Quantum Field Theory and Quantum-Monte Carlo Approaches to Bose-Einstein Condensation

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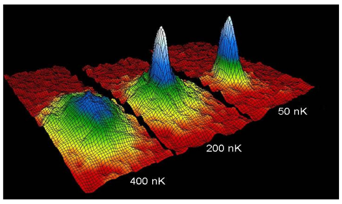
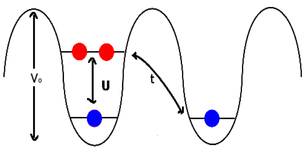
1. Introduction

Since 1995, when Bose-Einstein condensation in ultracold atomic gases has been realized experimentally, there has been quite a number of substantial breakthroughs [1-4]. Today, systems of ultracold bosonic and/or fermionic quantum gases allow for experimental control on a very high level concerning, e.g. the underlying trap geometry and the interaction strength. Moreover, they lend themselves to precise theoretical calculations of their static and dynamic properties, thus leading to highly accurate comparisons of experiment and theory. Ultracold atomic and molecular matter can be employed to provide practically ideal realizations of paradigmatically important many-body models considered in various fields, such as atomic and molecular physics, solid-state physics, and even nuclear physics. Our present Volkswagen project investigates with optical lattices and dirty bosons two of these examples where a detailed study of ultracold atomic matter provides important insights into condensed matter physics.

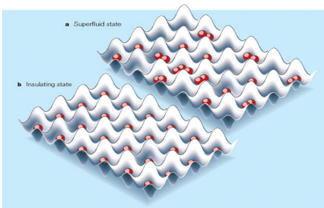


2. Optical Lattices

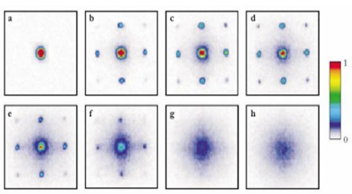
Bose-Hubbard model [5,6]:



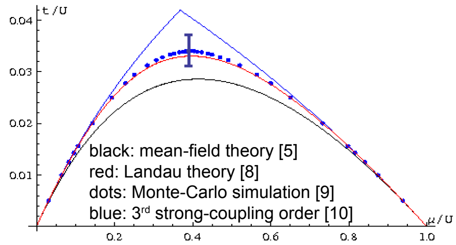
Quantum phase transition: theoretical



Quantum phase transition: experimental [7]



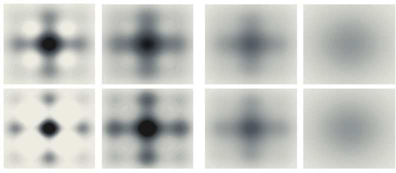
Strong-coupling expansion: Landau theory [8]



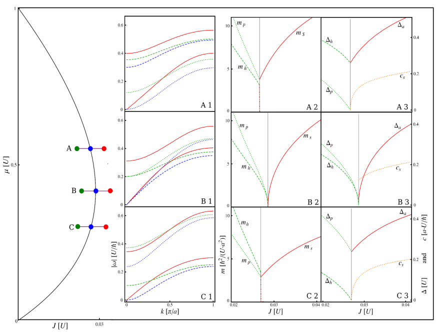
Extension to higher orders [11-13] and finite temperature [14]

Our first subproject: weak-coupling expansion

1) Bogoliubov theory does not yield quantum phase transition [15]
2) Next order is calculated perturbatively [16]
3) Variational resummation [17-22]
4) Time-of-flight absorption picture [23]

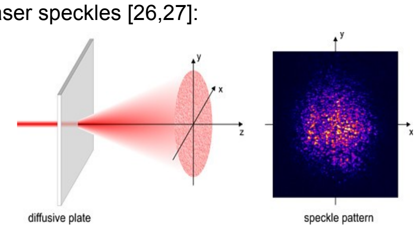


5) Application: thermometer
6) Excitation spectra [24,25]:

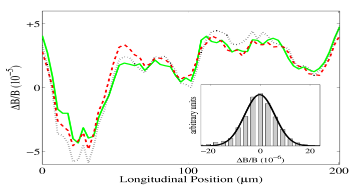


3. Dirty Bosons

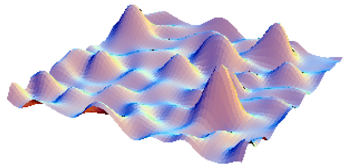
Laser speckles [26,27]:



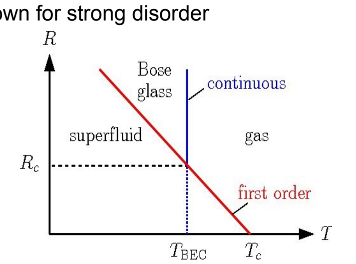
Wire traps [28,29]:



Disorder potential: fragmented condensates



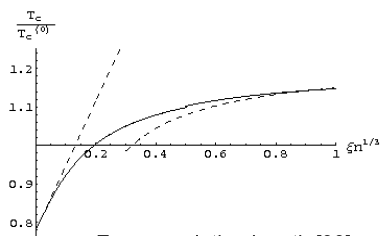
Qualitative phase diagram [30]:
Unknown for strong disorder



superfluid order parameter [31]
Bose-glass order parameter [30] in analogy to Edwards-Anderson order parameter for spin glasses [32]

Our second subproject: critical temperature

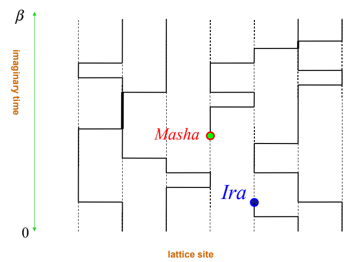
1) Perturbative shift is calculated



2) Non-perturbative term possible [34,35]
3) Occurrence of infrared divergencies is at present investigated
4) Variational resummation [17-22]

Our third subproject: numerical Approaches

1) Quantum Monte-Carlo method: lattice path-integrals for bosons [36,37]
2) Worm algorithm: draw and erase [38-40]



3) Calculate numerically critical temperatures for vanishing of condensate and superfluid density
4) Compare with analytical results from second subproject

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