BEC-BCS Crossover: Mean-Field Theories and Beyond

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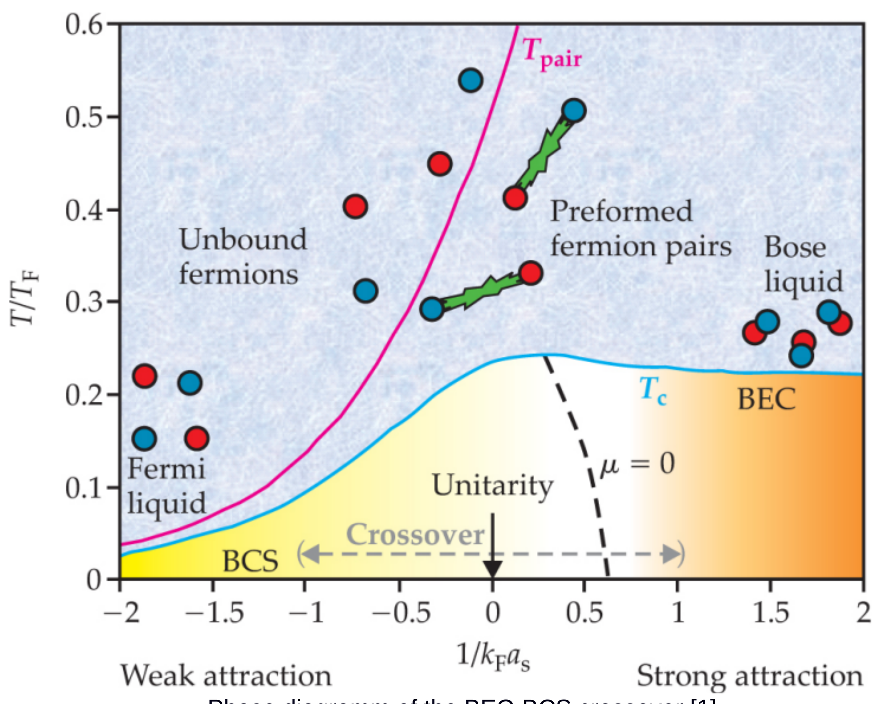
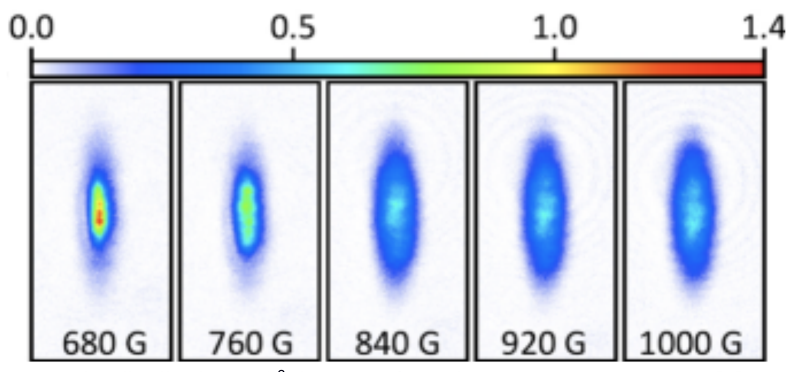
Introduction

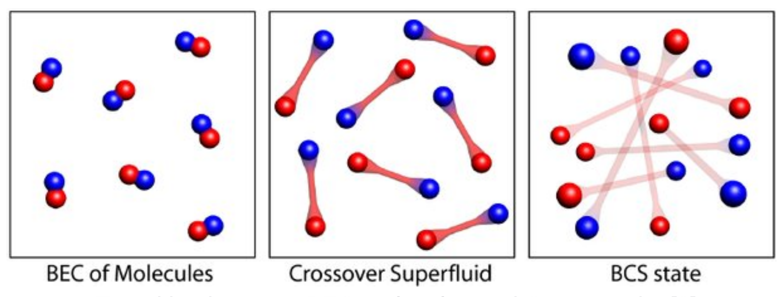


Phase diagram of the BEC-BCS crossover [1].

- Preforming of fermionic pairs below $T_{\rm pair}$ (Mean field).
- Condensation of Cooper-Pairs (BCS) or Cooper-Dimers (BEC) below the critical temperature T_C obtained by Gaussian fluctuations.



Density distributions of ^{6}Li atoms along the BEC-BCS crossover [2].



Transition between BEC molecules and Cooper pairs [3].

- Established mean-field theory relies on a functional integral approach.
- There one uses a Hubbard-Stratonovich transformation in combination with a saddle-point approximation.
- This approach only includes free fermions and dimer states.

Mean-Field Theories

Grand canonical many-body Hamiltonian operator

$$\hat H = \int d^3x \sum_\sigma \hat\psi^\dagger_\sigma(\mathbf{x})\left[-\frac{\hbar^2}{2m}\nabla^2 - \mu\right]\hat\psi_\sigma(\mathbf{x}) + g_b \hat\psi^\dagger_\uparrow(\mathbf{x})\hat\psi^\dagger_\downarrow(\mathbf{x})\hat\psi_\downarrow(\mathbf{x})\hat\psi_\uparrow(\mathbf{x})$$

Mean-Field (MF) **Approach**

Construct MF Hamiltonian that approaches the many-body Hamiltonian well enough: $\hat H - \hat H_{\rm MF} \in \mathcal{O}(\varepsilon)$

Approximate partition function as $Z \approx Z_{\rm MF} = {\rm Tr}\{e^{-\beta \hat H_{\rm MF}}\}$

Contact interaction with attractive coupling strength g_b

Hartree-Fock-Bogoliubov (HFB) Approximation

$$\hat\psi^\dagger_\uparrow\hat\psi^\dagger_\downarrow\hat\psi_\downarrow\hat\psi_\uparrow \approx +\langle\hat\psi^\dagger_\uparrow\hat\psi_\uparrow\rangle\hat\psi^\dagger_\downarrow\hat\psi_\downarrow + \langle\hat\psi^\dagger_\downarrow\hat\psi_\downarrow\rangle\hat\psi^\dagger_\uparrow\hat\psi_\uparrow - \langle\hat\psi^\dagger_\uparrow\hat\psi_\uparrow\rangle\langle\hat\psi^\dagger_\downarrow\hat\psi_\downarrow\rangle \longrightarrow \text{Hartree}$$

$$-\langle\hat\psi^\dagger_\uparrow\hat\psi_\downarrow\rangle\hat\psi^\dagger_\downarrow\hat\psi_\uparrow + \langle\hat\psi^\dagger_\downarrow\hat\psi_\uparrow\rangle\hat\psi^\dagger_\uparrow\hat\psi_\downarrow + \langle\hat\psi^\dagger_\uparrow\hat\psi_\downarrow\rangle\langle\hat\psi^\dagger_\downarrow\hat\psi_\uparrow\rangle \longrightarrow \text{Fock}$$

$$+\langle\hat\psi^\dagger_\uparrow\hat\psi^\dagger_\downarrow\rangle\hat\psi_\downarrow\hat\psi_\uparrow + \langle\hat\psi_\downarrow\hat\psi_\uparrow\rangle\hat\psi^\dagger_\uparrow\hat\psi^\dagger_\downarrow - \langle\hat\psi^\dagger_\uparrow\hat\psi^\dagger_\downarrow\rangle\langle\hat\psi_\downarrow\hat\psi_\uparrow\rangle \longrightarrow \text{Bogoliubov}$$

Different MF-Theories by selection of channels

Channels	Bogoliubov (B)	Hartree-Fock (HF)	HFB
Theory	BCS MF-Theory	Interacting Fermi gas	Interacting Superfluid
Order Parameters	$\Delta = g_b\langle\hat\psi_\downarrow\hat\psi_\uparrow\rangle$	$\varphi_\sigma = g_b\langle\hat\psi^\dagger_\sigma\hat\psi_\sigma\rangle, \phi = g_b\langle\hat\psi^\dagger_\downarrow\hat\psi_\uparrow\rangle$	Δ, φ_σ with $\phi = 0$
Diagonalization	Bogoliubov Trafo	Unitary Trafo	Bogoliubov Trafo

Bogoliubov channel gives regular BCS theory ⇒ Superfluidity

Hartree-Fock describes interaction effects ⇒ Contact interaction: No Fock contribution [4]

HFB allows interactions in superfluid phase ⇒ Separation of thermal and superfluid density

Including different channels is important

Beyond MF

- Expand $Z = {\rm Tr}\{e^{-\beta(\hat H_{\rm MF} + [\hat H - \hat H_{\rm MF}])}\}$ in orders of $\hat V = \hat H - \hat H_{\rm MF}$.

$$Z = Z_{\rm MF}\Big[1 - \underbrace{\frac{1}{\hbar}\int_0^{\hbar\beta} d\tau \langle\hat V(\tau)\rangle_{\rm MF}}_{Z_1} + \underbrace{\frac{1}{2\hbar^2}\int_0^{\hbar\beta}\int_0^{\hbar\beta} d\tau d\tau' \langle\hat{\mathcal{T}}[\hat V(\tau)\hat V(\tau')]\rangle_{\rm MF}}_{Z_2} + \dots\Big]$$

Correlation functions by a thermal Wick theorem

- Conversion to Dirac picture and construction of Dyson series.
- Truncation at zeroth order yields MF theory.
- To Do: Comparison of second order series results of free energy with Gaussian fluctuations above the MF saddle point for the functional integrals [5].

$$W \approx W_{\rm MF} + \frac{1}{\hbar\beta}Z_1 - \frac{1}{2\hbar^2\beta}(Z_2 - Z_1^2)$$

Density Profiles: LDA

Trapped Fermi Gas: Local density approximation (LDA)

In an experiment: Confinement of the gas by some harmonic potential in three dimensions:

$$V(\mathbf{x}) = \frac{m}{2}\sum_{j=1}^3 \omega_j^2 x_j^2 \quad \rightarrow \quad \text{LDA: } \mu(\mathbf{x}) = \mu - V(\mathbf{x})$$

Spatial dependent self-consistency equations have to be solved.

HFB equations in a harmonic trapping potential

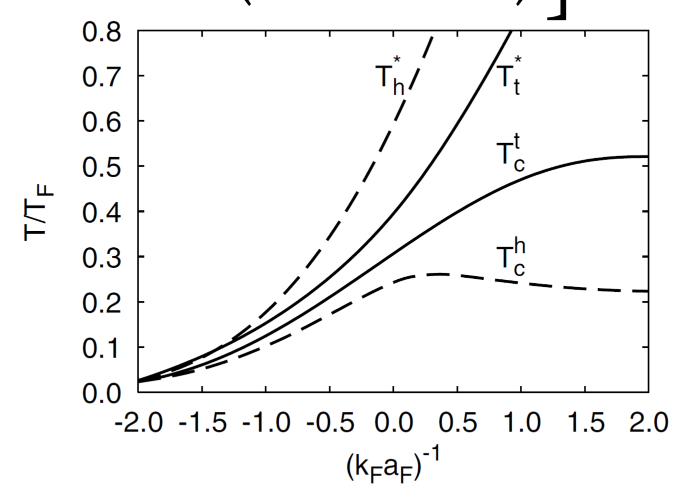
$$\frac{\Delta(\mathbf{x})}{g} = \frac{\Delta(\mathbf{x})}{2}\frac{1}{V}\sum_{\mathbf{k}}\left[\frac{2m}{\hbar^2\mathbf{k}^2} - \frac{\tanh\left(\frac{\beta}{2}E(\mathbf{k},\mathbf{x})\right)}{E(\mathbf{k},\mathbf{x})}\right] \quad (1)$$

$$n(\mathbf{x}) = \frac{1}{V}\sum_{\mathbf{k}}\left[1 - \frac{\epsilon_{\mathbf{k}}(\mathbf{x}) + \frac{g}{2}n(\mathbf{x})}{E(\mathbf{k},\mathbf{x})}\tanh\left(\frac{\beta}{2}E(\mathbf{k},\mathbf{x})\right)\right] \quad (2)$$

Hartree-Bogoliubov dispersion

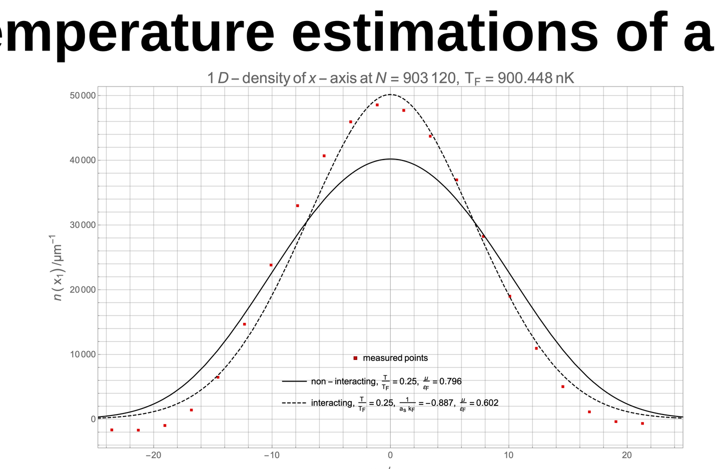
$$\epsilon_{\mathbf{k}}(\mathbf{x}) = \frac{\hbar^2\mathbf{k}^2}{2m} - \mu(\mathbf{x})$$

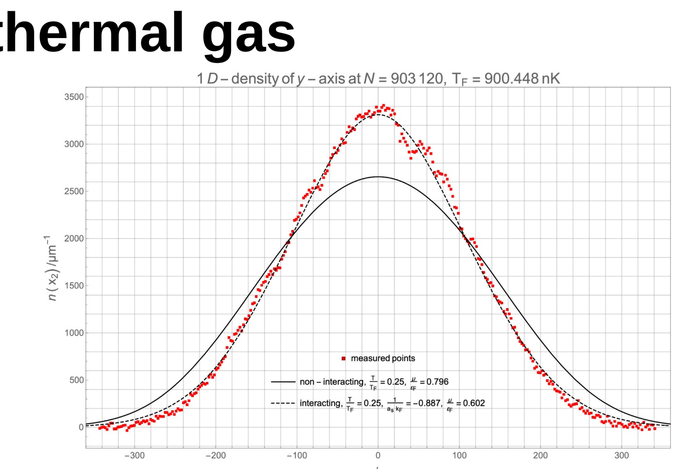
$$E(\mathbf{k},\mathbf{x}) = \sqrt{\left(\epsilon_{\mathbf{k}}(\mathbf{x}) + \frac{g}{2}n(\mathbf{x})\right)^2 + \Delta^2(\mathbf{x})}$$



Critical temperature of homogeneous(h) and trapped(t) gas [6].

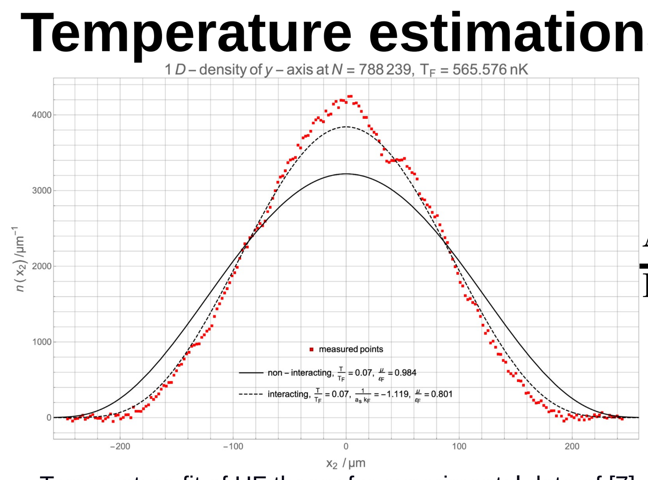
Temperature estimations of a thermal gas


Temperature fit for x_1 axis of experimental data of [7].

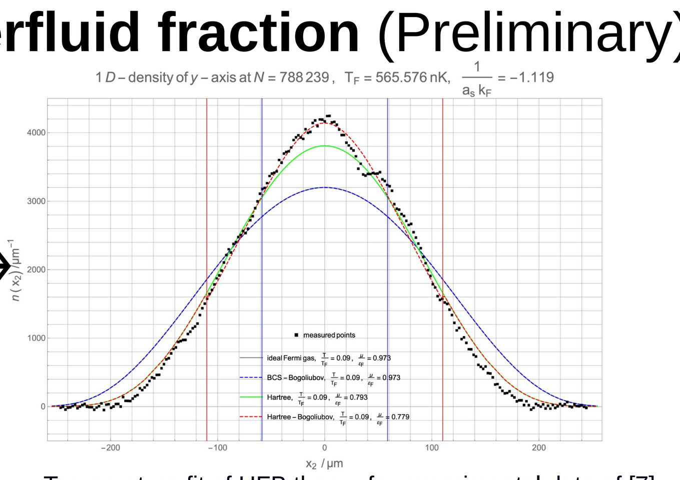

Temperature fit for x_2 axis of experimental data of [7].

- Fit of experimental data [7] to minimze mean square error between data and theoretical estimation of the HF theory.
- Location of minimal total error yields temperature T for known particle number N.
- Good estimations above critical temperature ⇒ Direct temperature measurement.
- For lower temperatures one needs also to include Bogoliubov equation (1).

Temperature estimations with superfluid fraction (Preliminary)


Temperature fit of HF theory for experimental data of [7].

Addition of Bogoliubov channel →


Temperature fit of HFB theory for experimental data of [7].

Conclusion

- MF theory is sufficient to directly measure the temperature of the weakly interacting Fermi gas above $T_{\rm pair}$
- Interactions play a crucial role above the critical temperature in the BCS regime.
- Below $T_{\rm pair}$ and towards unitarity the HFB MF theory improves the regular BCS MF theory.
- Closer to unitarity fluctuations get so important that MF theories fail.

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