Crossover from Adiabatic to Sudden Quench Dynamics for Time-of-Flight Imaging Measurements in BECs

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Abstract

Time-of-flight imagining is one of the standard techniques used in experiments with Bose-Einstein condensates (BECs) to measure and study their physical properties. Here we investigate effects of a controlled time-dependent quench of a trapping potential on Time-of-Flight (TOF) images in a ^{87}Rb condensate. To this end we model the following experimental protocol: initially the condensate is in the ground state and then the frequencies of a cylindrically-symmetric harmonic trapping potential are quenched during a given time interval. This will generate a BEC dynamics within the intriguing crossover from adiabatic to sudden quench dynamics, which affects the TOF images made immediately afterwards. We study both numerically and variationally such effects of quenching of a trapping potential, as well as necessary modifications to the algorithm used for reconstructing the density profile of a BEC cloud. The obtained results are relevant for new experiments, which are performed e.g. at the Center of Applied Space Technology and Microgravity (ZARM) at the University of Bremen [1] and offer a glimpse into the non-equilibrium BEC physics.

Introduction

- **Mean-field description of a BEC [2] in a harmonic trapping potential** $V(\mathbf{r},t)$

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t) + g|\Psi(\mathbf{r},t)|^2\Psi(\mathbf{r},t) \tag{1}$$

- **Scaling approach [3, 4]**

  The classical trajectory of each particle in the BEC cloud is scaled like

$$R_j = \lambda_j(t)R_j(0) \quad (j = x, y, z). \tag{2}$$

The spatial density evolves like

$$\rho(\mathbf{r},t) = \frac{1}{\lambda_x(t)\lambda_y(t)\lambda_z(t)}\,\rho\left[\left\{r_j/\lambda_j(t)\right\}_{j=x,y,z}, 0\right]. \tag{3}$$

From Newton's second law, the self-consistence equations of scaling factors can be obtained,

$$m\ddot{\lambda}_j R_j(0) - (\partial_{r_j})V[R(t),t] + \frac{1}{\lambda_x(t)\lambda_y(t)\lambda_z(t)}(\partial_{r_j}V)[R(0),0]. \tag{4}$$

Dynamically conserving trap symmetry

A dynamically conserving trap symmetry is characterized by the condition $\frac{\omega_x(t)}{\omega_y(t)} = \frac{\omega_x(0)}{\omega_y(0)}$. Take a 2D BEC as a typical example and define the scaling factors for quantum mechanical calculations as $\langle x\rangle_+ = \frac{\int_0^{+\infty}dx\int_{-\infty}^{+\infty}x\rho(x,y,t)dy}{\int_0^{+\infty}dx\int_{-\infty}^{+\infty}x\rho(x,y,0)dy}$ and $\langle y\rangle_+ = \frac{\int_0^{+\infty}dy\int_{-\infty}^{+\infty}y\rho(x,y,t)dx}{\int_0^{+\infty}dy\int_{-\infty}^{+\infty}y\rho(x,y,0)dx}$

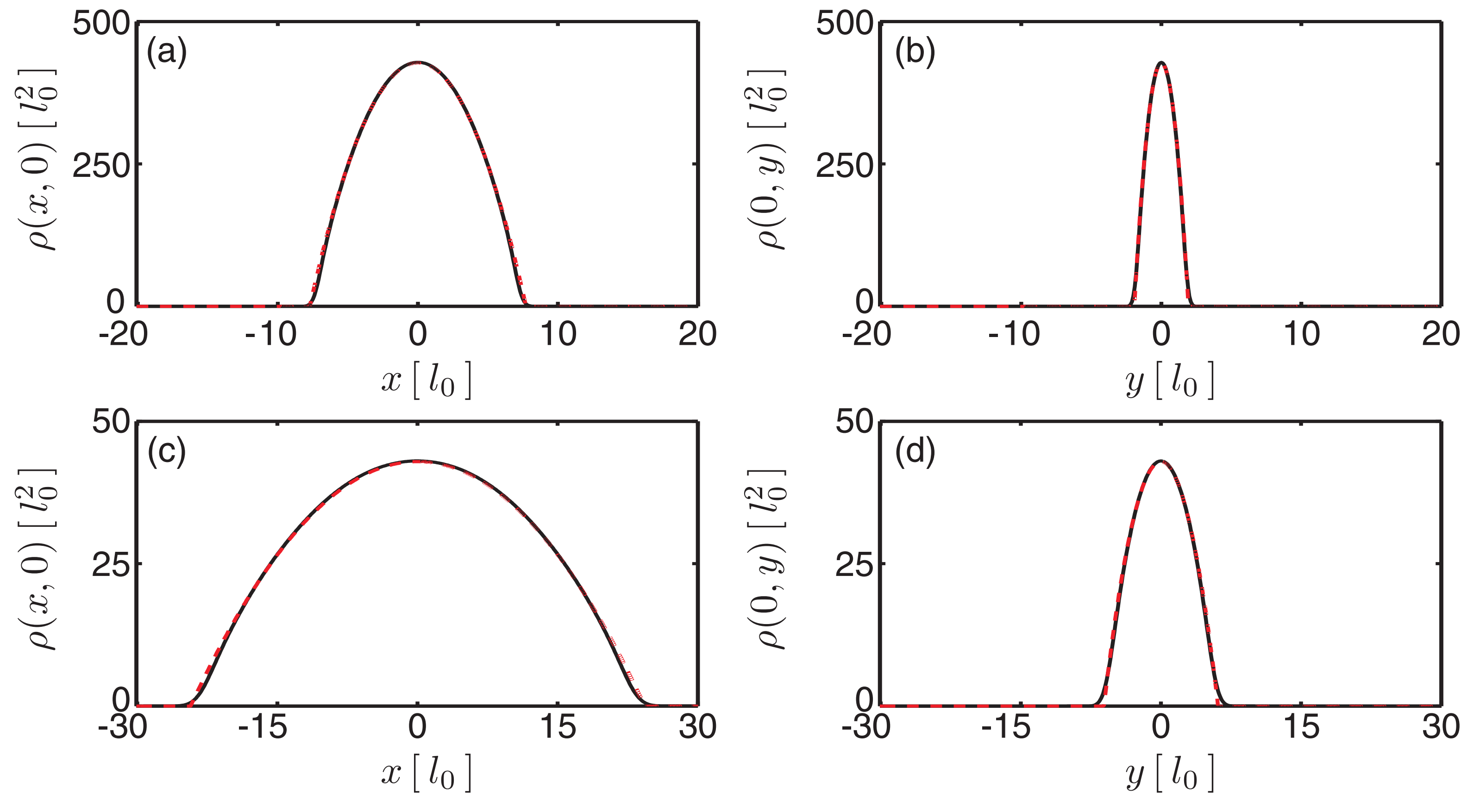

Fig. 1: Comparison of full quantum mechanical and scaling calculations for the condensate under a controllable quench where trap frequencies vary as $\omega_j(t) = \omega_j(0)e^{-\alpha t}$ with $\alpha = 0.1\omega_x(0)$ for $N = 10^4$ atoms. (a) and (b) depict the *initial* density profile and (c) and (d) the density profile at $t = \frac{46}{\omega_x(0)}$. Dark solid lines in (a), (b), (c) and (d) are obtained by numerically solving GPE, while red dashed lines in (a) and (b) represent the Thomas-Fermi solution, and in (c) and (d) represent scaling solutions. Length unit l_0 is a harmonic oscillator length for the frequency $\sqrt{\omega_x(0)\omega_y(0)}$.

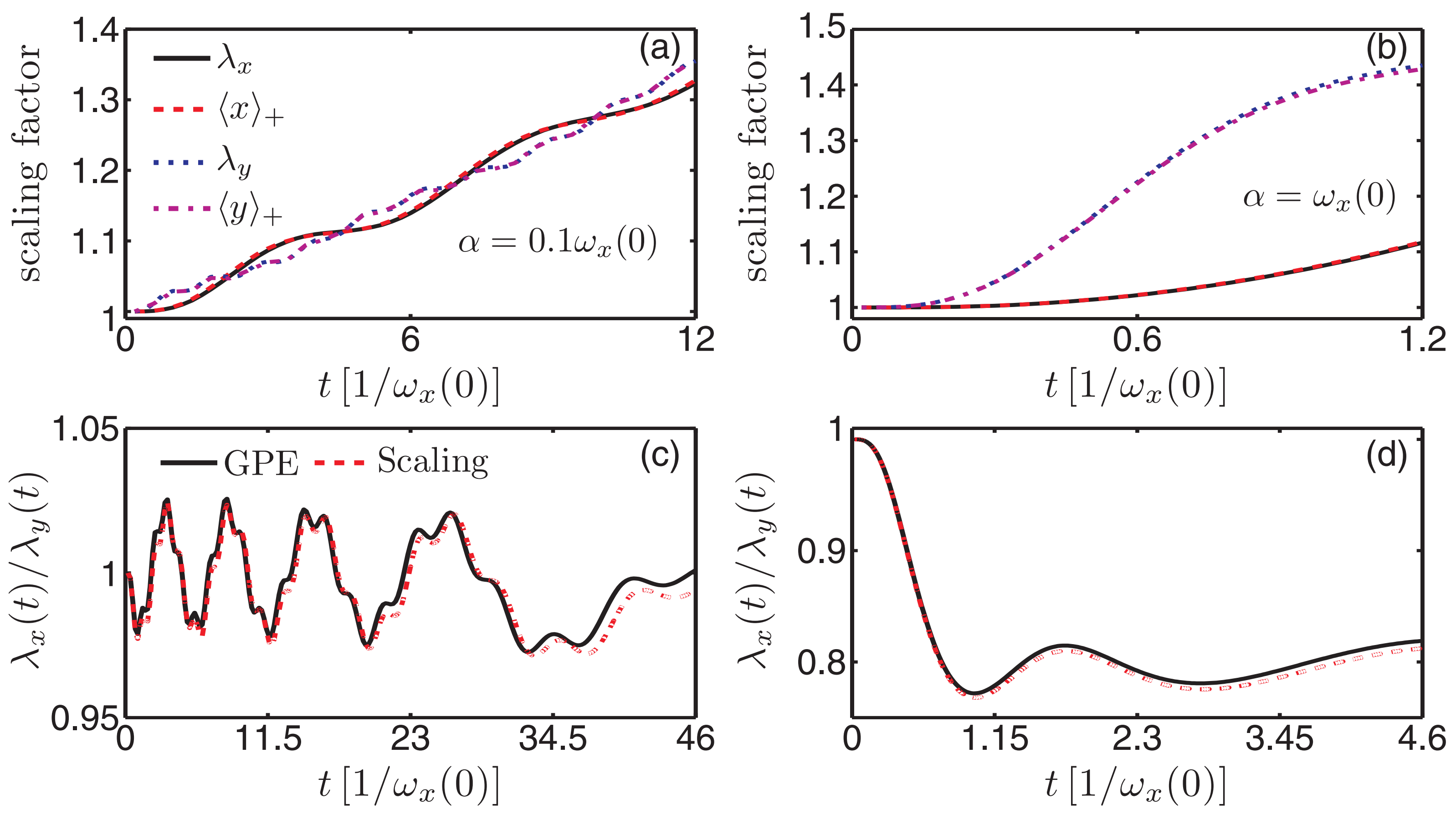

Fig. 2 Illustration of the time evolution of the scaling factors as well as the width of the condensate ((a) and (b)) and its aspect ratio ((c) and (d)), under the variation of trap frequencies shown in Fig. 1, with respect to different potential quenches: (a) and (c) correspond to a weak quench where $\alpha = 0.1\omega_x(0)$, while (b) and (d) to a strong quench with $\alpha = \omega_x(0)$.

Dynamically breaking trap symmetry

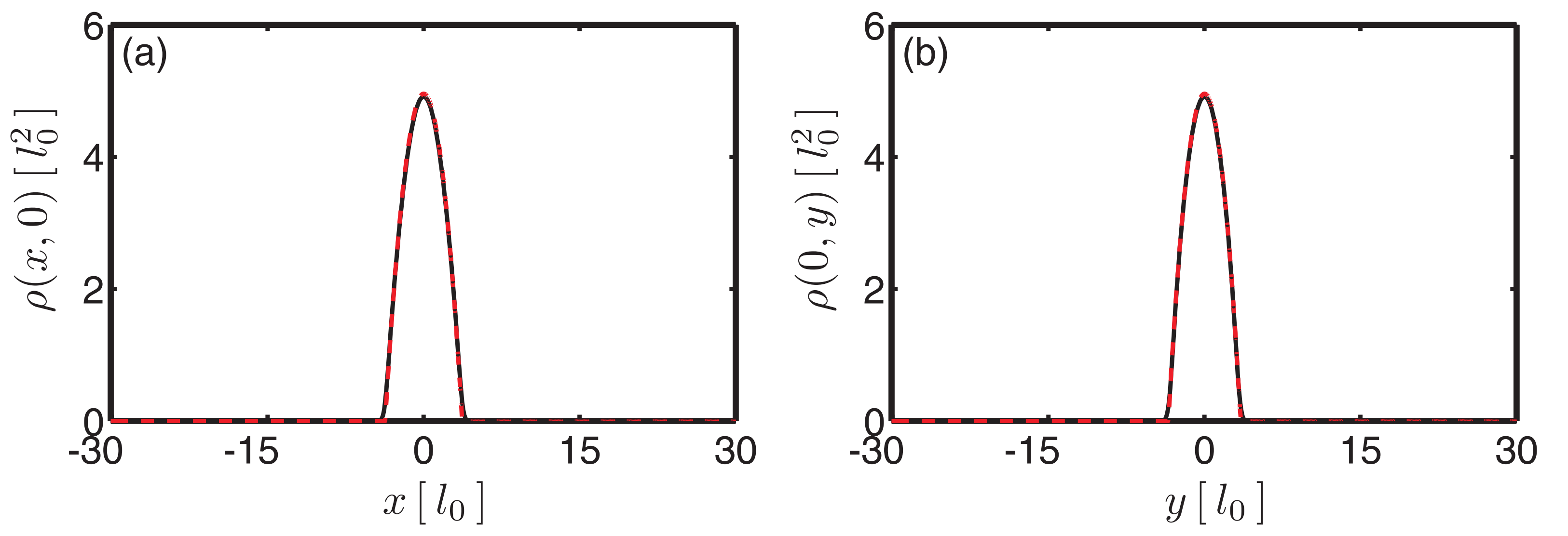

Fig. 3: Comparison of full quantum mechanical and scaling calculations for the condensate under a controllable quench where the trap frequencies vary as $\omega_x(t) = \omega_x(0)e^{\alpha t}$ and $\omega_y(t) = \omega_y(0)e^{-\alpha t}$ for $N = 10^4$ atoms. (a) and (b) depict the density profile at $t = \frac{1.4}{\omega_x(0)}$ with $\alpha = \omega_x(0)$. Dark solid lines are obtained by numerically solving GPE, while red dashed lines correspond to scaling solutions.

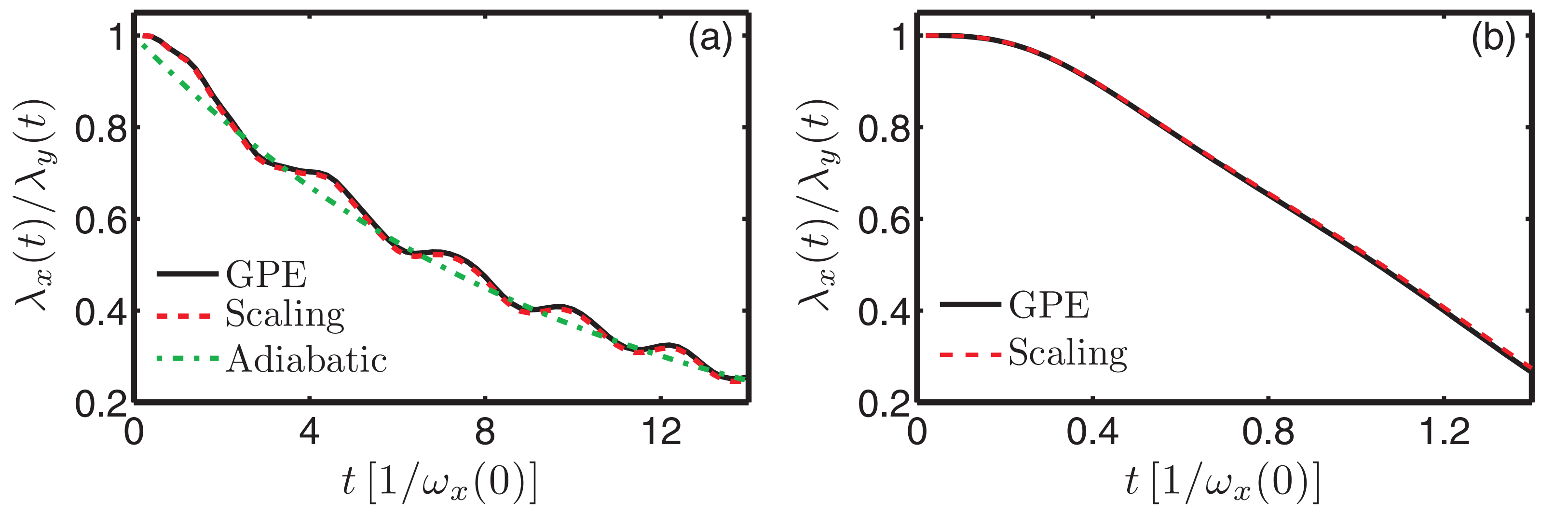

Fig. 4: Illustration of the time evolution of the aspect ratio, under the broken dynamic ratio of trap frequencies shown in Fig. 3, with respect to different potential quenches: (a) corresponds to a weak quench where $\alpha = 0.1\omega_x(0)$, while (b) to a strong quench with $\alpha = \omega_x(0)$. Adiabatic solution refers to the one at infinitesimal α, which is equivalent to the ground-state solution where $\omega_j(0)$ is substituted by $\omega_j(t)$.

Conclusions and outlook

- Scaling approach can be applied to quench dynamics of anisotropic trapped BEC.
- Some excitations induced by the quench can not be captured by the scaling treatment (see Fig. 5).

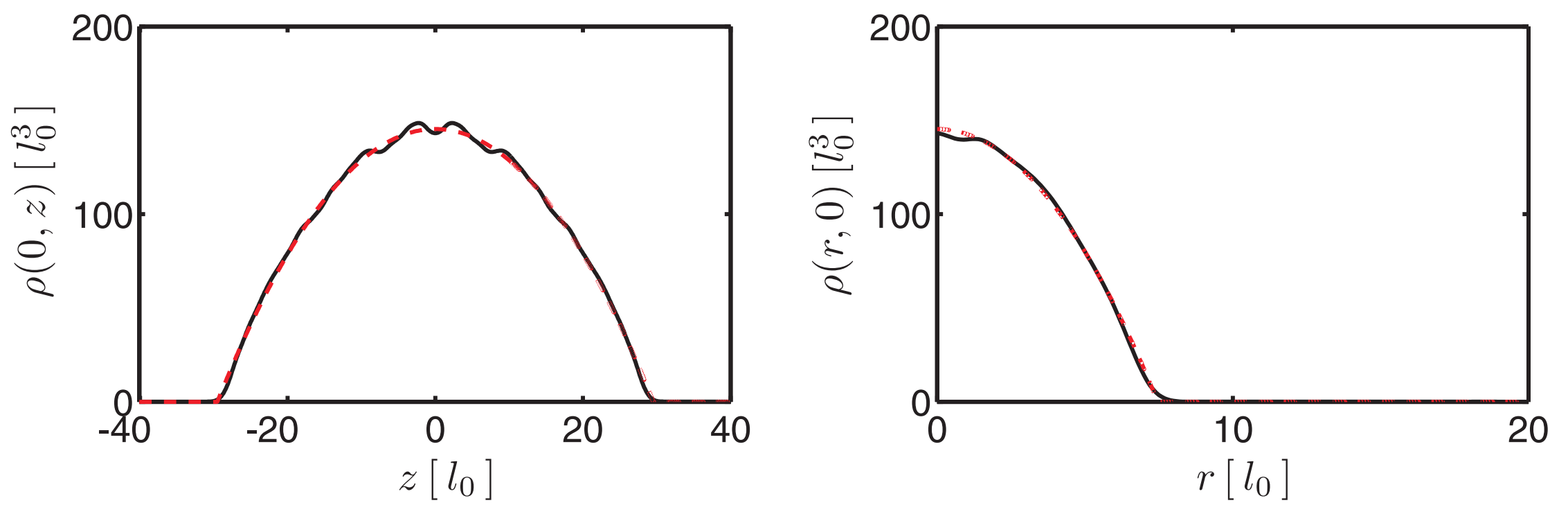

Fig. 5: Density distribution of 3D BEC with the same variation of trap frequencies as in Fig. 1. Dark solid lines are obtained by numerically solving GPE and red dashed line from scaling solutions.

- In the future, explore the validity of scaling method in periodically driven dynamics of trapped BEC.
- Moreover, investigate the possibility of this method for non-equilibrium dynamics of multiple BECs [5].

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Acknowledgments

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