Deformation of the Fermi Surface

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Abstract

In the presence of isotropic interactions, the Fermi surface (FS) of an ultracold Fermi gas is spherical. Introducing anisotropic and long-range dipole-dipole interaction (DDI) to the system deforms the Fermi surface to an ellipsoid, as was experimentally observed in a degenerate dipolar Fermi gas of erbium atoms [1]. The deformation is caused by the interplay between the strong magnetic DDI and the Pauli exclusion principle. It was also observed that the atomic cloud follows the rotation of the dipoles when the direction of the external magnetic field is changed, keeping the major axis always parallel to the direction of the maximum attraction of the DDI. Here we present a generalization of the previous Hartree-Fock mean-field theory [2,3], where the magnetic field was assumed to be parallel to one of the harmonic trap axes. We now extend our calculations for an arbitrary orientation of the magnetic field. In order to obtain the ground state and analyze the resulting deformation of the Fermi surface, we minimize the total energy of the system, which enables us to determine its Thomas-Fermi radii and momenta. These analytical and numerical calculations are in agreement with observations from the Innsbruck experiment [1] and are relevant for understanding similar ongoing experiments with ultracold fermionic dipolar atoms.

Experiment

- ⋆ Recent experiment [1] measured FS deformation in atomic Er sample
- ⋆ Number of atoms and trap frequencies - two sets of parameters:

^{167}Er	$N(\times 10^4)$	ω_x (Hz)	ω_y (Hz)	ω_z (Hz)
Case 1	6.7	579	91	611
Case 2	6.3	428	91	459

- ⋆ The imaging angle is $\alpha = 28°$
- ⋆ The orientation of the external magnetic field $\mathbf{B}$ is described by the spherical angles θ and $\varphi = 14°$
- ⋆ The system is in the collisionless regime at very low temperature

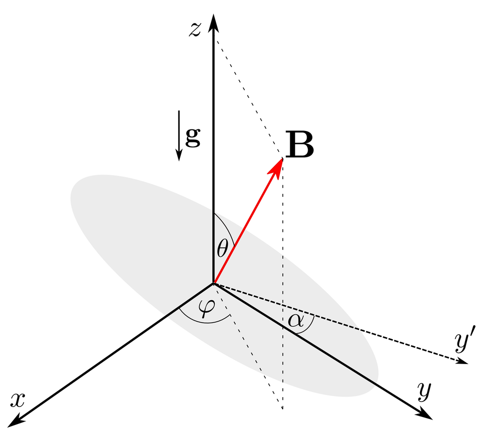

Fig. 1. Experimental setup in Ref. [1].

Global equilibrium and total energy

- ⋆ Ansatz for the Wigner distribution function in global equilibrium is a Heaviside function

$$\nu^0(\mathbf{r},\mathbf{k}) = \mathrm{H}\left(1 - \sum_{i,j} r_i \mathbb{A}_{ij} r_j - \sum_{i,j} k_i \mathbb{B}_{ij} k_j\right),$$

where matrix elements $\mathbb{A}_{ij}$ ($\mathbb{B}_{ij}$) describe the atomic cloud (Fermi surface) in real (momentum) space

- ⋆ Matrix $\mathbb{A}$ is determined by the trap potential, while matrix $\mathbb{B}$ is determined by the orientation of dipoles, and has a diagonal form $\mathbb{B}' = \mathbb{R}^T \mathbb{B} \mathbb{R}$ in the rotated frame

$$\mathbb{A} = \begin{pmatrix} R_x^2 & 0 & 0 \\ 0 & R_y^2 & 0 \\ 0 & 0 & R_z^2 \end{pmatrix}, \quad \mathbb{B}' = \begin{pmatrix} K_x'^2 & 0 & 0 \\ 0 & K_y'^2 & 0 \\ 0 & 0 & K_z'^2 \end{pmatrix}, \quad \mathbb{R} = \begin{pmatrix} \cos\theta\cos\varphi & -\sin\varphi & \sin\theta\cos\varphi \\ \cos\theta\sin\varphi & \cos\varphi & \sin\theta\sin\varphi \\ -\sin\theta & 0 & \cos\theta \end{pmatrix},$$

where R_i and K_i' denote equilibrium radii and momenta of the atomic cloud and FS in direction i

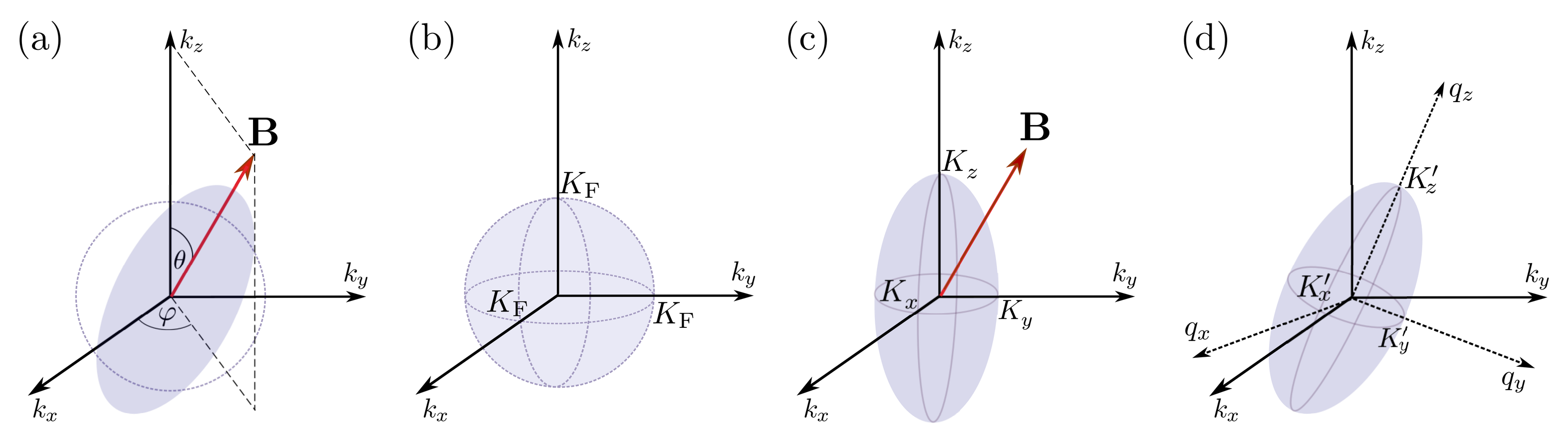

Fig. 2. Schematic illustrations of various ansaätze for the deformed Fermi surface.

- ⋆ The total number of fermions is

$$N = \frac{1}{48} R_x R_y R_z K_x' K_y' K_z'$$

- ⋆ Total energy of the system in the Hartree-Fock mean-field theory

$$E = \frac{N}{8}\sum_j \left(\frac{\hbar^2 K_j'^2}{2M} + \frac{M\omega_j^2 R_j^2}{2}\right) + \frac{48N^2 c_0}{8\bar{R}^3} f\left(\frac{K_z'}{K_x'}, \frac{K_z'}{K_y'}\right) - \frac{48N^2 c_0}{8\bar{R}^3}\left[\mathbb{R}_{xz}^2 f\left(\frac{R_y}{R_x}, \frac{R_z}{R_x}\right) + \mathbb{R}_{yz}^2 f\left(\frac{R_x}{R_y}, \frac{R_z}{R_y}\right) + \mathbb{R}_{zz}^2 f\left(\frac{R_x}{R_z}, \frac{R_y}{R_z}\right)\right],$$

where c_0 is the interaction strength and $f(x,y)$ is anisotropy function [**?**, 2, 3]

- ⋆ The variational parameters R_i and K_i' are determined by minimizing the total energy
- ⋆ Cylindrical symmetry remains in a plane perpendicular to the direction of the external field, i.e., Thomas-Fermi momenta in that plane are equal ($K_x' = K_y'$)
- ⋆ Deformation of the Fermi surface is described by $\Delta = K_z'/K_x' - 1$
- ⋆ Total energy of the ideal noninteracting trapped Fermi gas is $E_0 = \frac{3}{4} N E_{\mathrm{F}} = \frac{3}{4} N \hbar\bar{\omega}(6N)^{1/3}$
- ⋆ Different models compared in terms of relative deviation of the total energy $\delta E = \frac{E - E_0}{E_0}$

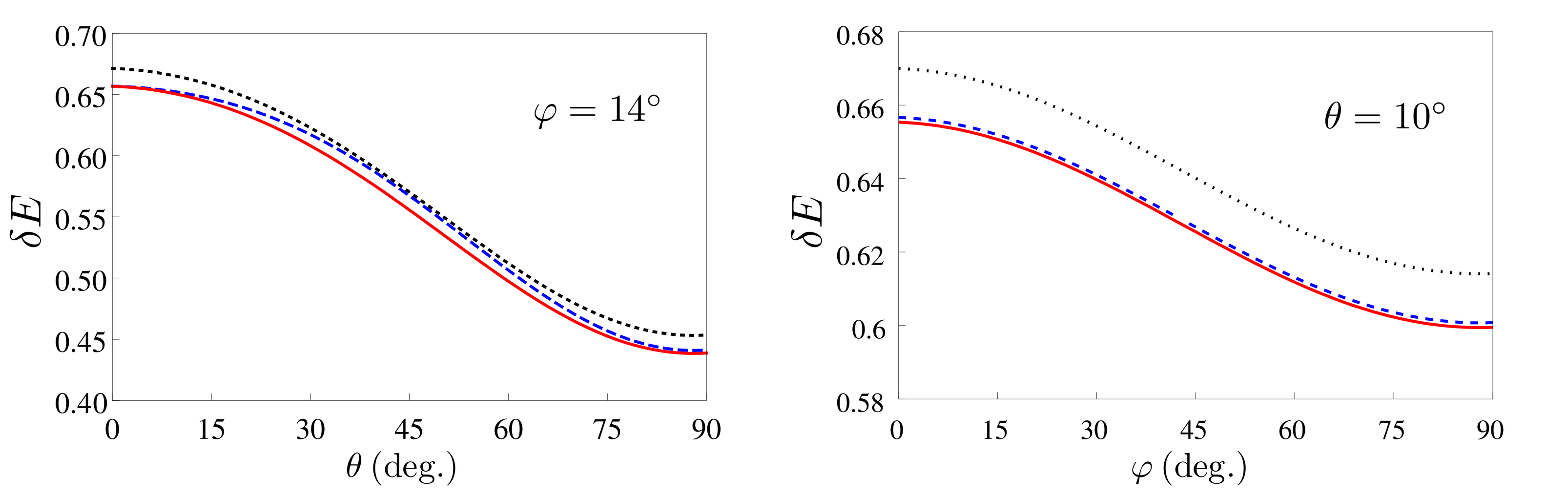

Fig. 3. δE as a function of θ (left) and φ (right) for Case 1. Dotted black lines correspond to the model depicted in Fig. 2(b), blue dashed lines to the model from Fig. 2(c), and solid red lines to the model from Fig. 2(d).

Fermi Surface Deformation

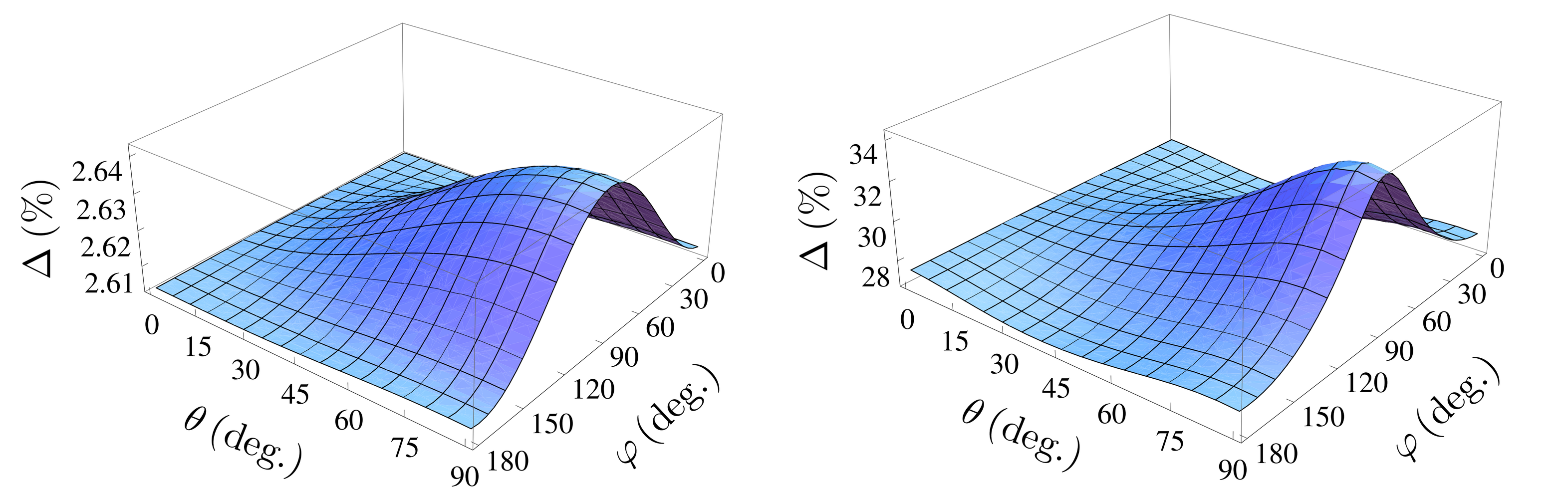

Fig. 4. Angular dependence of the FS deformation Δ for ^{167}Er atoms with the magnetic dipole moment $\mu = 7\,\mu_{\mathrm{B}}$ (left) and ^{40}K^{87}Rb molecules with the electric dipole moment $d = 0.25$ D (right) for Case 1.

- ⋆ Deformation of FS is measured in terms of the momentum-space aspect ratio

$$A_{\mathrm{K}} = \sqrt{\frac{K_x'^2 \sin^2\theta + K_z'^2 \cos^2\theta}{K_x'^2[1 - \sin^2\theta(\cos^2\varphi\cos^2\alpha + \sin^2\varphi\sin^2\alpha)] + K_z'^2 \sin^2\theta(\cos^2\varphi\cos^2\alpha + \sin^2\varphi\sin^2\alpha)}}$$

- ⋆ Time-of-flight ballistic expansion implies $A_{\mathrm{K}} = \lim_{t\to\infty} A_{\mathrm{R}}^{\mathrm{bal}}(t)$

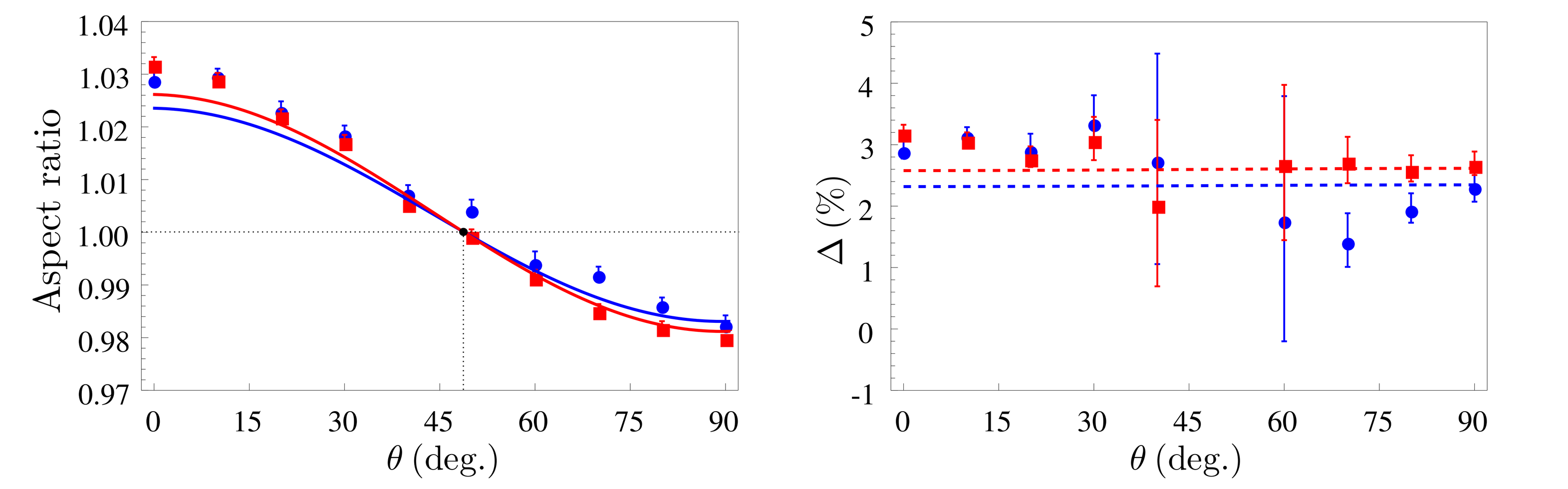

Fig. 5. Theoretical results for A_{K} (solid lines, left) and Δ (dashed lines, right), as well as experimental results (circles and squares) for $A_{\mathrm{R}}(t = 12\,\mathrm{s})$ for ^{167}Er. Red lines and symbols correspond to Case 1, blue ones to Case 2.

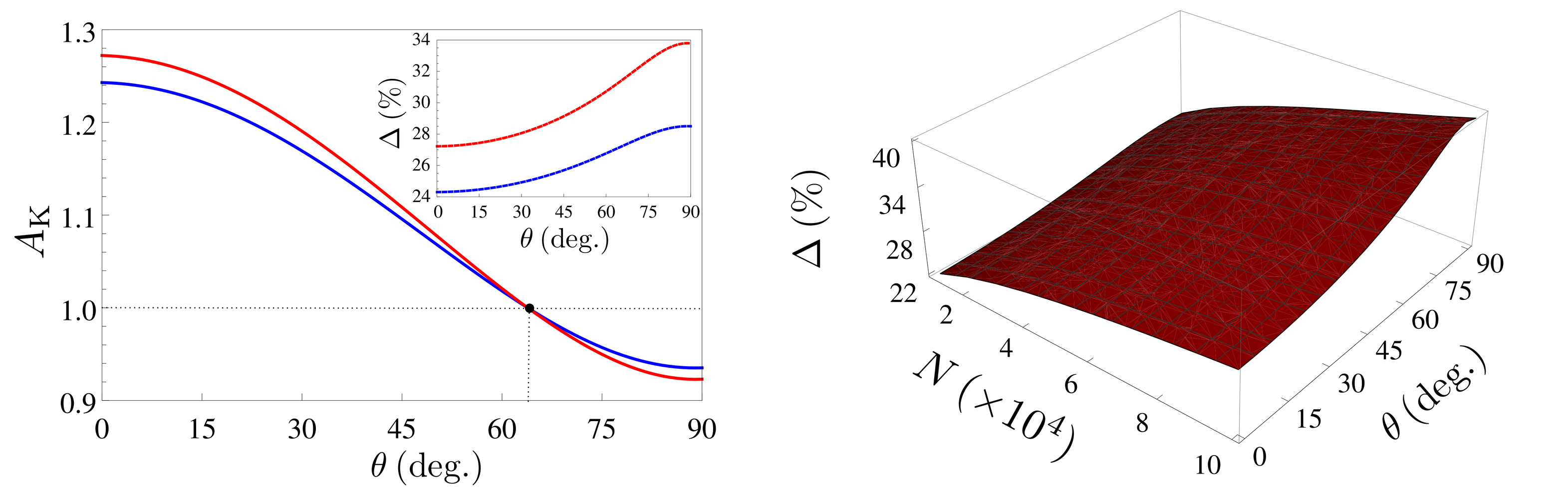

Fig. 6. Left: theoretical results for A_{K} and Δ (inset) for ^{40}K^{87}Rb molecules for $\varphi = 90°$ in Case 1 (red) and Case 2 (blue). Right: FS deformation Δ as a function of the number of molecules N and θ for fixed $\varphi = 90°$ in Case 1.

- ⋆ Intersection point $A_{\mathrm{K}} = 1$, taking into account that $K_z' > K_x'$, implies

$$\sin^2\theta^* = \frac{1}{1 + \cos^2\varphi^*\cos^2\alpha + \sin^2\varphi^*\sin^2\alpha}$$

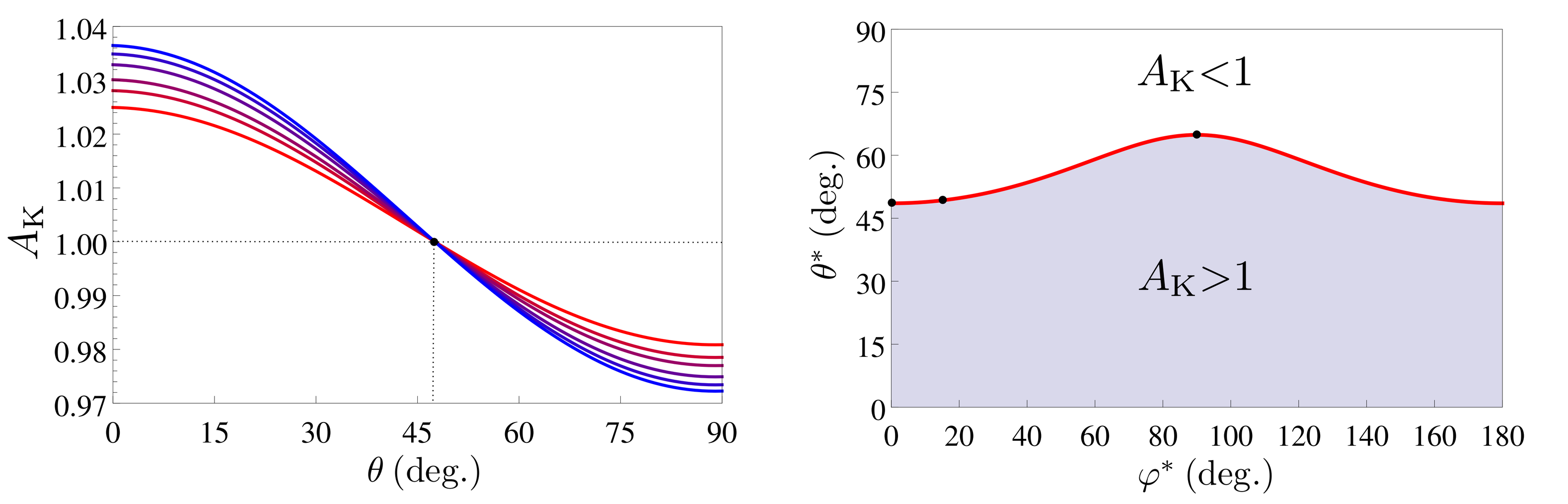

Fig. 7. Left: aspect ratio A_{K} for ^{167}Er as a function of angle θ for $\varphi = 0°$, $N = 7 \times 10^4$ and different trap geometries, $f_x = f_z = 500$ Hz, $f_y = 100n$ Hz, $n \in \{1, 2, 3, 5, 7, 9\}$ (top to bottom on the left hand side). Right: angle θ^* as a function of angle ϕ^*.

Conclusions and outlook

- ⋆ Fermi surface deformation is a consequence of the dipole-dipole interaction
- ⋆ Description of the TOF dynamics of a system with arbitrary orientation of the external field, from the collisionless to the hydrodynamic regime
- ⋆ Obtained equations of motion for scaling parameters applicable to other non-equilibrium scenarios, e.g., parametric modulation of trap frequencies or strength of the dipolar interaction

References

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