Anisotropic Superfluidity of Bosons in optical Kagome Superlattice

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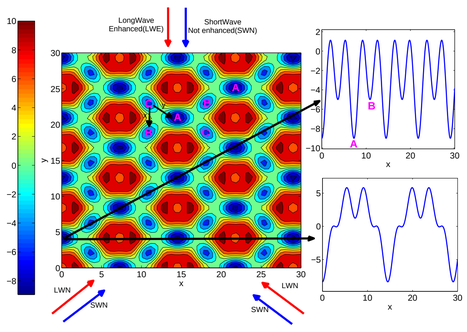
Abstract

We study the boundaries of the quantum phase transitions for the extended Bose-Hubbard model with bosons on a Kagome superlattice which can be implemented by enhancing the long wavelength laser in one direction of the optical lattice [1]. To this end we combine the virtues of a Mean-Field theory with the Laudau theory of Ref. [2] and work out of a decoupled effective potential method. By comparing the corresponding analytic results with extensive quantum Monte Carlo simulations, we find that several striped solids emerge in this system. Due to the blockade effect of such a striped order, the resulting superfluid density turns out to be anisotropic and thus, reveals its tensional property [3]. Finally, we discuss the complete quantum phase diagram.

Model



From the left figure, we know that the Kagome supperlattice can be formed by enhancing the long wavelength laser of one direction. If such enhancement is quite small and the temperature is sufficiently low so that we only need to consider the lowest band, then the system can be described by a Bose-Hubbard model

$$H = -\sum_{\langle i,j\rangle} J\hat{a}_i^\dagger \hat{a}_j + U/2\sum_i \hat{n}_i\left(\hat{n}_i - 1\right) - \mu\sum_i \hat{n}_i - \Delta\mu\sum_A \hat{n}_A,$$

where J represents the hopping between nearest neighbor sites, U is the on-site repulsion, μ is the chemical potential of the system, and $\Delta\mu$ is the additional chemical potential on the sublattice A which has a deeper local potential.

Decoupled Effective Potential Method

Firstly, we decouple the system into three sublattices labelled with A, B and C. Secondly, we introduce the source terms for each sublattice. Then we get the decoupled Hamiltonian with additional source terms

$$\begin{aligned} H = &-J\sum_{AB}\left(\hat{a}_A^\dagger\hat{a}_B + \hat{a}_B^\dagger\hat{a}_A\right) - J\sum_{AC}\left(\hat{a}_A^\dagger\hat{a}_C + \hat{a}_C^\dagger\hat{a}_A\right) - J\sum_{BC}\left(\hat{a}_B^\dagger\hat{a}_C + \hat{a}_C^\dagger\hat{a}_B\right) + U/2\sum_A \hat{n}_A\left(\hat{n}_A - 1\right) \\ &+ U/2\sum_B \hat{n}_B\left(\hat{n}_B - 1\right) + U/2\sum_C \hat{n}_C\left(\hat{n}_C - 1\right) - \mu\sum_A \hat{n}_A - \mu\sum_B \hat{n}_B - \mu\sum_C \hat{n}_C - \Delta\mu\sum_A \hat{n}_A \\ &+ \sum_A\left(j_A^*\hat{a}_A + j_A\hat{a}_A^\dagger\right) + \sum_B\left(j_B^*\hat{a}_B + j_B\hat{a}_B^\dagger\right) + \sum_C\left(j_C^*\hat{a}_C + j_C\hat{a}_C^\dagger\right). \end{aligned}$$

Thus, the free energy is

$$\begin{aligned} F = N_s\Bigg(&F_0(t) + \sum_{P_A=1}^{\infty} C_{2pA}(J)|j_A|^{2P_A}\sum_{P_B=1}^{\infty} C_{2pB}(J)|j_B|^{2P_B}\sum_{P_C=1}^{\infty} C_{2pC}(J)|j_A|^{2P_C}\Bigg) \\ &+ N_s\Bigg(\sum_{P_{AB}=1}^{\infty} C_{pAB}(J)\left(j_A^*j_B + j_B^*j_A\right)^{P_{AB}} + \sum_{P_{AC}=1}^{\infty} C_{pAC}(J)\left(j_A^*j_C + j_C^*j_A\right)^{P_{AC}}\Bigg) \\ &+ N_s\Bigg(\sum_{P_{BC}=1}^{\infty} C_{pBC}(J)\left(j_B^*j_C + j_C^*j_B\right)^{P_{BC}}\Bigg). \end{aligned}$$

Because the order parameter is

$$\psi_A = <a_A> = \frac{1}{N_s}\frac{\partial F}{\partial j_A^*}, \quad \psi_B = <a_B> = \frac{1}{N_s}\frac{\partial F}{\partial j_B^*}, \quad \psi_C = <a_C> = \frac{1}{N_s}\frac{\partial F}{\partial j_C^*},$$

we can define the effective potential as

$$\Gamma = F/N_s - \psi_A^* j_A - \psi_A j_A^* - \psi_B j_B^* - \psi_B^* j_B - \psi_C^* j_C - \psi_C j_C^*.$$

In order to get the phase boundary of the system, we have to calculate the second order expansion parameter of Γ. Taking a_{2pA} for example

$$a_{2pA} = \frac{\partial^2\Gamma}{\partial\psi_A\partial\psi_A^*} = -N_s\frac{1}{\left(\frac{\partial^2 F}{\partial j_A\partial j_A^*}\right)} = -\frac{1}{C_{2pA}},$$

if we expand C_{2pA} in the order of hopping parameter J, we can get a series as

$$C_{2pA} = \sum_{n=0}^{\infty}(-J)^n\alpha_{2pA}^n.$$

Then, making a truncation at the first order of J, we can get

$$a_{2pA} = -\frac{1}{C_{2pA}} = -\frac{1}{\left(\alpha_{2pA}^0\right)^2}(\alpha_{2pA}^0 + J\alpha_{2pA}^1).$$

Following the steps above, we can get the other second order coefficients in the matrix.

$$\begin{pmatrix} a_{2pA} & a_{2pAB} & a_{2pAC} \\ a_{2pAB} & a_{2pB} & a_{2pBC} \\ a_{2pAC} & a_{2pBC} & a_{2pC} \end{pmatrix}$$

Although the system has striped solid phases, there is no sudden change in the system (considering the symmetry), so we expect that the phase transition of the system is second order. And the phase transition happens when the determinant of the above matrix is zero.

Phase boundary

Following the steps above and making use of the diagram calculation developed in Ref. [2], we can get the phase boundaries of the system

$$\tilde{J} = \frac{\lambda_1 + \sqrt{\lambda_1^2 + 8\lambda_1\lambda_2}}{8\lambda_1\lambda_2},$$

in which

$$\lambda_1 = \frac{n_A}{n_A - 1 - \tilde{\mu} - \Delta\tilde{\mu}} + \frac{n_A + 1}{\tilde{\mu} + \Delta\tilde{\mu} - n_A}, \quad \lambda_2 = \frac{n_B}{n_B - 1 - \tilde{\mu}} + \frac{n_B + 1}{\tilde{\mu} - n_B}$$

and

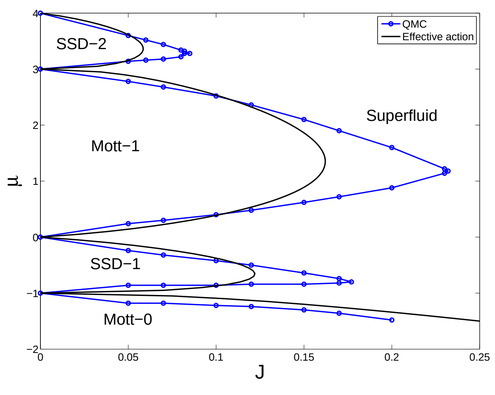
$$\tilde{J} = J/U, \quad \tilde{\mu} = \mu/U, \quad \Delta\tilde{\mu} = \Delta\mu/U$$

where n_A and n_B are the occupation number of the deep and shallow lattice sites, respectively. When $\Delta\mu$ is zero so that the system reduces to the normal optical lattice, the phase boundary is

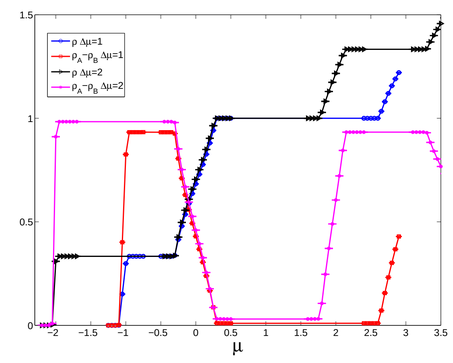
$$4\tilde{J}\left(\frac{n}{n_{-}1 - \tilde{\mu}} + \frac{n+1}{\tilde{\mu} - n}\right) + 1 = 0$$

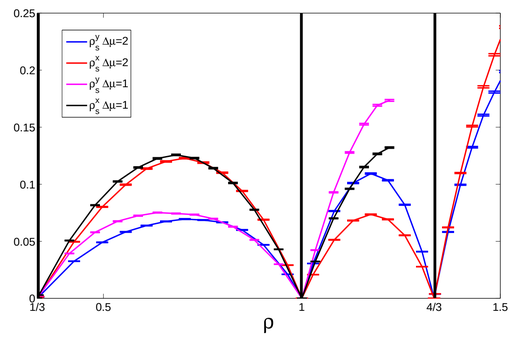
which coincides with the normal effective potential result. In addition, comparing with the normal decoupled mean-field, the decoupled effective potential method shows the higher accuracy.

Results



The left figure is the phase diagram at $\Delta\mu = 1$. We can find the decoupled effective potential method matches well with the quantum Monte Carlo simulation, and we expect a better result when considering higher hopping orders.





We also use the quantum Monte Carlo method to calculate the densities in different sublattices, the total density and the superfluid density in different directions at $J = 0.1$ (see the two figures above). In the left figure, the plateaus indicate the incompressible solid phase with different filling and the difference between the density in different sublattices hints the striped order in some of them. In the right figure on the canonical ensemble, we found that the superfluid density is anisotropic in this system. Between $1/3$ striped solid and Mott-1 insulator, the superfluid density in x direction is larger than y direction, and between Mott-1 insulator and $4/3$ striped solid, the superfluid density in y direction is larger.

Conclusions

We developed a decoupled effective-potential method to study a bosonic Kagome superlattice system. After comparing its results with quantum Monte Carlo simulation, we found that there exists the striped solids and got the whole phase diagram. In the regions between different solid phases, we found the anisotropic superfluidity which indicates the tensional property of the superfluid density.

Acknowledgements

We acknowledge Hanse-Wissenschaftskolleg (HWK) for financial support in this conference. This work is supported by Chinese Scholarship Council (CSC) and the German Research Foundation (DFG) via the SFB/Transregio 49.

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