Numerical Studies of Dye-Mediated Photon-Photon Interaction in Condensates of Light [arXiv:1801.00155]

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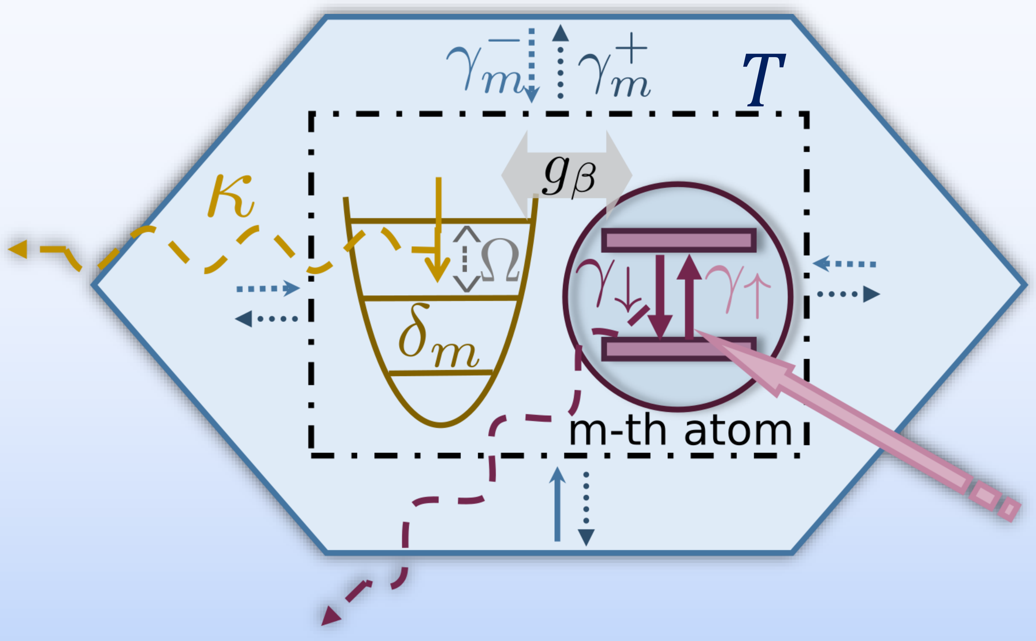
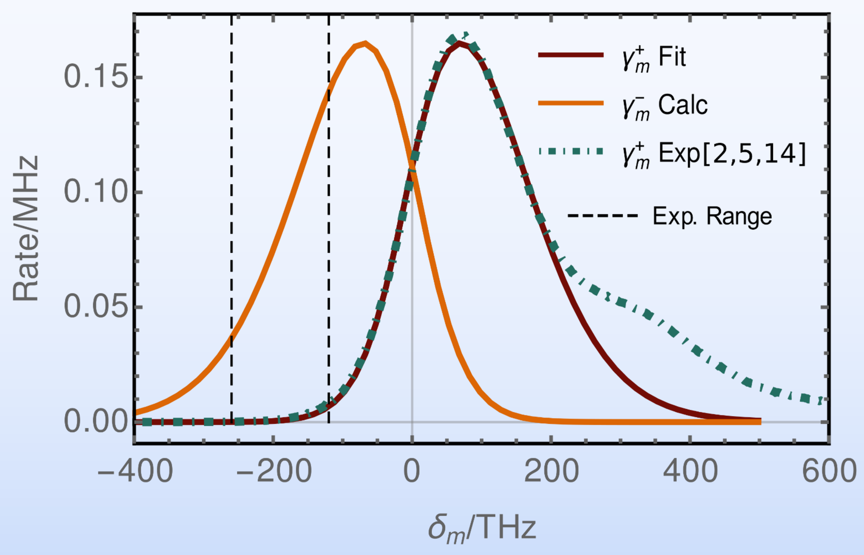
We numerically investigate the properties of the photon condensate with a dye-mediated photon-photon interaction. To this end we extend the model of a Lindblad master equation by an additional interaction between the dye molecules and the cavity modes. Our focus lies on the resulting dimensionless photon interaction strength $\tilde{g}$, which we determine using the occupation of the thermal cloud in the steady state and compare the determined value for $\tilde{g}$ with the literature. On top we investigate how the $\tilde{g}$ depends on the system parameters such as the effective temperature of the dye, and the number of the dye molecules.

Master Eq. for Photon Condensation [1-4]



Symbol	Value
T	300 K
g_β	0.02 GHz
κ	3.5 GHz
δ_1	-260 THz
N	10^9
M	$\sum_{\ell=1}^{L} d_\ell$
L	101
Ω	1.4 THz



- See Poster [6]: $\gamma_m^\pm = \gamma_m^\pm(g,\eta,\omega_c,\Delta)$
- Fit exp. Data for γ_m^+
- Calculate γ_m^-
- γ_m^+/γ_m^- verifies Kennard-Stepanov relation [5,7]

g	2.46 GHz	ω_c	20.5 THz
η	0.6	Δ	3487 THz

Symbol	Description
$\mathcal{L}$	Lindblad-dissipator
$a_m^{(\dagger)}/\sigma_j^{(\pm)}$	Operators of photons/dye molecules
g_β	Bath dressed cavity-dye coupling
$\kappa,\gamma_\uparrow,\gamma_\downarrow$	Rates, environment
$\gamma_m^\pm$	Rates, dye property
δ_m	Cavity mode detuning
d_ℓ	Level degeneracy
Δ	Dye transition frequency
η,ω_c,g	Dye model parameters
N	# of dye molecules
M	# of cavity modes

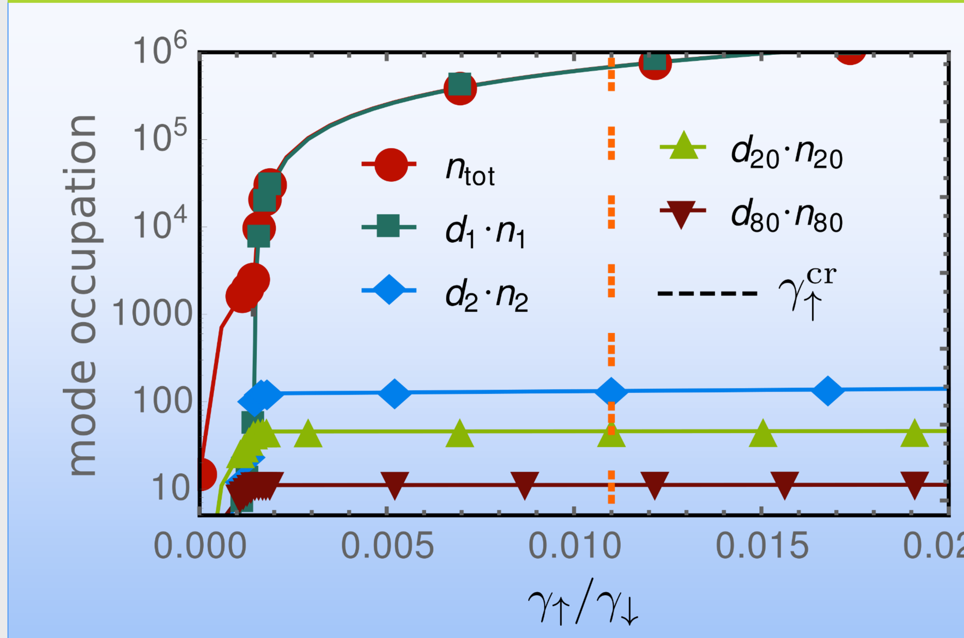
$$\dot{\rho}_{\mathrm{D,C}} = -i\left[\sum_{m=1}^{M}\delta_m a_m^\dagger a_m + g_\beta\sum_{m=1}^{M}\sum_{j=1}^{N}\left(a_m^\dagger\sigma_j^- + a_m\sigma_j^+\right), \rho_{\mathrm{D,C}}\right] - \left\{\sum_{m=1}^{M}\frac{\kappa}{2}\mathcal{L}[a_m] + \sum_{j=1}^{N}\left(\frac{\gamma_\uparrow}{2}\mathcal{L}[\sigma_j^+] + \frac{\gamma_\downarrow}{2}\mathcal{L}[\sigma_j^-]\right) + \sum_{m=1}^{M}\sum_{j=1}^{N}\left(\frac{\gamma_m^+}{2}\mathcal{L}[a_m\sigma_j^+] + \frac{\gamma_m^-}{2}\mathcal{L}[a_m^\dagger\sigma_j^-]\right)\right\}\rho_{\mathrm{D,C}}$$

Master Eq. → Mean-Field with 2. order Cumulant Expansion [8] → U(1) Symmetry: take only gauge invariant quantities → Eq-Set.: $\langle a_m^\dagger a_m\rangle = n_m, \langle\sigma_1^z\rangle, \langle a_m\sigma_1^+\rangle, \langle a_k^\dagger a_m\rangle, \langle\sigma_1^\dagger\sigma_2\rangle$ → Level Degeneracy d_ℓ: $d_\ell = 2\cdot\ell$ → $\tilde{g} = \frac{m}{\hbar^2}g_{\mathrm{GP}}$ [9], g_{GP}: Interaction in GP Eq.

Procedure to determine $\tilde{g}$

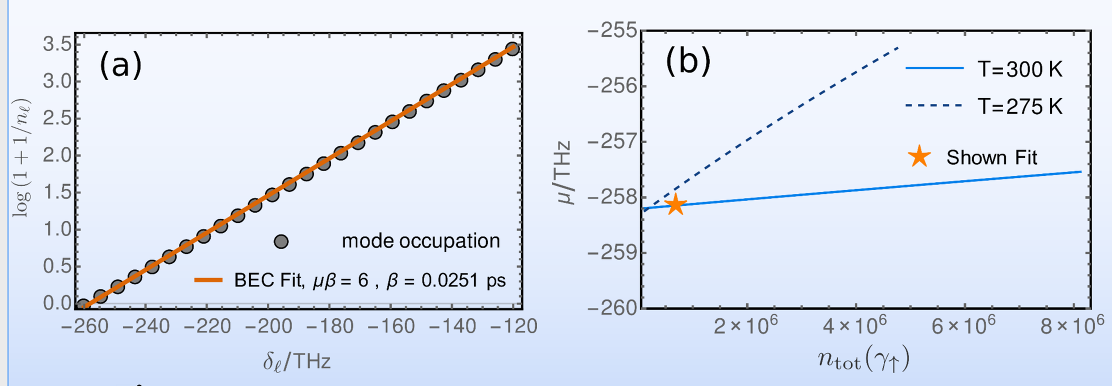
- Level degeneracy d_ℓ: $\sum_{m=1}^{M} a_m^\dagger a_m = \sum_{\ell=1}^{L} d_\ell \cdot a_\ell^\dagger a_\ell$
- Steady state of the 2. order eq.-set
  - Mode occupation n_ℓ
  - Thermalized state [1,2, 10]
  - Extract μ by BEC – Fit
  - Use different parameters
- Perturbative solution of the GP-Equation
  - $\mu \approx \hbar\Omega + \frac{\tilde{g}\hbar\Omega}{2\pi}\cdot n_{\mathrm{tot}}$
  - Slope $\frac{\tilde{g}\hbar\Omega}{2\pi}$ encodes $\tilde{g}$
- Combination: Dimensionless photon-photon interaction strength $\tilde{g}$

Occupation vs. Pumping



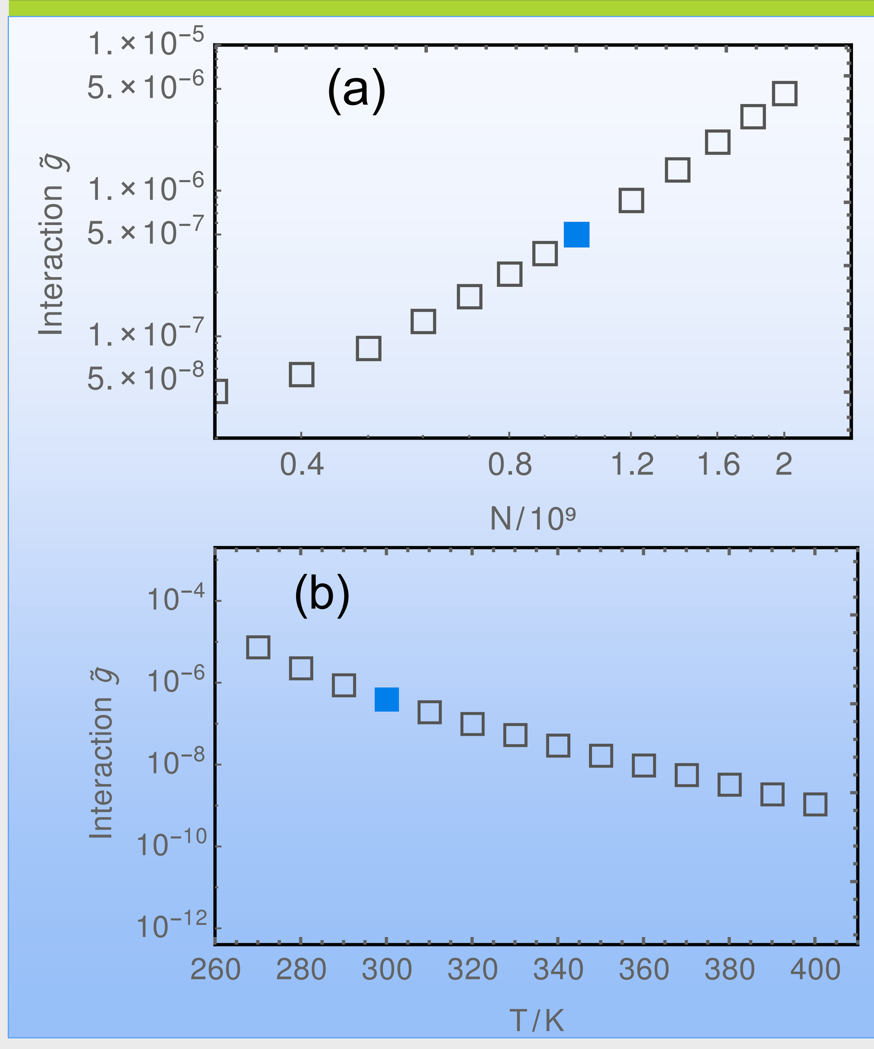
- Condensation onset at $\gamma_\uparrow^{\mathrm{cr}}$ with $n_{\mathrm{tot}}^{\mathrm{cr}} = 2800$
- From a non-interacting theory: $n_{\mathrm{tot}}^{\mathrm{cr}} = 2640$

Chemical Potential μ



- Fix $\gamma_\uparrow$
- Fit the BEC distribution $n_\ell = \frac{1}{\exp[\beta(\delta_\ell - \mu)] - 1}$
- Obtain slope $\frac{\tilde{g}\hbar\Omega}{2\pi}$
- Here, slope $\frac{\tilde{g}\hbar\Omega}{2\pi} \neq 0$ in contrast to [1] → Interaction

$\tilde{g}$ Dependency



Discussion

- We find
  - Strong $\tilde{g}$ dependency on the number of dye molecules N and effective dye-temperature T
  - In exp. case: $\tilde{g} \sim 10^{-7}$ [blue point]
  - Note, we have dye mediated contribution
- In a real experiment:
  - thermo-optical effect dominates, $\tilde{g}_{\mathrm{Exp}}^{\mathrm{Therm}} \approx 10^{-4} - 10^{-2}$ [2,5,10,11]
  - Kerr contribution: $\tilde{g}_{\mathrm{Exp}}^{\mathrm{Kerr}} \sim 10^{-7}..10^{-8}$ [5]
- Agrees with theoretical prediction for dye-mediated interaction from [12]
- Outlook
  - $\tilde{g}$ polarization dependency [13]
  - Extension with spatial dimension

References

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