Low-Lying Excitation Modes of a Dipolar Fermi Gas: From Collisionless to Hydrodynamic Regime

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Motivation: By means of the Boltzmann-Vlasov equation (BVE) we investigate dynamical properties of a trapped dipolar Fermi gas at zero temperature. In order to determine an approximative solution, we follow Ref. [1] and rescale both space and momentum variables, thus obtaining ordinary differential equations for the respective scaling parameters. Then, we procced by linearizing these equations around the equilibrium in order to study the low-lying excitations of the system. Within the relaxation-time approximation for the collisional integral, our approach is able to describe the low-lying excitations all the way from the collisionless [2] to the hydrodynamic [3,4] regime.

Equation of Motion of the Fermi Gas

⋆ Many-body Hamiltonian

$$\hat{H} = \int d^3r \hat{\psi}^\dagger(\mathbf{r}) \left[-\frac{\hbar^2\nabla^2}{2m} + U(\mathbf{r}) \right] \hat{\psi}(\mathbf{r}) + \frac{1}{2} \int d^3r d^3r' V_{\text{int}}(\mathbf{r}-\mathbf{r}') \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}^\dagger(\mathbf{r}') \hat{\psi}(\mathbf{r}') \hat{\psi}(\mathbf{r})$$

⋆ Field operators satisfy anticommutation relations

$$\{\hat{\psi}(\mathbf{r}), \hat{\psi}^\dagger(\mathbf{r}')\} = \delta(\mathbf{r}-\mathbf{r}'), \quad \{\hat{\psi}(\mathbf{r}), \hat{\psi}(\mathbf{r}')\} = 0, \quad \{\hat{\psi}^\dagger(\mathbf{r}), \hat{\psi}^\dagger(\mathbf{r}')\} = 0$$

⋆ Dynamics described by Wigner function

$$\nu(\mathbf{r},\mathbf{p},t) = \int d^3s e^{\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{s}} \text{Tr}\left[\hat{\rho}(t) \hat{\psi}^\dagger\left(\mathbf{r}+\frac{\mathbf{s}}{2}\right) \hat{\psi}\left(\mathbf{r}-\frac{\mathbf{s}}{2}\right) \right]$$

spatial density is given by

$$n(\mathbf{x},t) = \int \frac{d^3k}{(2\pi)^3} \nu(\mathbf{x},\mathbf{k},t)$$

⋆ Time evolution of ν governed by Boltzmann-Vlasov equation derived by applying perturbation theory in interaction up to second order and by assuming a weakly varying trapping potential [5]

$$\left\{ \frac{\partial}{\partial t} + \frac{\mathbf{p}}{m}\cdot\frac{\partial}{\partial \mathbf{r}} - \frac{\partial\left[U(\mathbf{r}) + U_{\text{mf}}(\mathbf{r},\mathbf{p})\right]}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{p}} + \frac{\partial U_{\text{mf}}(\mathbf{r},\mathbf{p})}{\partial \mathbf{p}} \cdot \frac{\partial}{\partial \mathbf{r}} \right\} \nu(\mathbf{r},\mathbf{p},t) = I_{\text{coll}}[\nu](\mathbf{r},\mathbf{p},t)$$

with Hartree-Fock mean field terms

$$U_{\text{mf}}(\mathbf{r},\mathbf{p}) = \int d^3x V_{\text{int}}(\mathbf{r}-\mathbf{x}) n(\mathbf{x},t) - \int \frac{d^3p'}{(2\pi\hbar)^3} \nu(\mathbf{r},\mathbf{p}',t) \tilde{V}_{\text{int}}(\mathbf{p}-\mathbf{p}')$$

⋆ Description of collision integral in terms of relaxation time approximation [6]

$$I_{\text{coll}} = -\frac{\nu - \nu^{\text{le}}}{\tau}$$

with relaxation into local equilibrium $I_{\text{coll}}[\nu^{\text{le}}] = 0$

Scaling Ansatz

⋆ Perform the scaling ansatz with global equilibrium ν^0 [1]

$$\nu(\mathbf{x},\mathbf{q},t) \to \Gamma \nu^0\left(\mathbf{r}(t),\mathbf{k}(t)\right)$$

with rescaled variables $r_i = \frac{x_i}{b_i(t)}$, $k_i = \frac{1}{\Theta_i^{\frac{1}{2}}(t)}\left[q_i - \frac{m\dot{b}_i(t)x_i}{\hbar b_i(t)}\right]$ and factor $\Gamma^{-1} = \prod_i b_i \Theta_i^{\frac{1}{2}}$

⋆ Taking moments of the rescaled BVE leads to ordinary differential equations for respective scaling parameters ($r_i k_i$ for b_i's and k_i^2 for Θ_i's)

⋆ Ansatz for the global equilibrium distribution function ν^0

$$\nu^0(\mathbf{r},\mathbf{k}) = \Theta\left(1 - \sum_j \frac{r_j^2}{R_j^2} - \sum_j \frac{k_j^2}{K_j^2}\right)$$

⋆ Consider dipole-dipole interaction and harmonic trap

$$V_{\text{int}}(\mathbf{x}) = \frac{C_{\text{dd}}}{4\pi\left|\mathbf{x}\right|^3}\left(1 - 3\cos^2\vartheta\right) \qquad U(\mathbf{x}) = \frac{m}{2}\sum_j \omega_j^2 x_j^2$$

where $C_{\text{dd}} = \mu_0 m^2$ in magnetic case and $C_{\text{dd}} = 4\pi d^2$ in electric case with electric (d) and magnetic dipole moments (m)

⋆ Differential equations for scaling parameters

$$\ddot{b}_i + \omega_i^2 b_i - \frac{\hbar^2 K_i^2 \Theta_i}{m^2 b_i R_i^2} + \frac{48Nc_0}{m b_i R_i^2 \prod_j b_j R_j}\left[f\left(\frac{b_x R_x}{b_z R_z}, \frac{b_y R_y}{b_z R_z}\right) - b_i R_i \frac{\partial}{\partial b_i R_i} f\left(\frac{b_x R_x}{b_z R_z}, \frac{b_y R_y}{b_z R_z}\right)\right]$$

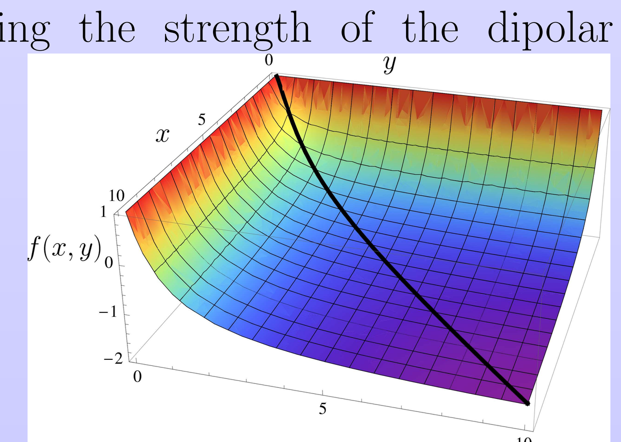
$$- \frac{48Nc_0}{m b_i R_i^2 \prod_j b_j R_j}\left[f\left(\frac{\Theta_z^{\frac{1}{2}}K_z}{\Theta_x^{\frac{1}{2}}K_x}, \frac{\Theta_z^{\frac{1}{2}}K_z}{\Theta_y^{\frac{1}{2}}K_y}\right) + \Theta_i^{\frac{1}{2}}K_i \frac{\partial}{\partial \Theta_i^{\frac{1}{2}}K_i} f\left(\frac{\Theta_z^{\frac{1}{2}}K_z}{\Theta_x^{\frac{1}{2}}K_x}, \frac{\Theta_z^{\frac{1}{2}}K_z}{\Theta_y^{\frac{1}{2}}K_y}\right)\right] = 0,$$

$$\dot{\Theta}_i + 2\frac{\dot{b}_i}{b_i}\Theta_i = -\frac{1}{\tau}\left\{\Theta_i - \Theta_i^{\text{le}} - \frac{< r_i^2 >^0 m^2}{< k_i^2 >^0 \hbar^2}\left[\dot{b}_i^{\text{le}} - \frac{\dot{b}_i b_i^{\text{le}}}{b_i}\right]^2\right\},$$

where f is the anisotropy function, ω_i the trap frequency in the i-th direction and coupling constant c_0 measuring the strength of the dipolar interaction

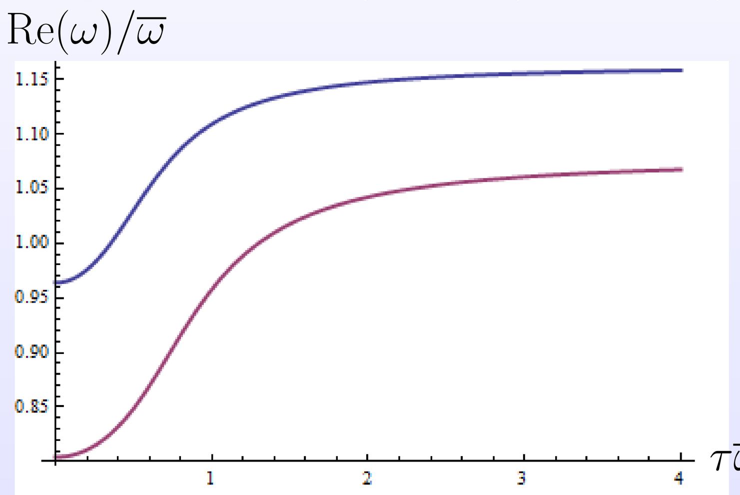
⋆ Determining the scaling parameters Θ_i^{le} via a minimization of Hartree-Fock energy

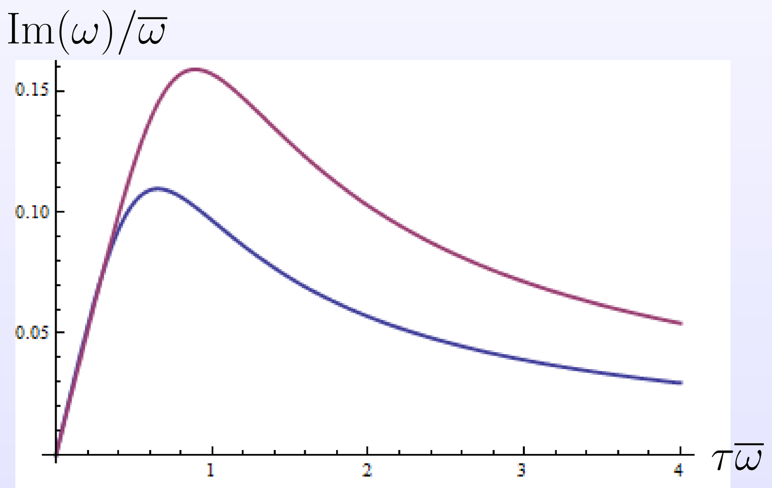
⋆ Plot of the anisotropy function [7–10]



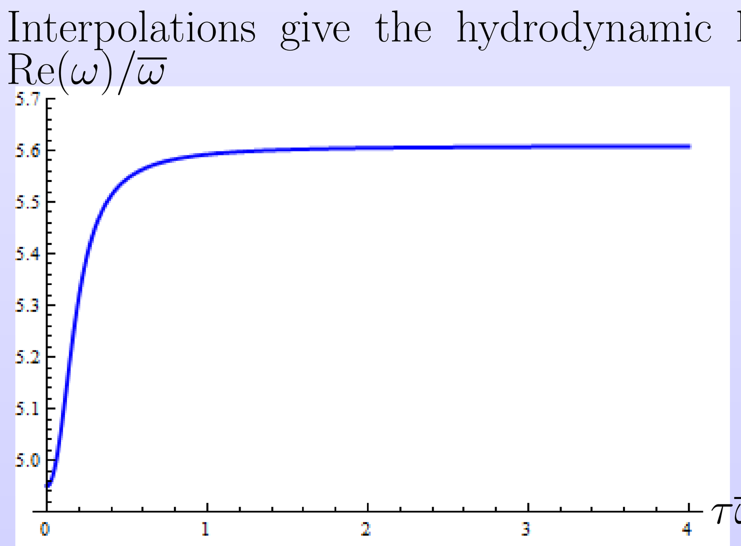
Frequency-interpolation under assumption of collisions do not change spatial distribution ($b_i = b_i^{\text{le}}$)

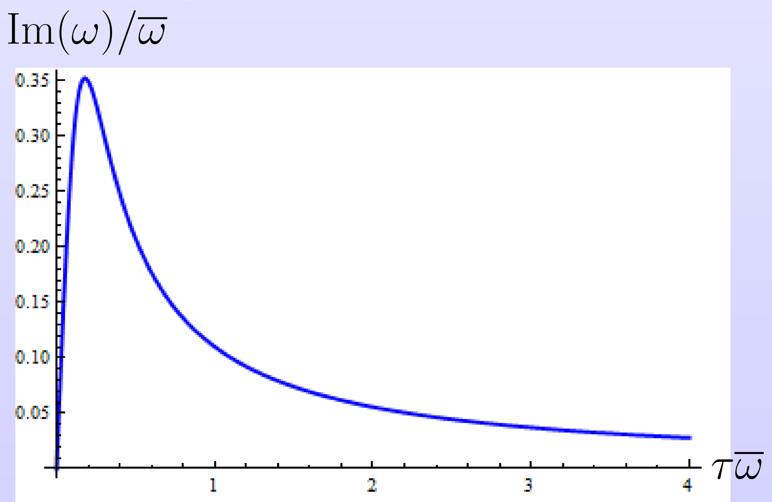
⋆ Plots of real and imaginary parts of three-dimensional quadrupole, radial quadrupole and monopole mode for cylinder symmetric system over relaxation time





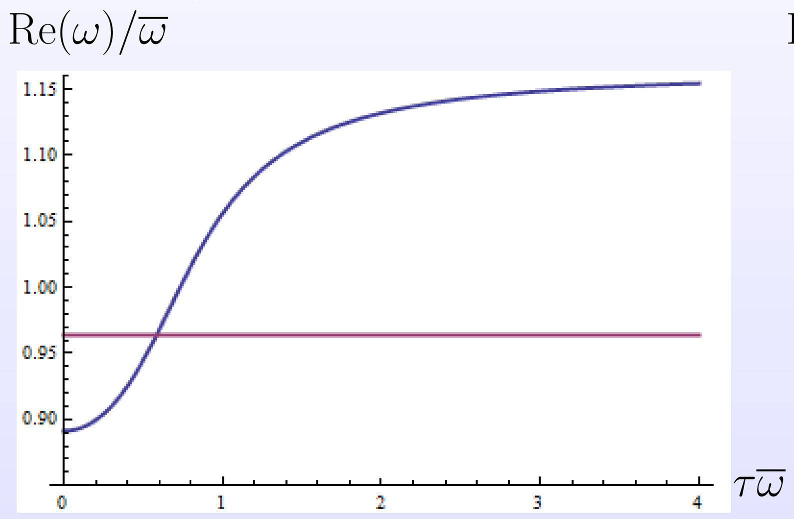
⋆ Interpolations give the hydrodynamic limit of [3,4] and the collisionless limit of [2]

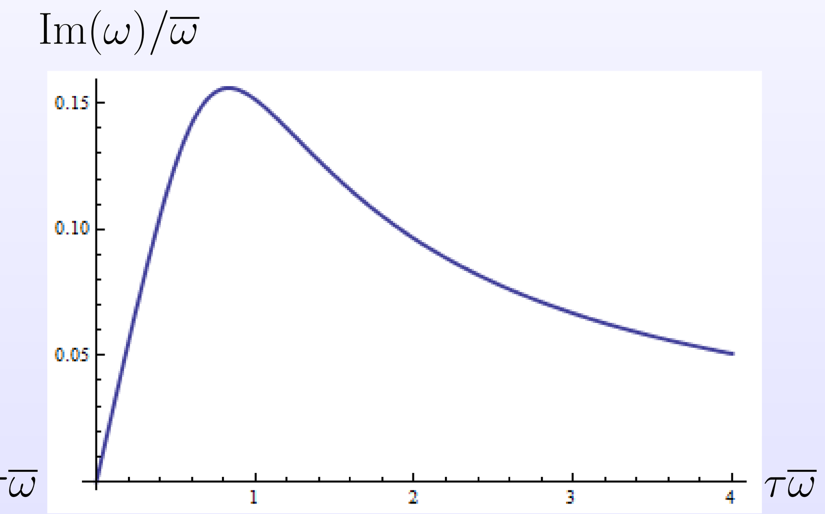




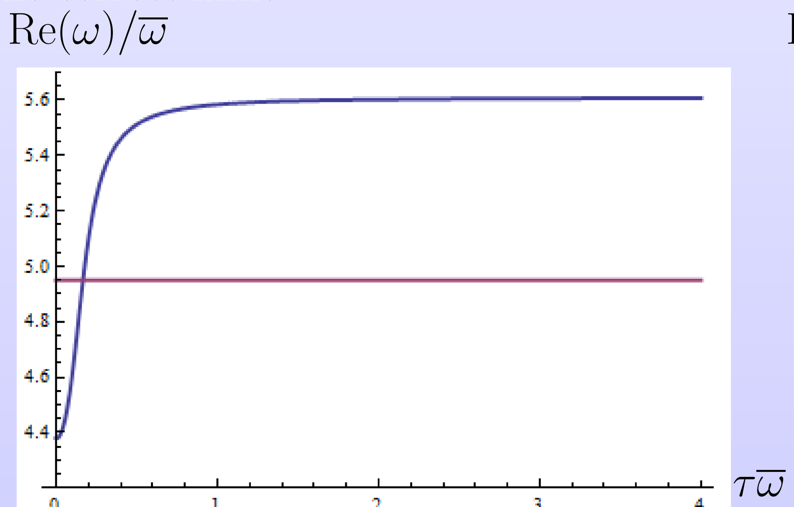
Frequency-interpolation under assumption of collisions can change spatial distribution ($b_i \neq b_i^{\text{le}}$)

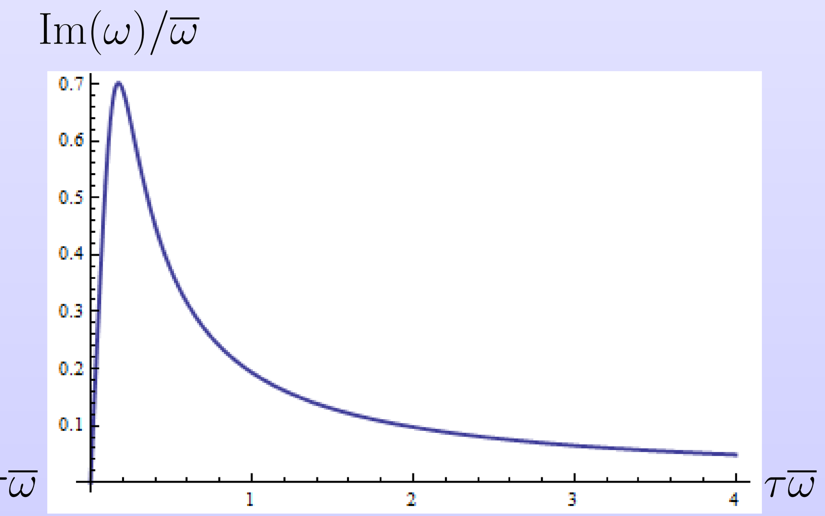
⋆ Plots of real and imaginary part of three-dimensional quadrupole and monopole mode for a cylinder symmetric system over relaxation time





⋆ The hydrodynamic limit is different from [3,4], so that we included horizontal lines showing the correct limit





Numerical values: $\lambda = 5$ and dimensionless dipole-dipole strength $\epsilon_{\text{dd}} = \frac{C_{\text{dd}}}{4\pi}\sqrt{\frac{m^3\overline{\omega}}{\hbar^5}}N^{\frac{1}{6}} = \frac{4}{3}$ where $\overline{\omega}$ is the geometric average of trap fequencies

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References

[1] P. Pedri, D. Guéry-Odelin, and S. Stringari. Dynamics of a classical gas including dissipative and mean-field effects. *Phys. Rev. A* **68**, 043608 (2003).

[2] T. Sogo, L. He, T. Miyakawa, S. Yi, H. Lu, and H. Pu. Dynamical properties of dipolar Fermi gases. *New J. Phys.* **11**, 055017 (2009).

[3] A. R. P. Lima and A. Pelster. Collective Motion of polarized dipolar Fermi gases in the hydrodynamic regime. *Phys. Rev. A* **81**, 021606(R) (2010).

[4] A. R. P. Lima and A. Pelster. Dipolar Fermi gases in anisotropic traps. *Phys. Rev. A* **81**, 063629 (2010).

[5] E. Zaremba, T. Nikuni, and A. Griffin. Dynamics of Trapped Bose Gases at Finite Temperatures. *J. Low Temp. Phys.* **116**, 277 (1999).

[6] A. Griffin, T. Nikuni, and E. Zaremba. *Bose-Condensed Gases at Finite Temperatures*. Cambridge University Press (2009).

[7] K. Glaum, A. Pelster, H. Kleinert, and T. Pfau. Critical Temperature of Weakly Interacting Dipolar Condensates. *Phys. Rev. Lett.* **98**, 080407 (2007).

[8] K. Glaum and A. Pelster. Bose-Einstein Condensation Temperature of Dipolar Gas in Anisotropic Harmonic Trap. *Phys. Rev. A* **76**, 023604 (2007).

[9] A. R. P. Lima and A. Pelster. Quantum Fluctuations in Dipolar Bose Gases. *Phys. Rev. A* **84**, 041604(R) (2011).

[10] A. R. P. Lima and A. Pelster. Beyond Mean-Field Low-Lying Excitations of Dipolar Bose Gases. *arXiv*:1111.0900.