Cloud Shape of Dipolar Fermi Gases

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Abstract

In a recent time-of-flight (TOF) expansion experiment for ultracold polarized fermionic erbium atoms it was shown that the Fermi surface has an ellipsoidal shape [1]. It was also observed that the Fermi surface follows a rotation of the dipoles, which is induced by changing the direction of the external magnetic field, keeping the major axis always parallel to the direction of maximal attraction of the dipole-dipole interaction. Here we present a theory for determining the cloud shape in both real and momentum space by extending the work of Ref. [2], where the magnetic field is oriented along one of the harmonic trap axes, to an arbitrary orientation of the magnetic field. In order to analyze the cloud shape within TOF dynamics, we solve analytically the corresponding Boltzmann-Vlasov equation by using a suitable rescaling of the equilibrium distribution [3]. The resulting ordinary differential equations of motion for the scaling parameters are solved numerically in the collisionless regime at zero temperature and turn out to agree with the observations in the Innsbruck experiment [1].

Experiment

- ⋆ The sample in experiment [1] contains $N = 7\cdot 10^4$ fermionic ^{167}Er atoms
- ⋆ Atoms are confined into harmonic trap with frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi(579, 91, 611)$ Hz
- ⋆ The imaging axis forms an angle of $\alpha = 28°$ with respect to the y-axis in the xy-plane
- ⋆ The magnetic field $\vec{B} = B(\sin\beta\cos\gamma, \sin\beta\sin\gamma, \cos\beta)^T$ is characterised by spherical angles β and $\gamma = 14°$
- ⋆ The system is in the collisionless regime at very low temperature

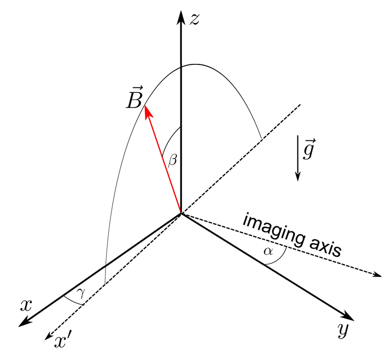


Fig. 1. Experimental setup in Ref. [1].

Global equilibrium and total energy

- ⋆ Ansatz for the global equilibrium distribution function ν^0:

$$\nu^0(\mathbf{r},\mathbf{k}) = \Theta\left[1 - \sum_{ij} r_i a_{ij} r_j - \sum_{ij} k_i b_{ij} k_j\right],$$

where matrix elements a_{ij} (b_{ij}) describe the atomic cloud (Fermi surface) in real (momentum) space

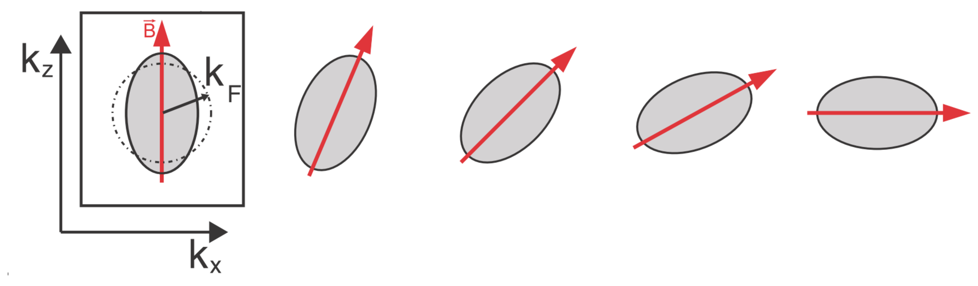


Fig. 2. Schematic illustrations of the deformed Fermi surface [1].

- ⋆ Matrix $\mathbb{A} = \{a_{ij}\}$ is determined by the trap potential, while matrix $\mathbb{B} = \{b_{ij}\}$ in the frame $x'y'z'$, in which z'-axis is parallel to dipole moments, has a diagonal form:

$$\mathbb{A} = \begin{pmatrix} R_x^2 & 0 & 0 \\ 0 & R_y^2 & 0 \\ 0 & 0 & R_z^2 \end{pmatrix}, \quad \mathbb{B}' = \begin{pmatrix} K_x'^2 & 0 & 0 \\ 0 & K_y'^2 & 0 \\ 0 & 0 & K_z'^2 \end{pmatrix},$$

where R_i and K_i' denote equilibrium sizes of atomic cloud and Fermi surface in corresponding direction

- ⋆ The total number of fermions is:

$$N = \frac{1}{48} R_x R_y R_z K_x' K_y' K_z'.$$

- ⋆ Total energy of the system in the Hartree-Fock mean-field theory:

$$E_{\rm tot} = \frac{N}{8}\sum_j \frac{\hbar^2 K_j'^2}{2m} + \frac{N}{8}\frac{m}{2}\sum_j \omega_j^2 R_j^2 - \frac{48 N^2 c_0}{8\bar{R}^3} f_{\rm H}\left(\frac{R_x}{R_z}, \frac{R_y}{R_z}\right) + \frac{48 N^2 c_0}{8\bar{R}^3} f_{\rm F}\left(\frac{K_z'}{K_x'}, \frac{K_z'}{K_y'}\right),$$

where $c_0 = \frac{2^{10} C_{\rm dd}}{3^4\cdot 5\cdot 7\cdot \pi^3}$ is a coupling constant measuring the strength of the dipolar interaction, $f_{\rm H}(x,y) = 1 - 3\sin^2\beta\cos^2\gamma f_1(x,y) - 3\sin^2\beta\sin^2\gamma f_2(x,y) + 3\cos^2\beta f_3(x,y)$, $f_{\rm F}(x,y) = 1 + 3 f_3(x,y)$ and f_i are modified anisotropy functions [4]

- ⋆ The variational parameters R_i and K_i' are determined by minimizing the total energy

Boltzmann-Vlasov equation and saling ansatz

- ⋆ Dynamics of the system described by Boltzmann-Vlasov equation:

$$\frac{\partial\nu(\mathbf{x},\mathbf{q},t)}{\partial t} + \frac{\hbar\mathbf{q}}{M}\nabla_{\mathbf{x}}\nu + \frac{1}{\hbar}\nabla_{\mathbf{q}}U(\mathbf{x},\mathbf{q},t)\nabla_{\mathbf{x}}\nu(\mathbf{x},\mathbf{q},t) - \frac{1}{\hbar}\nabla_{\mathbf{x}}U(\mathbf{x},\mathbf{q},t)\nabla_{\mathbf{q}}\nu(\mathbf{x},\mathbf{q},t) = I_{\rm coll}[\nu](\mathbf{x},\mathbf{q},t),$$

where $U(\mathbf{x},\mathbf{q},t) = U_{\rm ext}(\mathbf{x}) + \int d\mathbf{x}' V_{\rm int}(\mathbf{x}-\mathbf{x}')n(\mathbf{x}',t) - \int\frac{d\mathbf{q}'}{(2\pi\hbar)^3}\tilde{V}_{\rm int}(\mathbf{q}-\mathbf{q}')f(\mathbf{x},\mathbf{q}',t)$ is a Hartree-Fock mean field potential

- ⋆ Boltzmann-Vlasov equation is solved using a scaling ansatz of the equilibrium distribution [3]

$$\nu(\mathbf{x},\mathbf{q},t) \to \Gamma(t)\nu^0(\mathbf{r}(t),\mathbf{k}(t)),$$

with rescaled variables

$$r_i(t) = \frac{x_i}{b_i(t)} \quad\text{and}\quad k_i'(t) = \frac{1}{\sqrt{\theta_i(t)}}\left[q_i - \frac{m\dot{b}_i(t)x_i}{\hbar b_i(t)}\right].$$

- ⋆ Normalization factor: $\Gamma(t)^{-1} = \prod_i b_i(t)\sqrt{\theta_i(t)}$
- ⋆ Collisionless regime: $I_{\rm coll}[\nu](\mathbf{r},\mathbf{p},t) = 0$ and $\theta_i(t) = 1/b_i^2(t)$
- ⋆ Equations of motion for scaling parameters b_i:

$$\ddot{b}_i + \omega_i^2 b_i - \frac{\hbar^2 K_i'^2}{m^2 b_i^3 R_i^2} + \frac{48 N c_0}{m b_i R_i^2 \prod_j b_j R_j}\left[f_{\rm H}\left(\frac{b_x R_x}{b_z R_z}, \frac{b_y R_y}{b_z R_z}\right) - b_i R_i \frac{\partial}{\partial b_i R_i} f_{\rm H}\left(\frac{b_x R_x}{b_z R_z}, \frac{b_y R_y}{b_z R_z}\right)\right]$$

$$- \frac{48 N c_0}{m b_i R_i^2 \prod_j b_j R_j}\left[f_{\rm F}\left(\frac{b_x K_z'}{b_z K_x'}, \frac{b_y K_z'}{b_z K_y'}\right) + \frac{K_i'}{b_i}\frac{\partial}{\partial K_i' b_i} f_{\rm F}\left(\frac{b_x K_z'}{b_z K_x'}, \frac{b_y K_z'}{b_z K_y'}\right)\right] = 0.$$

Time-of-flight expansion

- ⋆ Dynamics in the real space can be described in terms of aspect ratios:

$$A_{\rm R}(t) = \sqrt{\frac{\langle r_z^2\rangle}{\langle r_x^2\rangle\cos^2\alpha + \langle r_y^2\rangle\sin^2\alpha}}, \quad \langle r_i^2\rangle = R_i^2 b_i^2(t).$$

- ⋆ Dynamics in the momentum space can be described in terms of aspect ratios:

$$A_{\rm K}(t) = \sqrt{\frac{\langle k_z^2\rangle}{\langle k_x^2\rangle\cos^2\alpha + \langle k_y^2\rangle\sin^2\alpha}}, \quad \langle k_i^2\rangle = \frac{1}{8}\left(\frac{K_i^2}{b_i^2(t)} + \frac{m^2 R_i^2 \dot{b}_i^2(t)}{\hbar^2}\right).$$

Aspect ratios for $\beta = 0°$ and $\beta = 90°$: $\lim_{t\to\infty} A_{\rm R}(t) = \lim_{t\to\infty} A_{\rm K}(t)$

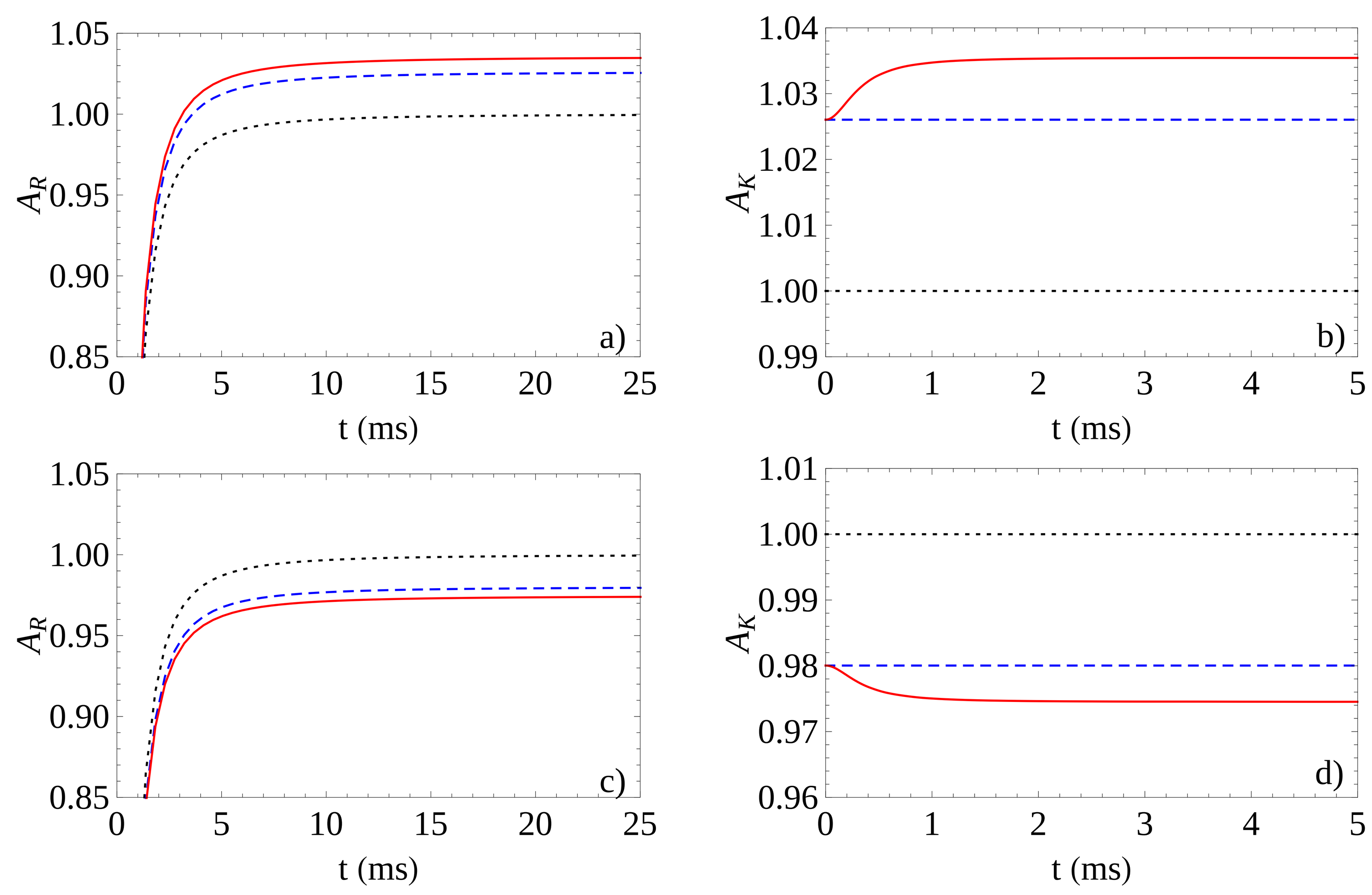
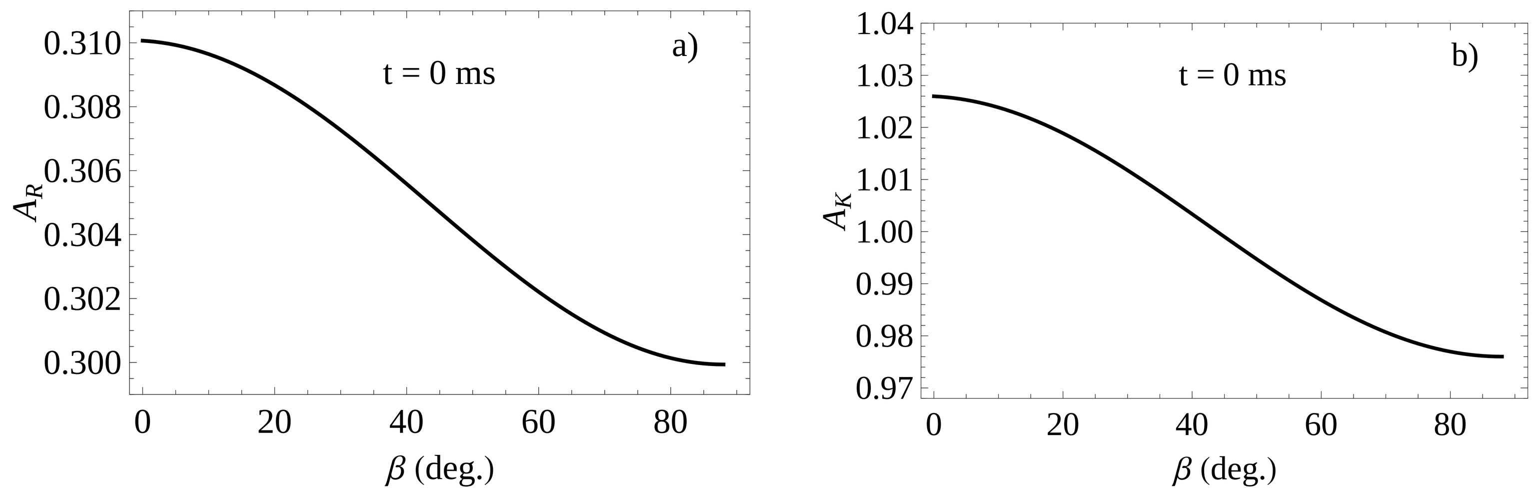


Fig. 3. Time evolution of aspect ratios in real space a) and c) and momentum space b) and d) for non-ballistic expansion (red solid line) and ballistic expansion (blue dashed line). Graphs a) and b) correspond to $\beta = 0°$, while graphs on c) and d) correspond to $\beta = 90°$. Black dotted line represents the noninteracting Fermi gas.

β dependence of aspect ratios



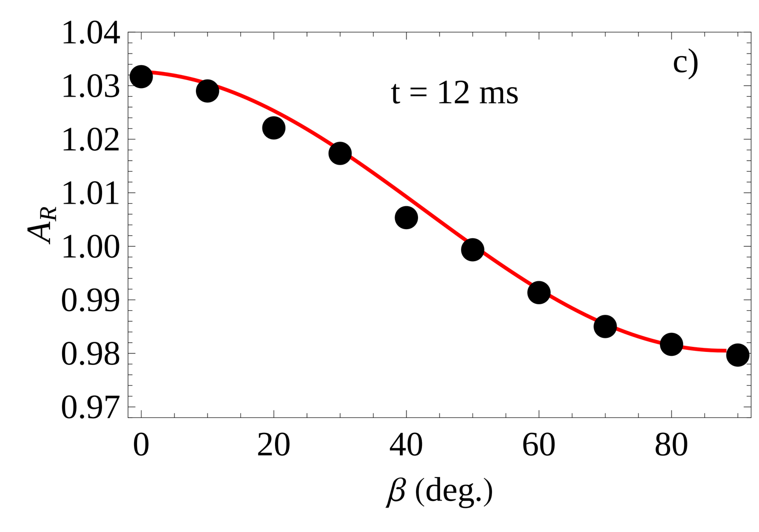


Fig. 4. Aspect ratios as functions of angle β in real space a) and c) and momentum space b). Red line in c) represents our theoretical result, while black dots are taken from experiment [1].

Conclusions and outlook

- ⋆ Fermi surface deformation is a consequence of dipole-dipole interaction
- ⋆ TOF dynamics for magnetic field with arbitrary orientation all the way from from collisionless to hydrodynamic regime
- ⋆ Applying equations of motion for scaling parameters for other non-equilibrium cases, e.g., for a parametric modulation of trap frequencies or strength of dipolar interaction

References

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