Mapping slow evolution onto periodic driving of Bose gases in the mean-field regime

Etienne Wamba, Axel Pelster and James R. Anglin

Fachbereich Physik and Research Center OPTIMAS, TU Kaiserslautern, Germany

Abstract

By using our recent results on the exact mappings between time evolutions of different quantum many-body systems [1], we construct a mean-field model of many-body systems with rapid periodic driving. The single-particle potential and the inter-particle interaction strength are both time-dependent at once, in a related way. We map the evolutions of the model system onto evolutions with slowly varying parameters. Such a mapping between a Floquet evolution and a static or slow process allows us to investigate non-equilibrium many-body dynamics and examine how rapidly driven systems may avoid heating up, at least when mean-field theory is still valid. From that special but interesting case, we learn that rapid periodic driving may not yield heating because the time evolution of the system has a kind of hidden adiabaticity, inasmuch as it can be mapped exactly onto that of an almost static system.

Motivation

Current experiments with periodically driven systems show that in some interesting regimes, the system will not heat up while an appropriate driving is acting on it [2]. Besides, some experiments with periodically modulated interactions are harder to achieve due to the delicacy of the Feshbach resonance width.

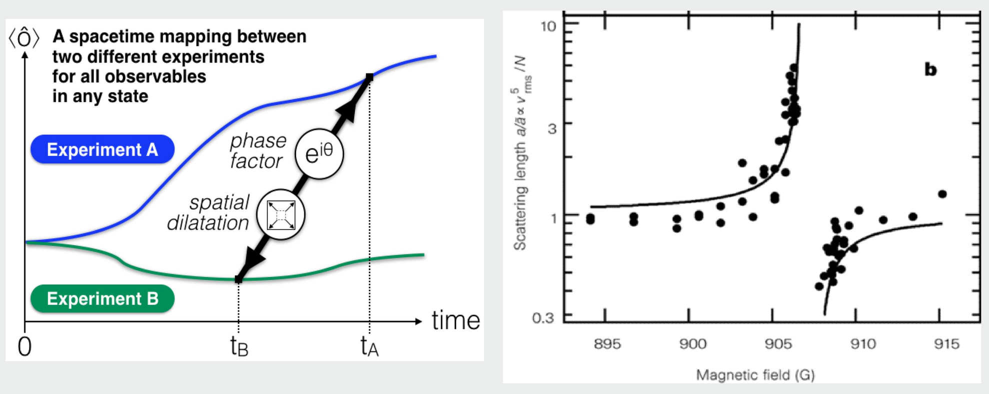

Figure 1: Mapping of two experiments (left) and Feshbach resonance (right).

Can we apply our mapping in [1] to
- probe how heating can be avoided by periodically driven systems?
- study the detailed dynamics of systems with periodic interactions from static ones?

Our mapping reveals that among experiments with periodically driven systems, it exists a special class where the system does not heat up, because it exactly corresponds to a static evolution (with obviously no heating), and thereby showing that heating will not occur in the driven system.

Equation of motion

The dynamics of the system can be described through the Heisenberg equation of motion for the bosonic field operator $\hat{\psi}(\mathbf{r}, t)$ as follows

$$i\hbar\frac{\partial}{\partial t}\hat{\psi}(\mathbf{r},t) = \left(-\frac{\hbar^2}{2M}\nabla^2 + V(\mathbf{r},t)\right)\hat{\psi}(\mathbf{r},t) + \int d^D\mathbf{r}' U(\mathbf{r},\mathbf{r}',t)\hat{\psi}^\dagger(\mathbf{r}',t)\hat{\psi}(\mathbf{r}',t)\hat{\psi}(\mathbf{r},t). \quad (1)$$

Using the quantum field, any measurement on the quantum gas experiment can be explained in terms of an N-point function

$$\mathcal{F}(\mathbf{R},\mathbf{R}',t) = \left\langle \left[\prod_{i=1}^{N}\hat{\mathbf{\Psi}}^\dagger(\mathbf{r}'_i,t)\right]\left[\prod_{i=1}^{N}\hat{\mathbf{\Psi}}(\mathbf{r}_i,t)\right]\right\rangle, \quad (2)$$

where $\mathbf{R} = (\mathbf{r}_1, \dots, \mathbf{r}_N)$ and $\mathbf{R}' = (\mathbf{r}'_1, \dots, \mathbf{r}'_N)$.

Mapping scheme

- Suppose the interparticle interaction potentials are homogeneous of degree D (contact interaction) in their space component: $U(\lambda\mathbf{r},\mathbf{r}',t) = \lambda^{-D}U(\mathbf{r},\mathbf{r}',t)$, with $U(\mathbf{r},\mathbf{r}',t) = g(t)\delta(\mathbf{r}-\mathbf{r}')$.
- Consider a first quantum field $\hat{\psi}_B$ (describing Experiment B) that evolves following (1), and construct a second quantum field $\hat{\psi}_A$ such that:

$$\hat{\psi}_B(\mathbf{r},t) = \lambda^{D/2}\, e^{-i\frac{M}{2\hbar\lambda}\frac{d\lambda}{dt}\mathbf{r}^2}\, \hat{\psi}_A(\lambda\mathbf{r}, \int_0^t \lambda(t')^2 dt'). \quad (3)$$

  Then $\hat{\psi}_A$ (describing Experiment A) also satisfies (1) with the potentials:
  - $U_A(\mathbf{r},\mathbf{r}',t) \longmapsto U_B(\mathbf{r},\mathbf{r}',t) = \lambda^{2-D}U_A(\mathbf{r},\mathbf{r}',t)$;
  - $V_A(\mathbf{r},t) \longmapsto V_B(\mathbf{r},t) = \lambda^2\left[V_A(\lambda\mathbf{r}, \int_0^t \lambda(t')^2dt') + \frac{1}{2}Mf(t)\mathbf{r}^2\right]$,

  where $f(t) = \lambda\left(\frac{1}{\lambda^2}\frac{d}{dt}\right)^2\lambda$. The function $\lambda(t)$ is then *arbitrary*, except for the initial conditions $\lambda = 1$ and $\dot{\lambda} = 0$ which must be satisfied.
- Let $\omega_A/\omega_B = \gamma$; we choose

$$\lambda(t) = \left[(1-\gamma^2)\cos(2\omega_B t)/2 + (1+\gamma^2)/2\right]^{-1/2} \quad (4)$$

  such that

$$\begin{cases} g_A = \text{const.} \\ V_A = \frac{M}{2}\gamma^2\omega_B^2\mathbf{r}^2 \end{cases} \Longrightarrow \begin{cases} g_B = g_A\lambda(t)^{2-D} \\ V_B = \frac{M}{2}\omega_B^2\mathbf{r}^2 \end{cases} \quad (5)$$

Results: evolution of the density

- We cannot plot quantum fields; but our mapping is valid also for mean fields. Consider a cigar-shaped BEC ($D = 1$) with contact interparticle interactions.
- Numerically solve the corresponding Gross-Pitaevskii equation.

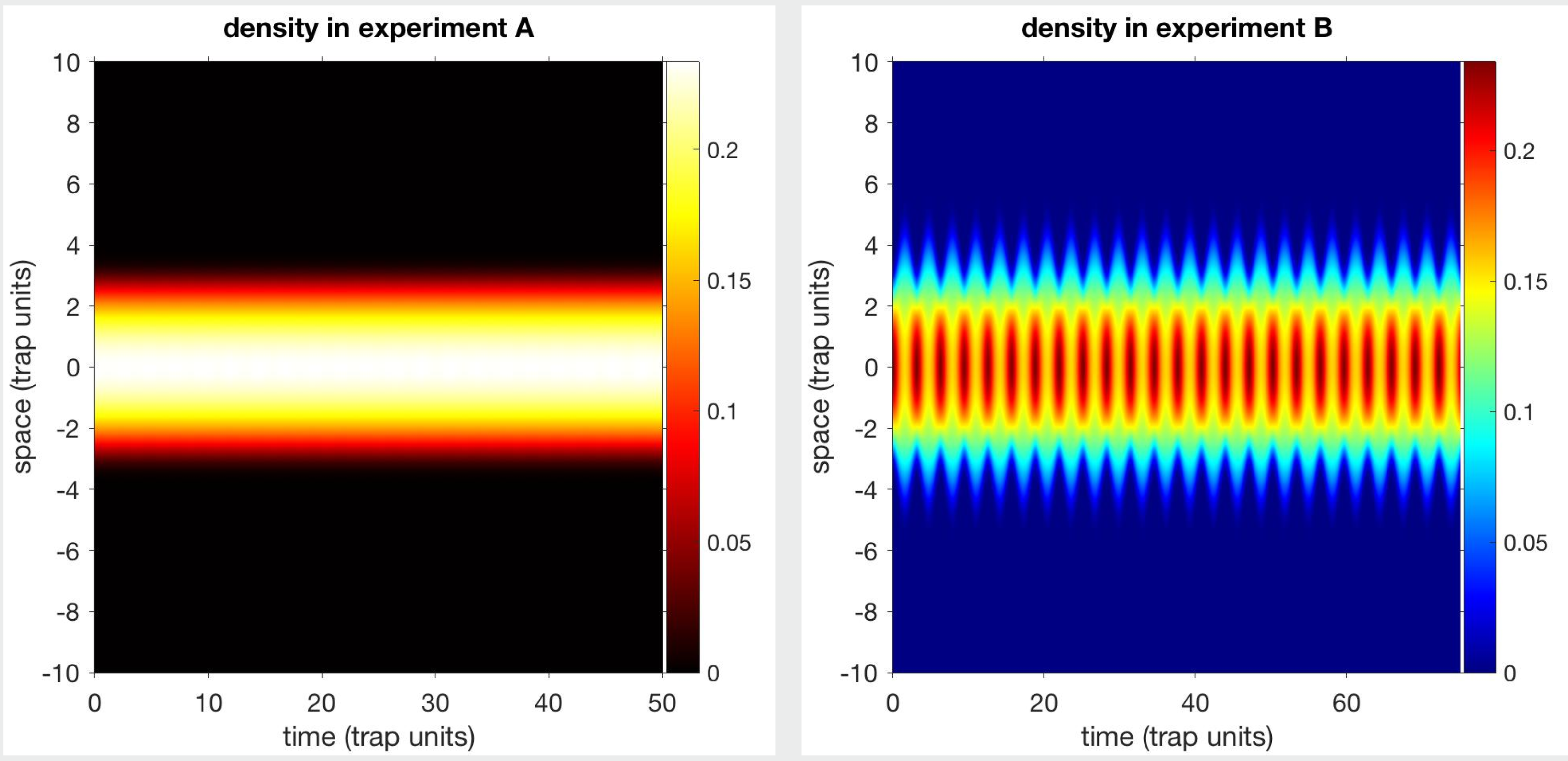

Figure 2: Density evolution in time and space in two different experiments, namely a static problem A (left) and a Floquet problem B (right). We used $\gamma = 1.5, \omega_B = 1, g_A = 1$.

- Starting from the data computed in one experiment, say ρ_A in (A), we may deduce ρ_B using the mapping (3). Then the result ρ_B^{map} is compared to the result ρ_B of direct (numerical) experiment. Repeat the same process for the other experiment.

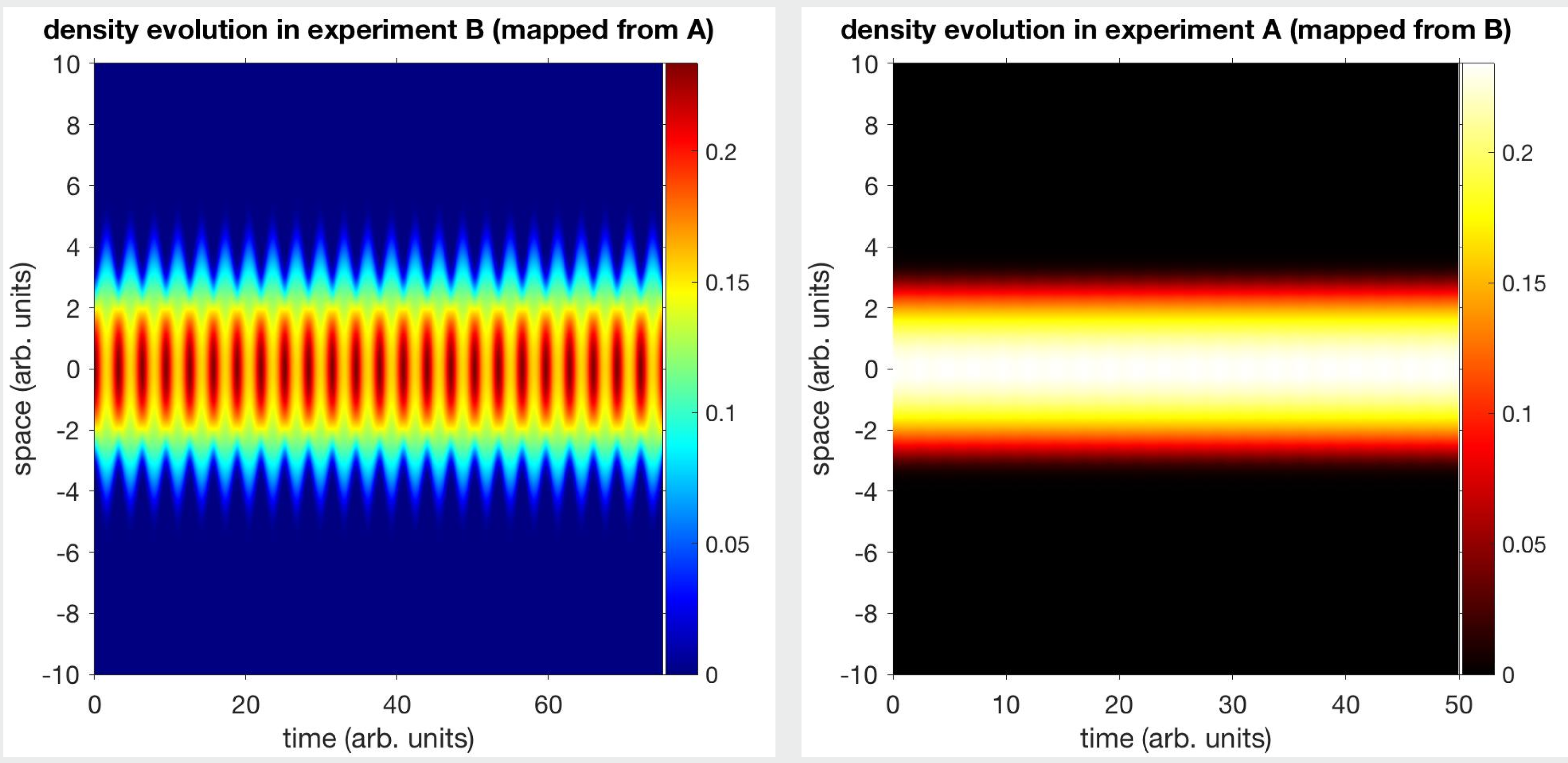

Figure 3: Mapped version of the density evolution in time and space in the two experiments above. Remark that the time and space spans are not the same.

- It clearly appears that ρ_B^{map} and ρ_B are the same; so the mapping exactly reproduces the computed data.
- Since the static evolution A produces no heating, we expect no heating in the periodically driven system B.
- We expect the result to still be valid even if mean-field breaks down.

Conclusion

In summary, our exact mapping for closed quantum systems reveals that a family of experiments with periodically driven systems may avoid heating up, because such experiments can exactly be mapped onto a static evolution. Moreover, our results suggest that a static evolution can mimick an evolution with time dependent interactions.

References & Acknowledgments

[1] E. Wamba, A. Pelster, and J. R. Anglin, Phys. Rev. A **94**, 043628 (2016).

[2] A. Eckardt, Rev. Mod. Phys. **89**, 011004 (2017).

[3] One image comes from Nature **392**, 151 (1998).

Acknowledgments

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Web: http://www.physik.uni-kl.de/anglin/ Email: wamba@rhrk.uni-kl.de