QUANTUM PHASE DIAGRAM OF BOSONS WITH MODULATED SCATTERING LENGTH IN OPTICAL LATTICE

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Abstract

We study a homogeneous system of spinless bosons in a cubic optical lattice of arbitrary dimension, where the s-wave scattering length is periodically modulated with some amplitude and frequency in the vicinity of a Feshbach resonance [1]. To this end we follow Ref. [2] and perform a similar analysis as for shaken lattices in order to map the driven system for large enough frequencies to an effective time-independent one. Subsequently, we calculate the transition line between the Mott-Insulator and Superfluid phase both with a Landau theory extended for the driven system [2,3] and within a Mean-Field theory for the effective time-independent system [4]. Although the respective results deviate from each other, they coincide for a large particle number per site.

Model

The periodically driven system is described by a time-dependent Hamiltonian of the form:

$$\hat{H}(t) = -\sum_{ij} J_{ij}\hat{a}_i^\dagger \hat{a}_j + \sum_i \left[f_i(\hat{n}_i) + A g_i(\hat{n}_i)\cos\omega t\right].$$

The local independent part reads

$$f_i(\hat{n}_i) = \frac{U}{2}\left(\hat{n}_i^2 - \hat{n}_i\right) - \mu \hat{n}_i$$

and the periodic modulation of the s-wave scattering length is modeled by [1]

$$g_i(\hat{n}_i) = \frac{1}{2}\left(\hat{n}_i^2 - \hat{n}_i\right).$$

Within Floquet theory in the extended Hilbert space, in which the time t is regarded as a coordinate, we can find the Floquet functions

$$|n_i, m(t)\rangle = e^{i\omega t}\prod_i e^{-\frac{A g_i(\hat{n}_i)}{\hbar\omega}\sin\omega t}|n_i\rangle.$$

In the large frequency limit [2], transitions between states with $m \neq m'$ are highly suppressed, so we can find an effective time-independent Hamiltonian [4]

$$\hat{H} = -\sum_{ij} J_{ij}\hat{a}_i^+ J_0\left[\frac{A}{\hbar\omega}(\hat{n}_j - \hat{n}_i)\right]\hat{a}_j + \sum_i f_i(\hat{n}_i).$$

Strong-coupling method for low-dimensional system

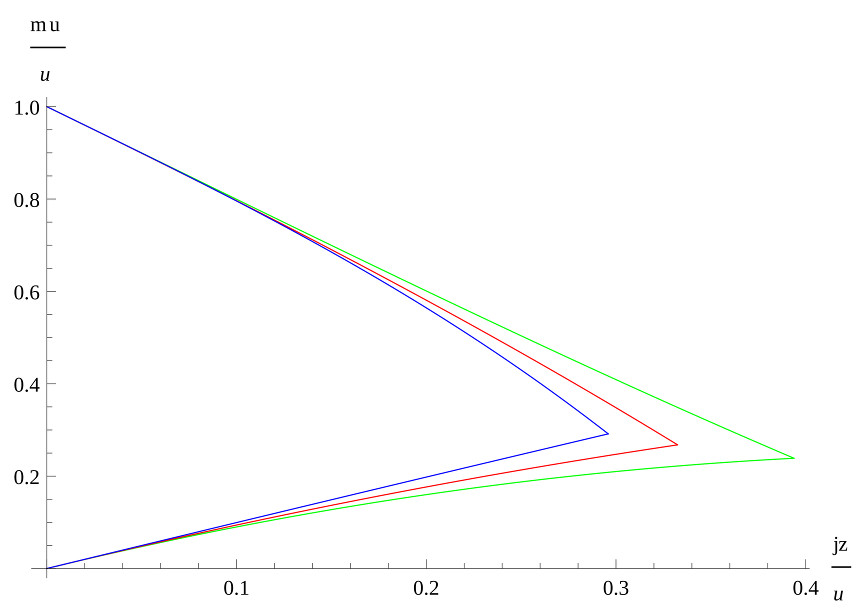
For the filling number $n = 1$, we calculate the 3rd order of strong-coupling expansion method to get the upper phase boundary:

$$\frac{\mu}{U} = 1 - 2\frac{J}{U}z - \frac{J^2}{U^2}\left\{2z^2 J_0^2\left(\frac{A}{\hbar\omega}\right) + z\left[-6J_0^2\left(\frac{A}{\hbar\omega}\right) + 1.5J_0^2\left(\frac{2A}{\hbar\omega}\right)\right]\right\} - \frac{J^3}{U^3}\left\{6z^3J_0^2\left(\frac{A}{\hbar\omega}\right) + z^2\left[6J_0^2\left(\frac{A}{\hbar\omega}\right)J_0\left(\frac{2A}{\hbar\omega}\right) - 24J_0^2\left(\frac{A}{\hbar\omega}\right) - 1.5J_0\left(\frac{2A}{\hbar\omega}\right)\right] + z\left[18 - 6J_0\left(\frac{2A}{\hbar\omega}\right)\right]\right\}$$

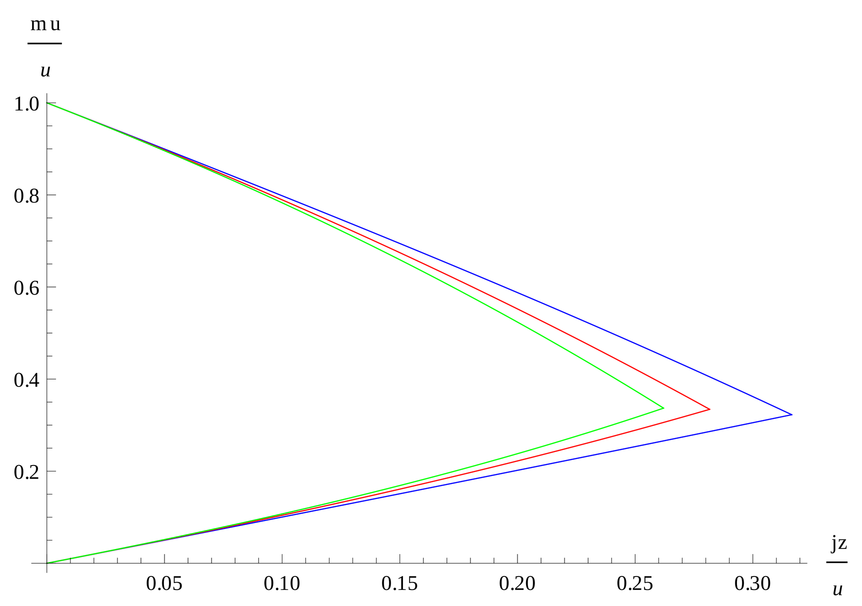
and the lower one:

$$\frac{\mu}{U} = z\frac{J}{U} + \frac{J^2}{U^2}J_0^2\left(\frac{A}{\hbar\omega}\right)\left(2z^2 - 6z\right) - \frac{J^3}{U^3}J_0^2\left(\frac{A}{\hbar\omega}\right)\left(6z^3 - 18z^2 + 12z\right).$$

In the undriven case $A = 0$, the lower and the upper line coincide with the usual ones from Ref. [5]. As the strong-coupling result is good at low dimensions in the undriven case, we use it to calculate the 1d and 2d phase boundary for the driven system:



In the above figure, the green, red, blue line, respectively, represent $\frac{A}{\hbar\omega} = 0$, $\frac{A}{\hbar\omega} = 1$, $\frac{A}{\hbar\omega} = 2$ for one dimension. It represents an unreasonable result as a larger driving amplitude seems to increase the superfluid phase.



Correspondingly, the phase boundary in 2d is reasonable as the picture above shows the green, red, blue line, respectively, for $\frac{A}{\hbar\omega} = 0$, $\frac{A}{\hbar\omega} = 1$, $\frac{A}{\hbar\omega} = 2$.

Effective action theory

In order to include a possible breaking of the system phase symmetry, we include source terms to the effective Hamiltonian $\hat{H}$ [3]

$$\hat{H}(j_i^*, j_i) = -\sum_{ij} J_{ij}\hat{a}_i^\dagger J_0\left[\frac{A}{\hbar\omega}(\hat{n}_j - \hat{n}_i)\right]\hat{a}_j + \sum_i f_i(\hat{n}_i) + \sum_i\left(j_i^*\hat{a}_i + j_i\hat{a}_i^\dagger\right).$$

In first order of J, the correlation function $G_{ij} = \langle\hat{a}_i^\dagger\hat{a}_j\rangle$ is given by $G_{ij} = G^{(0)}\delta_{ij} + G^{(1)}J_{ij}$ with

$$G^{(0)} = \frac{n+1}{f(n) - f(n+1)} + \frac{n}{f(n) - f(n-1)}$$

and

$$G^{(1)} = \left[\frac{n+1}{f(n) - f(n+1)} + \frac{n}{f(n) - f(n-1)}\right]^2 + \frac{2(n+1)n[J_0(\frac{A}{\hbar\omega}) - 1]}{[f(n) - f(n+1)][f(n) - f(n-1)]}.$$

Thus we get the first-order effective hopping

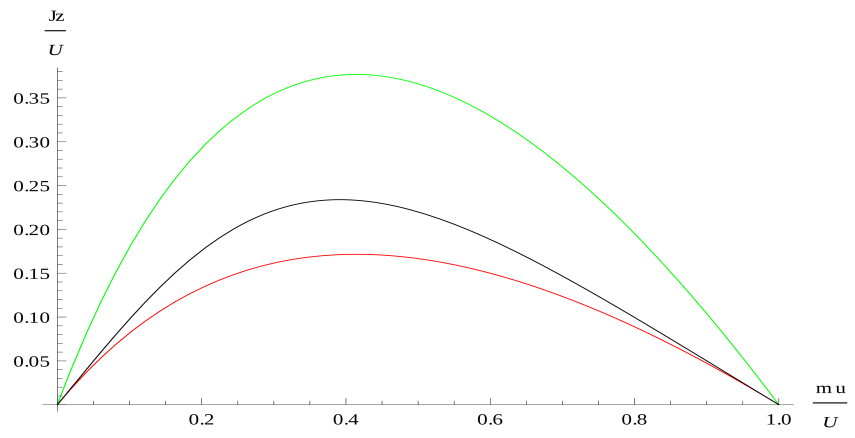
$$J_{ij}^{\text{eff}} = J_{ij}\left(1 + \frac{2(n+1)n[J_0(\frac{A}{\hbar\omega}) - 1]}{[f(n) - f(n+1)][f(n) - f(n-1)]}\left[\frac{n+1}{f(n) - f(n+1)} + \frac{n}{f(n) - f(n-1)}\right]^{-2}\right)$$

which reduces for a large filling number n to the mean-field effective hopping [2]

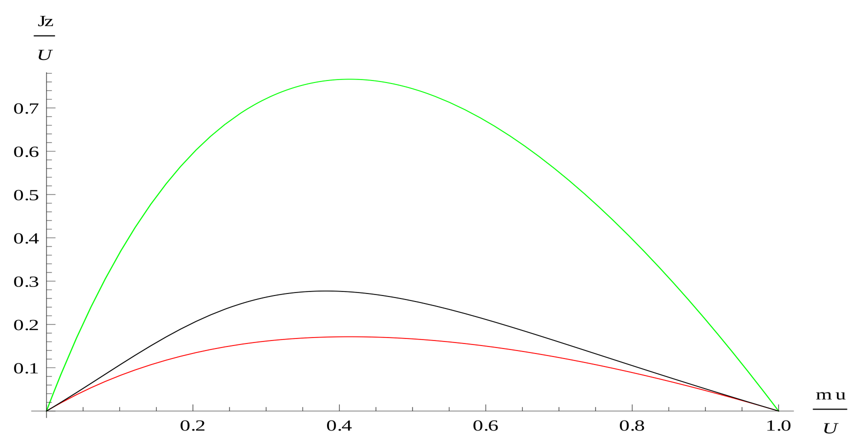
$$J_{ij}^{\text{eff}} = J_{ij}J_0\left(\frac{A}{\hbar\omega}\right).$$

For both cases we get the phase boundary by replacing J by J^{eff}

$$\frac{J^{\text{eff}}z}{U} = \frac{\left(n - \frac{\mu}{U}\right)\left(\frac{\mu}{U} - n + 1\right)}{1 + \frac{\mu}{U}}.$$



The red, green, black line, respectively, represent $\frac{A}{\hbar\omega} = 0$, $\frac{A}{\hbar\omega} = 1.6$ mean-field theory, $\frac{A}{\hbar\omega} = 1.6$ effective action theory for $d = 3$ and $n = 1$. We can see from the picture that the first-order effective action theory result differs from the mean-field theory result. But in the large filling number they coincide.



The red, green, black line, respectively, represent $\frac{A}{\hbar\omega} = 0$, $\frac{A}{\hbar\omega} = 2$ mean-field theory, $\frac{A}{\hbar\omega} = 2$ effective action theory for $d = 2$ and $n = 1$. From the above picture we can see that strong-coupling method and the effective action method get roughly the same critical point, while the mean-field theory result is too large. This suggests that the strong-coupling and the effective action method lead to a reasonable result in 2d.

Results

1. With Floquet theory in the high frequency limit, we get a time-independent effective Hamiltonian for the original driven system.
2. The strong-coupling expansion method seems to be wrong in 1d, but it is reasonable for 2d.
3. The mean-field theory and the first-order effective action theory get a different result, but they coincide for large filling numbers.

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