right)}{2\left(\theta_z^{\text{le}}K_z^2 - \theta_x^{\text{le}}K_x^2\right)}\right]$$

- Normalization factor: $\prod_i b_i^{\text{le}}\sqrt{\theta_i^{\text{le}}} = 1$

Time-of-flight Expansion

- The atomic cloud in the experiment [2] is imaged with an angle of 28° with respect to the y-axis.
- The magnetic field forms angle β with z-axis and it is in $x' - z'$ plane rotated for an angle of 14° with respect to the $x - z$ plane.
- We consider a system of dipolar fermions with the point dipoles aligned along the z-direction.
- The cloud aspect ratio A is defined as the ratio of the vertical and horizontal radius of the cloud in the imaging plane:

$$A(t) = \frac{R_z(t)}{\sqrt{R_x^2(t)\cos^2 28° + R_y^2(t)\sin^2 28°}}$$

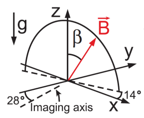

Figure 2. Experimental setup in Ref. [2].

- The equations of motion are solved while setting the harmonic restoring force to zero.

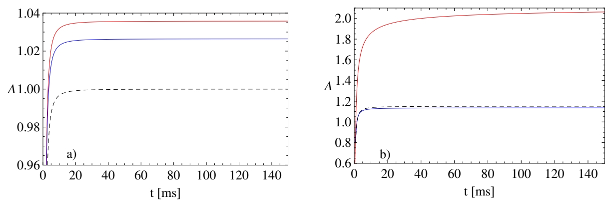

Figure 3. Time dependence of aspect ratio in a) collisionless and b) hydrodynamic regime for ballistic (blue line) and non-ballistic (red line) expansion. Dashed line represents non-interacting Fermi gas.

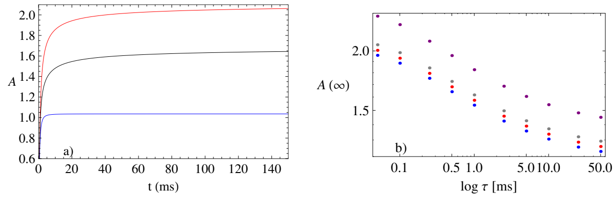

Figure 4. a) Time dependence of aspect ratio for non-ballistic expansion in collisionless regime (blue line), hydrodynamic regime (red line), and in the collisional regime for the relaxation time $\tau = 1/\omega$ (black line). b) Aspect ratio after infinitely long time as a function of relaxation time τ for ^{53}Cr (blue), ^{167}Er (red), ^{161}Dy (gray), and ^{40}K^{87}Rb (purple).

Conclusions and Outlook

- Analysis of time-of-flight expansion all the way from the collisionless to the hydrodynamic regime
- Determination of the time scale for approaching long-time limit from expansion data
- Estimate the relaxation time τ from experimental data
- Determine the dependence of A as a function on the angle β for different relaxation times and compare with experimental findings in Ref. [2]
- Microscopic determination of the relaxation time from collisional integral
- Applying equations of motion for a scaling parameters for other non-equilibrium cases, for instance, a parametric modulation of trap frequencies or strength of dipolar interaction

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